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SLIDING MODE DIFFERENTIATOR VIA IMPROVED ADAPTIVE NOTCH FILTER

JUAN WANG, HEHONG ZHANG, YUANLONG YU, ZHIHONG DAN, GAOXI XIAO, QIUMING GU, AND CHAO ZHAI

To tackle the difficulty in tuning the parameters of sliding mode differentiator (SMD), an improved adaptive notch filter based real-time parameter tuning scheme (denoted as ANF-SMD) is presented. Specifically, the integral feedback of the system output errors is introduced in constructing the cost function for the adaptive notch filter so as to estimate the real-time amplitude and frequency of given inputs. Then, upon the deterministic formula between the parameters of the SMD and the input signals, the parameters of the SMD can be adjusted adaptively as inputs vary. Simulation results show that the proposed ANF-SMD scheme performs well in signal filtering and differentiation estimation. The effectiveness of the proposed ANF-SMD is further experimentally verified on the pressure signal processing for the altitude ground test facility.

Keywords: sliding mode differentiator, parameter tuning scheme, adaptive notch filter, altitude ground test facility

Classification: 93A30, 93Cxx

1. INTRODUCTION

The real-time estimation of the filtering and differentiation of given noise-polluted signals is a big challenge in control theory and applications [10, 24, 30]. For example, the derivative control plays an important role in restraining the overshoot phenomenon for widely-used PID controller. The filtering and differential signals are indispensable in cooperative control of multi-agent systems, especially in the interaction and transfer of information [12, 13]. However, the differentiation of inputs is prone to noise corruption and usually cannot be extracted properly. To achieve effective differential signals for better control performance, a wide variety of differentiator algorithms have been designed and implemented. The classical one is the differentiator based on an inertia link [11]. It is not widely applied in practice, however, as it is quite sensitive to noises. In [25], a finite-time-convergent differentiator was proposed by adopting singular perturbation technique, which helps shorten convergence time and reduce the chattering amplitude. Other existing results include the high-gain observer-based differentiator designed in [7], two types of linear time-derivative trackers studied in [14] and so forth [5, 8, 31, 33, 34].

As a part of the extensive studies on differentiator algorithms, the sliding mode differentiator (SMD) has been widely applied in tackling real-life engineering problems [6, 26, 35] due to its advantageous smoothness in filtering and robustness against disturbances. The sliding mode control algorithms with the robust stability proposed in [1] and [20] provide a good way to design new advanced differentiators to process noise-polluted signals. Well-known examples include the SMD based on the super-twisting algorithm (STA) proposed by Levant [16]. The performance of the STA-SMD in tracking filtering and differentiation acquisition, however, relies heavily on the parameter setting [28]. Without a set of properly selected parameters, the STA-SMD may induce chattering phenomenon or large phase lag, even failing to estimate differential signals in certain cases. It is therefore of necessity to propose an effective real-time parameter tuning scheme to enhance the performance of the STA-SMD in filtering and differentiation acquisition. In [2, 17, 21, 22, 23, 32], some parameter tuning methods of the STA-SMD were proposed. For these methods, priori information including the upper bound on the derivative of the inputs has to be available for tuning the corresponding parameters; such restricts the applications of the STA-SMD.

To enable more efficient parameter tuning in SMD, an adaptive notch filter based parameter tuning scheme (denoted as ANF-SMD) is proposed in this work. The main contributions of this work are twofolds: 1) a new real-time parameter tuning scheme is proposed that can adapt to changed inputs; and 2) to acquire the effective and real-time amplitude and frequency of inputs to participate in parameter tuning, an improved adaptive notch filter (ANF) is proposed by including the integral feedback of system output errors in constructing the cost function. This work effectively overcomes the difficulty of parameter tuning in the SMD algorithm. Unlike the STA based differentiator, the parameters of the proposed ANF-SMD can be adjusted adaptively and effectively as inputs vary. Numerical simulations have been conducted on some different signals with noises as well as in experiments of pressure signal processing for the flight environment simulation system in an altitude ground test facility. The effectiveness of the proposed ANF-SMD are verified by such simulation results.

The rest of this paper is organized as follows: in Section II, the parameter tuning scheme for the SMD is presented. The algorithms including ANF and ANF-SMD are proposed in Section III. In Section IV, numerical simulations are conducted on some signals with noises to verify the effectiveness of the proposed ANF and ANF-SMD. In Section V, the experiment results of pressure signal processing for the flight environment simulation system are presented. Finally, Section VI concludes this paper.

2. PARAMETER TUNING SCHEME OF SLIDING MODE DIFFERENTIATOR

The effectiveness of the SMD in filtering and differentiation acquisition depends heavily on its parameter setting. When the amplitude and frequency of the input signals vary, the parameters of the differentiator need to be tuned accordingly and adaptively. Our previous work [27] has indicated that the parameters of the SMD can be determined directly according to a deterministic formula once the amplitude and frequency of the inputs are obtained. Real-time and effective estimation of the amplitude and frequency of the changed inputs therefore becomes critically important. In this section, a parameter

tuning scheme for SMD via an improved ANF is presented.

Given the input signal $f(t) = f_0(t) + \eta(t)$, where $\eta(t)$ is the bounded random white noises, and $f_0(t)$ is the unknown basic signal whose frequency and amplitude are variable. We shall develop a parameter tuning scheme for the SMD, which enables the differentiator to perform well in filtering and differentiation acquisition.

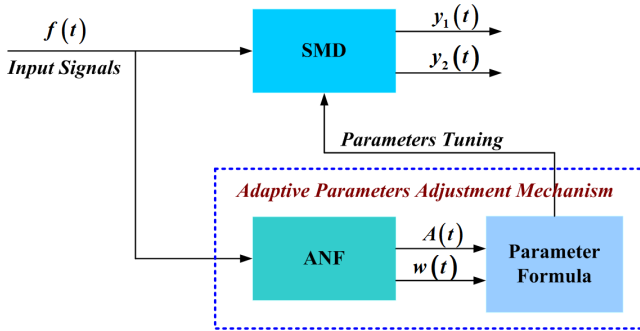


Fig. 1. Parameter tuning scheme of the SMD via an improved ANF.

Figure 1 illustrates the framework of the parameter tuning scheme of the SMD via an improved ANF. In particular, an improved ANF is designed to robustly estimate the real-time amplitude and frequency of the unknown and variable input signals $f(t)$. Then, the parameters of the SMD can be determined directly by the given parameter formula. The parameter formula provides the mathematical relationship between the parameters of the SMD and the amplitude and frequency of the signals. One can refer to our manuscript [27] for the detailed deduction of the deterministic parameter formula. For the clarity and completeness of our discussions, the main results are briefly summarized in the following theorem.

Theorem 2.1. Select the λ_0 and α_0 such that the SMD accurately filters and differentiates a basic signal $f_0(t) = \sin(t)$. Then, for any given input signal $f_m(t) = A_m \sin(w_m t)$, there are valid parameters λ_m and α_m that satisfy

$$\lambda_m = \sqrt{A_m} w_m \lambda_0, \alpha_m = A_m w_m^2 \alpha_0 \tag{1}$$

where, A_m and w_m are the amplitude and the frequency of the given input signal $f_m(t)$ respectively.

Proof. The theorem is proved by means of the time linear transformation and the coordination transformation on the given basic signal. Specifically, for the given input signal $f_m(\tau)$, the corresponding STA-RED can be constructed as follows

$$\begin{aligned} \dot{y}_1(\tau) &= y_2(\tau) - \lambda_m |y_1(\tau) - f_m(\tau)|^{\frac{1}{2}} \text{sign}(y_1(\tau) - f_m(\tau)) \\ \dot{y}_2(\tau) &= -\alpha_m \text{sign}(y_1(\tau) - f_m(\tau)) \end{aligned} \tag{2}$$

where, y_1 and y_2 are the filtered signal and the derivative signal of the given input signal $f_m(\tau)$, respectively. Similarly, with the given basic signal $f_0(t)$, the format of the STA-SMD is

$$\begin{aligned}\dot{x}_1(t) &= x_2(t) - \lambda_0 |x_1(t) - f_0(t)|^{\frac{1}{2}} \text{sign}(x_1(t) - f_0(t)) \\ \dot{x}_2(t) &= -\alpha_0 \text{sign}(x_1(t) - f_0(t)).\end{aligned}\quad (3)$$

To prove the theorem, the parameters λ_m and α_m in Eq. (2) need to be obtained. The time linear transformation and the coordinate transformation

$$\begin{aligned}t &= w_m \tau \\ y_1(\tau) &= A_m x_1(t) \\ y_2(\tau) &= A_m w_m x_2(t)\end{aligned}\quad (4)$$

are performed on Eq. (3) and we have

$$\begin{aligned}\frac{dy_1(\tau)}{d\tau} &= A_m \frac{dx_1(t)}{d\tau} = A_m \frac{dx_1(t)}{dt} \frac{dt}{d\tau} \\ \frac{dy_2(\tau)}{d\tau} &= A_m w_m \frac{dx_2(t)}{d\tau} = A_m w_m \frac{dx_2(t)}{dt} \frac{dt}{d\tau}.\end{aligned}\quad (5)$$

Then, there exist

$$\begin{aligned}\dot{y}_1(\tau) &= y_2(\tau) - \sqrt{A_m w_m} \lambda_0 |y_1(\tau) - f_m(\tau)|^{\frac{1}{2}} \cdot \text{sign}(y_1(\tau) - f_m(\tau)) \\ \dot{y}_2(\tau) &= -A_m w_m^2 \alpha_0 \text{sign}(y_1(\tau) - f_m(\tau))\end{aligned}\quad (6)$$

where

$$f_m(\tau) = A_m f(t) = A_m f(w_m \tau) = A_m \sin(w_m \tau).$$

Therefore, for any given input signal $f_m(\tau)$, the effective parameters of the corresponding STA-RED is

$$\lambda_m = \sqrt{A_m w_m} \lambda_0, \quad \alpha_m = A_m w_m^2 \alpha_0.\quad (7)$$

The proof of Theorem 2.1 has completed here. \square

For the input $f_0(t) = \sin(t)$, to ensure the robustness and accuracy of STA-SMD, the parameters are recommended to be selected as $\lambda_0 = 3$ and $\alpha_0 = 2$ according to the reference [16] and our own extensive simulations. According to Theorem 2.1, for any given inputs $f_m(t) = A_m \sin(w_m t)$, the algorithm structure of the STA-SMD can be presented as follows,

$$\begin{aligned}\dot{y}_1 &= y_2 - \lambda_m |y_1 - f_m|^{\frac{1}{2}} \text{sign}(y_1 - f_m) \\ \dot{y}_2 &= -\alpha_m \text{sign}(y_1 - f_m) \\ \lambda_m &= 3\sqrt{A_m} w_m \\ \alpha_m &= 2A_m w_m^2,\end{aligned}\quad (8)$$

where, A_m and w_m are the amplitude and the frequency of the given input signal $f_m(t)$ respectively. According to the updated STA-SMD in Eq. (8), once A_m and w_m are estimated by the proposed ANF, the parameters of the STA-SMD for any given signals can be directly determined by Eq. (8).

3. AN IMPROVED ADAPTIVE NOTCH FILTER AND ANF-SMD

In practical engineering, to apply the aforementioned parameter formula of the STA-SMD, we need to know the real-time frequency and amplitude of the input signals. A good solution is to design an adaptive notch filter (ANF) to estimate the amplitude and frequency of the inputs. However, there exist relatively big steady state errors in the estimation of frequency and amplitude of the given inputs by applying the commonly-used ANF [18, 19]. In order to reduce steady state errors in its estimation, the integral feedback of the system output errors is introduced for the first time in construction of the cost function for the ANF.

Consider a sinusoidal input signal with variable amplitude and frequency with white noise $\eta(t)$ i. e., $f(t) = f_0(t) + \eta(t)$. Therein,

$$f_0(t) = A_0 \sin(w_0 t + \delta_0),$$

where the amplitude A_0 , the frequency w_0 and the phase δ_0 are unknown and variable parameters. The amplitude A_0 and frequency w_0 of the signals are time-varying. The aim here is to design an ANF to estimate the signal $f_0(t)$. Let $y(t) = A(t) \sin(\phi(t))$ act as the estimation of $f_0(t)$, where

$$\phi(t) = \int_0^t w(\tau) d\tau + \delta(t).$$

The errors between the input signal $f(t)$ and its noise-filtered estimate $y(t)$ is

$$e(t) = f(t) - y(t) = f(t) - A(t) \sin(\phi(t)).$$

To minimize the error $e(t)$, the gradient descent method is applied. However, since the cost function is only composed of the error feedback, relatively big steady state errors may occur. To tackle the problem, the error integral feedback is introduced into the cost function. The new cost function is defined as follows

$$J(t, \Theta) = \frac{1}{2} \left(k_1 e(t) + k_2 \int_0^t e(\tau) d\tau \right)^2 := \frac{1}{2} \hat{e}(t)^2, \quad (9)$$

where, $\Theta = [A(t), w(t), \delta(t)]^T$, and $k_1 \geq 0$, $k_2 \geq 0$ are two constants. Applying the gradient descent method where the variables update in the direction of the negative gradient. We have

$$\dot{\Theta}(t) = -\mu \frac{\partial}{\partial \Theta} J(t, \Theta), \quad (10)$$

where

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{bmatrix}$$

$\mu_1 > 0$, $\mu_2 > 0$, and $\mu_3 > 0$ are the step sizes. Then the following differential equations can be obtained

$$\begin{aligned} \dot{A}(t) &= \mu_1 \hat{e}(t) \left(k_1 \sin(\phi(t)) + k_2 \int_0^t \sin(\phi(\tau)) d\tau \right) \\ \dot{w}(t) &= \mu_2 \hat{e}(t) \left(k_1 t A(t) \cos(\phi(t)) + k_2 \int_0^t A(\tau) \tau \cos(\phi(\tau)) d\tau \right). \end{aligned} \quad (11)$$

Without loss of generality, the terms $tA(t) = \tau A(\tau) > 0$ in Eq. (11) can be absorbed into the parameters μ_2 . For the total phase angle $\phi(t)$, we have,

$$\frac{d}{dt} \phi(t) = \frac{d}{dt} \left[\int_0^t w(\tau) d\tau + \delta(t) \right] = w(t) + \dot{\delta}(t).$$

Eventually, the improved ANF is obtained

$$\begin{aligned} \dot{A}(t) &= \mu_1 \hat{e}(t) \left(k_1 \sin(\phi(t)) + k_2 \int_0^t \sin(\phi(\tau)) d\tau \right) \\ \dot{w}(t) &= \mu_2 \hat{e}(t) \left(k_1 \cos(\phi(t)) + k_2 \int_0^t \cos(\phi(\tau)) d\tau \right) \\ \dot{\phi}(t) &= w(t) + \mu_3 \hat{e}(t) \left(k_1 \cos(\phi(t)) + k_2 \int_0^t \cos(\phi(\tau)) d\tau \right) \\ y(t) &= A(t) \sin(\phi(t)) \end{aligned} \quad (12)$$

where, $A(t)$, $w(t)$, and $\phi(t)$ are the estimates of the amplitude, frequency and phase angle of the signal $f_0(t)$, respectively. μ_1 , μ_2 , μ_3 , k_1 and k_2 are the tuning parameters, and

$$\hat{e}(t) = k_1 (f(t) - y(t)) + k_2 \int_0^t (f(\tau) - y(\tau)) d\tau.$$

Based on the improved ANF presented above, a differentiator that can process unknown and time-varying signal $f(t)$ can be designed. The outputs $A(t)$ and $w(t)$ of the proposed ANF are used to determine the parameters of the SMD according to Eq. (8). Then, the SMD via the improved ANF is proposed. The structure is shown as follows:

$$\left\{ \begin{array}{l} \hat{e}(t) = f(t) - y(t) + \int_0^t (f(\tau) - y(\tau)) d\tau \\ \dot{A}(t) = \mu_1 \hat{e}(t) \left(k_1 \sin(\phi(t)) + k_2 \int_0^t \sin(\phi(\tau)) d\tau \right) \\ \dot{w}(t) = \mu_2 \hat{e}(t) \left(k_1 \cos(\phi(t)) + k_2 \int_0^t \cos(\phi(\tau)) d\tau \right) \\ \dot{\phi}(t) = w(t) + \mu_3 \hat{e}(t) \left(k_1 \cos(\phi(t)) + k_2 \int_0^t \cos(\phi(\tau)) d\tau \right) \\ y(t) = A(t) \sin(\phi(t)) \\ \dot{y}_1(t) = y_2(t) - \lambda(t) |y_1(t) - f(t)|^{\frac{1}{2}} \text{sign}(y_1(t) - f(t)) \\ \dot{y}_2(t) = -\alpha(t) \text{sign}(y_1(t) - f(t)) \\ \lambda(t) = 3\sqrt{A(t)}w(t) \\ \alpha(t) = 2A(t)w^2(t) \end{array} \right. \quad (13)$$

where, $f(t)$ is the unknown input signal, $y(t)$ is the estimate of the basic signal $f_0(t)$, the state variables $y_1(t)$ and $y_2(t)$ are the desired trajectory and its derivative, respectively. $\mu_1 > 0$, $\mu_2 > 0$, $\mu_3 > 0$, $k_1 \geq 0$ and $k_2 \geq 0$ are the tuning parameters, $\lambda(t)$ and $\alpha(t)$ are adaptive parameters. In practice, by our extensive simulations, we find that once we set the parameters of μ_1 , μ_2 and μ_3 for the given input with the sampling period h_1 , one can approximately set the parameters of $\hat{\mu}_1 = \mu_1/\kappa$, $\hat{\mu}_2 = \mu_2/10\kappa$ and $\hat{\mu}_3 = \mu_3/0.4\kappa$ for the other given input with the sampling period h_2 , where $\kappa = h_2/h_1$. For any unknown and time-varying signals, their amplitude and frequency can be estimated by the first to fifth equations in Eq. (13), i.e, ANF algorithm. Then, the parameters of STA-SMD can be updated in real time according to the eighth to ninth equations in Eq. (13). With the adaptive parameter, the proposed ANF-SMD can effectively obtain real-time filtering and differentiation estimation of input signals.

The main algorithm complexity comes from the ANF algorithm that is used to estimate the approximate frequency and amplitude of the inputs. For ANF algorithm, by including the integral feedback of system output errors in reconstructing the cost function, the increase of the algorithm's running time is trivial, keeping at 0.5s. Also, by using the proposed deterministic parameter tuning formula, we significantly reduce the complexity and eliminates the cost paid in setting parameters.

Remark 3.1. The proposed ANF can adaptively estimate the frequency and amplitude of the signals collected by sensors in practice. Then the effective parameters can be directly confirmed according to the proposed parameter tuning formula. This ensures effective signal-tracking filtering and differentiation acquisition to participate in designing the feedback controller. For example, with good estimation of error value's filtering and the derivative, in PID control scheme, one can set a bigger proportional gain and derivative gain for achieving better control performance.

4. NUMERICAL SIMULATIONS

In this section, numerical simulations using Matlab are presented to verify the effectiveness of the proposed ANF algorithm and the real-time parameter tuning scheme. Specifically, the simulation tests include two parts: 1) the adaptability of the proposed ANF are tested by giving some input signals with variable frequency and amplitude, and 2) the advantages of the proposed ANF-SMD regarding signal-tracking filtering and differentiation acquisition are presented by comparison with commonly-used STA-SMD (see reference [16]), tracking differentiator based on discrete-time optimal control (DTOC-TD)(see reference [11]) and kth-order filter via SMs (see reference [9]).

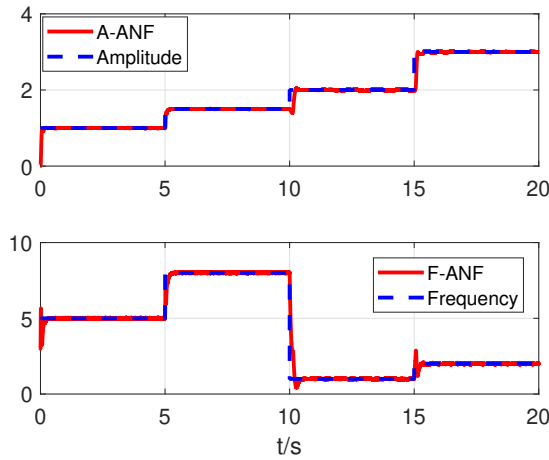


Fig. 2. Estimation performance of the proposed ANF in tracking $f_1(t)$ where SNR value is 40dB.

4.1. Performance characterization of the proposed ANF

In this part, the performances of the proposed ANF are studied. In simulations, the sampling period $h_1 = 0.001s$ is set for all investigations. As certain representative cases in real-life engineering, the following two types of time-varying signals (i. e., time-varying frequency and amplitude) with white noise are tested, where the signal-to-ratio (SNR) value of both signals is 40dB.

Example 1. Consider a time-varying signal $f_1(t)$ whose frequency range is [1Hz , 10Hz], i. e.,

$$f_1(t) = \begin{cases} \sin(10\pi t) & t \leq 5s \\ 1.5 \sin(16\pi t) & 5s < t \leq 10s \\ 2 \sin(2\pi t) & 10s < t \leq 15s \\ 3 \sin(4\pi t) & 15s < t \leq 20s \end{cases} .$$

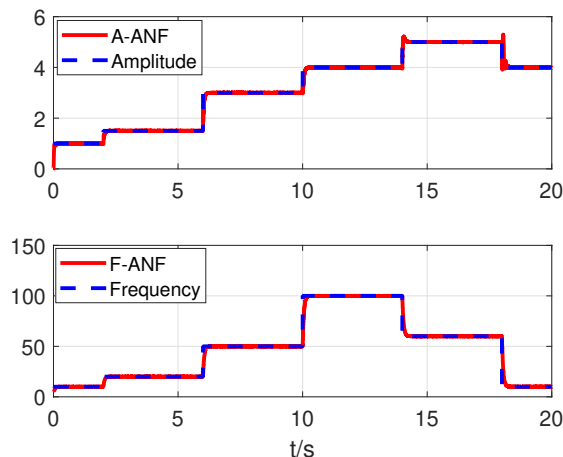


Fig. 3. Estimation performance of the proposed ANF in tracking $f_2(t)$ where SNR value is 40dB.

The parameters $\mu_1 = 80$, $\mu_2 = 10000$, $\mu_3 = 600$, $k_1 = 1$ and $k_2 = 0.05$ are set by the trial and error method for the proposed ANF algorithm. The simulation results are shown in Figure 2.

Example 2. Consider a time-varying signal $f_2(t)$ whose frequency range is [10Hz , 100Hz], i. e.,

$$f_2(t) = \begin{cases} \sin(20\pi t) & t \leq 2s \\ 1.5 \sin(40\pi t) & 2s < t \leq 6s \\ 3 \sin(100\pi t) & 6s < t \leq 10s \\ 4 \sin(200\pi t) & 10s < t \leq 14s \\ 5 \sin(120\pi t) & 14s < t \leq 18s \\ 4 \sin(20\pi t) & 18s < t \leq 20s \end{cases} .$$

We set the parameters $\hat{\mu}_1 = \mu_1/\kappa$, $\hat{\mu}_2 = \mu_2/10\kappa$, $\hat{\mu}_3 = \mu_3/0.4\kappa$, $k_1 = 1$ and $k_2 = 0.05$, where $\kappa = 1$. The simulation results are shown in Figure 3.

From Figures 2 and 3, we can see that the proposed ANF features, the smooth and fast response, high accuracy, and anti-chattering. It is insensitive to high frequency noise and has good filtering characteristics. For the time-varying signals, the proposed ANF has good adaptability and small steady state errors in terms of the estimation of frequency and amplitude. The real-time amplitude and frequency of the given signals can be accurately estimated.

4.2. Performance characterization of the proposed ANF-SMD

In this part, the characteristics of the proposed ANF-SMD in both filtering and differentiation estimation are analyzed. In particular, the proposed ANF-SMD (Eq. (13)) is compared with STA-SMD, DTOC-TD and SMs.

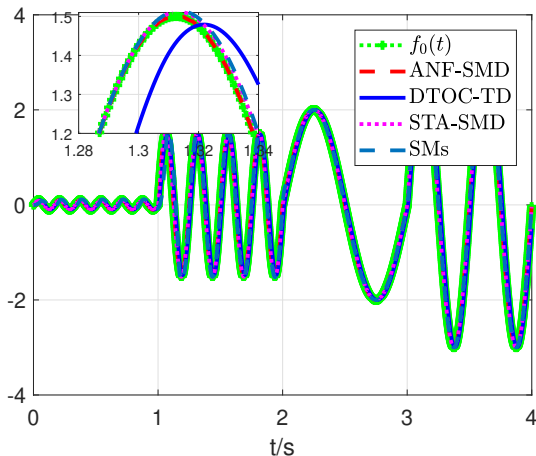


Fig. 4. Comparisons of signal-tracking filtering by the proposed ANF-SMD and other differentiator.

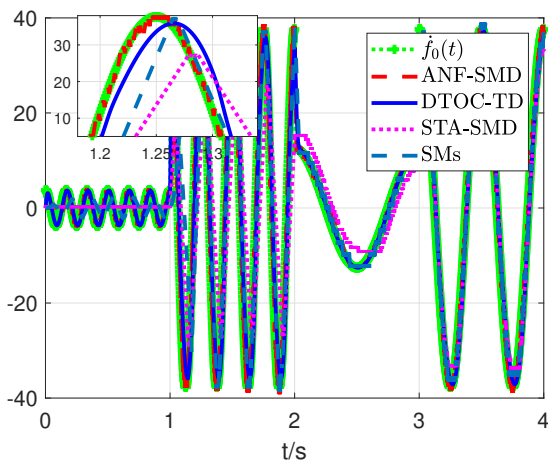


Fig. 5. Comparisons of differentiation acquisition by the proposed ANF-SMD and other differentiator.

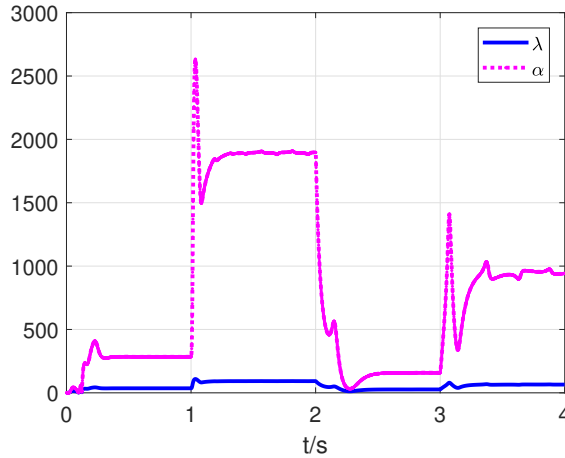


Fig. 6. The adaptive tuning process of parameters λ and α for the proposed ANF-SMD.

In all simulations, we choose the same sampling period $h_1 = 0.001s$ and the sinusoidal signal $f_0(t)$ with white noise, i. e.,

$$f_0(t) = \begin{cases} 0.1 \sin(12\pi t) & t \leq 1s \\ 1.5 \sin(8\pi t) & 1s < t \leq 2s \\ 2 \sin(2\pi t) & 2s < t \leq 3s \\ 3 \sin(4\pi t) & 3s < t \leq 4s \end{cases} .$$

All parameters are chosen by the trial and error method. For STA-SMD, the parameters $\lambda_0 = 140$ and $\alpha_0 = 450$ are set. The parameters $r = 3000$ and $c_0 = 5$ are set for DTOC-TD. For the proposed ANF-SMD, $\hat{\mu}_1 = \mu_1/\kappa$, $\hat{\mu}_2 = \mu_2/10\kappa$, $\hat{\mu}_3 = \mu_3/0.4\kappa$, $k_1 = 1$ and $k_2 = 0.05$ are selected, where $\kappa = 1$. For SMs, $\alpha_0 = 2500$, $\beta = 0.5$, $\gamma = 15$, $\lambda_0 = 6$, $\lambda_1 = 10$ and $\lambda_2 = 5$ are set. The comparison results in signal-tracking filtering and differentiation estimation are presented in Figures 4 and 5. The adjustment process of parameters λ and α are shown in Figure 6.

Form Figures 4 and 5, we can see that STA-SMD, DTOC-TD and SMs have relatively large phase delay compared with the proposed ANF-SMD. In terms of time-varying input signals, the proposed ANF-SMD is more accurate in signal filtering and differentiation acquisition than STA-SMD, DTOC-TD and SMs. Figure 6 shows that the parameters λ and α can be adjusted adaptively as input signals vary.

5. SIMULATION EXPERIMENTS

The altitude ground test facility is a highly complex test facility [4, 15, 29]. It is employed to test and evaluate the steady-state and transient performance of the tested aero-engines under altitude conditions. The flight environment simulation system is a core part of

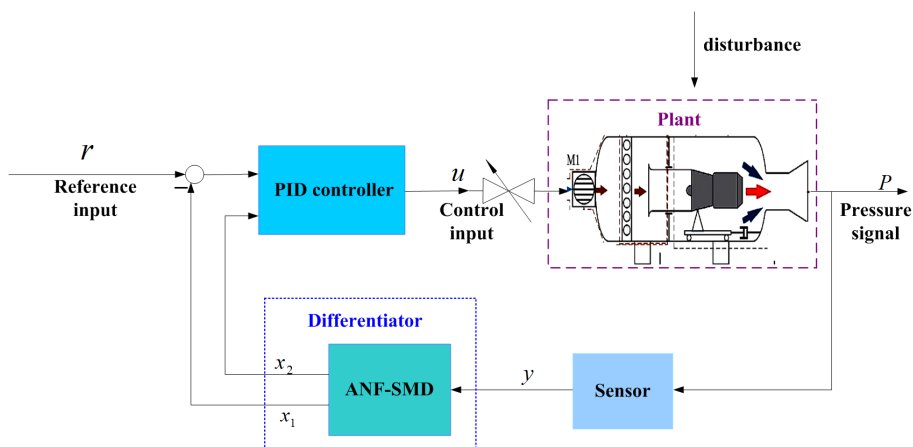


Fig. 7. Pressure control block diagram of the flight environment simulation system.

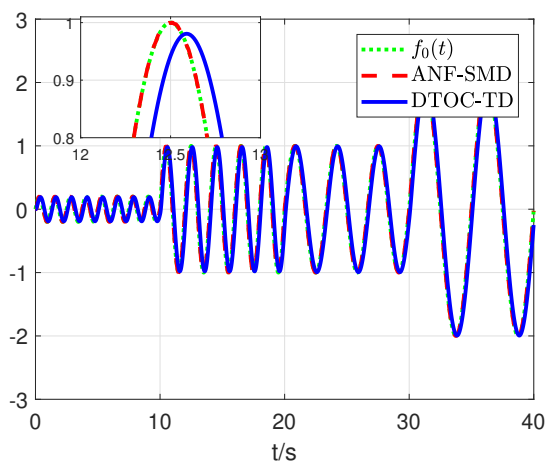


Fig. 8. Comparisons of signal-tracking filtering of the pressure signal by the proposed ANF-SMD and DTOC-TD.

the altitude ground test facility, which provides total pressure conditions as that occur during flight operation for the tested aero-engines at chamber. The flight environment simulation system control block diagram is depicted in Figure 7.

In practice, the PID controller is applied to adjust the pressure conditions at the inlet and outlet during the test. The pressure signals and their corresponding derivative signals are vital to constructing a PID controller. The pressure signals, however, feature a low frequency, a low sample frequency and are prone to be corrupted by noises. Therein,

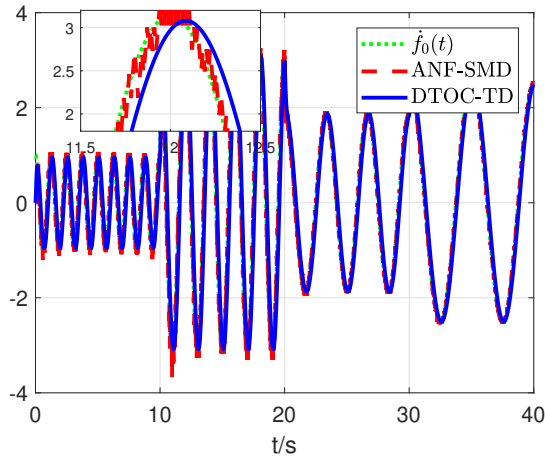


Fig. 9. Comparisons of differentiation acquisition of the pressure signal by the proposed ANF-SMD and DTOC-TD.

the frequency of the pressure signal is around 0.5Hz and the sample frequency is 100Hz, i. e., the sampling period $h_2 = 0.01s$. The effective filtering and derivative signals cannot be obtained by some typical differentiators. Therefore, the proposed ANF-RED is applied to the processing of pressure signals in the flight environment simulation system.

The proposed ANF-SMD has been tested in the flight environment simulation system and experimentally compared with the DTOC-TD (see reference [11]) by using Matlab. In the experiments, the parameters of DTOC-TD are given by the trial and error method, where $c_0 = 5$ and $r = 3000$. For the proposed ANF-SMD, the parameters $\hat{\mu}_1 = \mu_1/\kappa$, $\hat{\mu}_2 = \mu_2/10\kappa$, $\hat{\mu}_3 = \mu_3/0.4\kappa$, $k_1 = 1$ and $k_2 = 0.05$ are set, where $\kappa = 10$. The pressure signals processing comparison results including signal filtering and differentiation acquisition are shown in Figures 8 and 9, respectively.

From Figures 8 and 9, we can see that the proposed ANF-SMD, compared with the DTOC-TD, can produce better signal filtering and differentiation acquisition that meet the requirements of the altitude ground test facility. This indicates that the proposed ANF-SMD may be widely applied in the flight environment simulation systems to achieve improved stability and reliability.

6. CONCLUSION

In this paper, a parameter tuning scheme for the sliding mode differentiator (SMD) was proposed, and successfully applied in adjusting the parameters of super-twisting algorithm based SMD. The main issue of this parameter tuning scheme is that the amplitude and frequency of given inputs need to be known. To obtain the real-time amplitude and frequency of given random inputs in regulating the selection of the parameters of SMD, an improved adaptive notch filter (ANF) was proposed. In the design of ANF, the integral term of the system output errors was used to construct the cost function to

improve the estimation accuracy of the time-varying signals. Based on the parameter tuning formula and the proposed ANF, the parameters of the proposed ANF-SMD can be adaptively adjusted as the time-varying inputs in real time. The simulation results showed that the proposed ANF-SMD has a smaller phase lag and higher precision compared with DTOC-TD. The experiments carried out on the flight environment simulation system also verified that the proposed ANF-SMD is effective in signal filtering and differentiation acquisition. As inputs vary with different operating conditions in real-life engineering, the proposed real-time parameter tuning scheme enables the proposed ANF-SMD obtain a effective set of parameters for having good performances in filtering and differentiation acquisition. Future work shall include the stability proof of the proposed ANF-SMD, further suppression of chattering issue from the adoption of a boundary level in the proposed ANF-SMD, and developing new types of the differentiators and observers by adopting the proposed parameter tuning scheme.

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