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ON ADMISSIBLE GROUPS OF DIFFEOMORPHISMS

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ABSTRACT. In a number of papers the phenomenon of determining a smooth geometric structure on a manifold by the group of its automorphisms have been investigated. This seems to be a modern analogue of basic ideas of the Erlangen Program. We call such diffeomorphism groups admissible and try to describe them by imposing some axioms. Several examples are included.

1. INTRODUCTION

The phenomenon of determining a geometric structure on a manifold by the group of its automorphisms is a modern analogue of some basic ideas of the Erlangen Program of F.Klein [11]. This fact, which seems to be a deep feature of the geometry of manifolds, has been extensively commented in F.Takens' paper [18]. In this note diffeomorphisms groups fulfilling this phenomenon will be called admissible.

One of the first remarkable contributions to the Erlangen Program in terms of the modern geometry constitutes the paper by J.V.Whittaker [19]. He proved that under mild assumptions a topological manifold is completely defined by the group of its homeomorphisms. The techniques of this paper can be viewed as a basic tool in further attempts but now complicated infinite patching methods from [19] are useless since the fragmentation property has been proven. More recently R.P.Filipkiewicz [8] established that the group $Diff^r(M)$ of all C^r diffeomorphisms of a manifold M, $1 \leq r \leq \infty$, determines uniquely the topological and smooth structure of M. Next A.Banyaga in [3] showed analogous theorems for the group of automorphisms of a unimodular as well as a symplectic structure. A more general result has been proved in [14] by using a simplified pattern of the proof. This pattern has occured to be fruitful (cf.[15, 16]) as it gets rid the proof of theorems on the simplicity and perfectness of diffeomorphism groups (see for instance [7] and references in [14]), the theorems which are difficult and not known in important cases.

Our aim here is to describe a possibly large class of diffeomorphism groups fulfilling the above mentioned phenomenon. This is accomplished by imposing some axioms on these groups and by formulating a main theorem in section 2. (The term "axiom" is far-fetched here but it follows [7].) In the last section we prove the main theorem.

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All considerations are valid in the C^∞ (or C^r, $0 \le r \le \infty$) category, and are no longer true in the real analytic case.

2. Admissible Groups of Diffeomorphisms

Let M be a C^r manifold. Any subgroup $G = G(M)$ of $Diff^r(M)$ is said to be a group of C^r-diffeomorphisms of M. By $G(M)_c$ we denote the subgroup of compactly supported elements of $G(M)$. If $G(M)$ is locally contractible (this is satisfied by the "classical" groups of diffeomorphisms, i.e. the groups of symplectic diffeomorphisms, volume element preserving diffeomorphisms and contact diffeomorphisms, as well as the group of leaf preserving diffeomorphisms) then $G(M)_0$, the identity component of $G(M)_c$ in the C^r compact-open topology, consists of all $f \in G(M)_c$ such that there are a compact $K \subset M$ and an isotopy $F = \{f_t\}_{t \in I}$ in $G(M)$ connecting $f = f_1$ with the identity with each f_t stabilizing outside K.

Next for U open denote by $G(U)_c$ the group of diffeomorphisms of G compactly supported in U, and by $G(U)_0$ its identity component.

We start our axiomatization with the fragmentation property.

Axiom 1. For any $\mathcal{U} = \{U_j\}$ a finite open cover of *supp*(f), where $f \in G_0$, we have $f = f_1 \dots f_s$ where $f_i \in G(U_{j(i)})_0$ for $i = 1, \dots, s$.

It is well known that $Diff^r(M)$ satisfies Axiom 1 (cf. [13], for $r = 0$ [6]). The same holds for contact diffeomorphisms. Also for the kernel of the flux homomorphism of $G(M, \alpha)$, where α is a volume element or a symplectic form, we get this property as well (cf.[4]).

The following is clue in our pattern of the proof.

Axiom 2. For any $x \in M$ and a sufficiently small open ball U with the center at x there exists $f \in G_o$ with $Fix(f) = (M - U) \cup \{x\}$. In addition, for any $x \in U$, U open, there is $f \in G(U)_o$ such that $f(x) \neq x$.

Here $Fix(f) = \{x \in M | f(x) = x\}$. Again, this axiom is satisfied by "classical" groups of diffeomorphisms (cf.[3], [14]).

Definition. Let $dim(M) > 1$. Then $G(M)$ satisfies *T(n) property* if for any two n-tuples of n distinct points $x_1, \dots, x_n, y_1, \dots, y_n \in M$ there is $f \in G(M)$ such that $f(x_i) = y_i$.

If $dim(M) = 1$ then M is diffeomorphic either to the real line or to the circle. Then one can fixed an order either on M in the first case, or on $M - \{x\}$ with some fixed x in the second. In this case $G(M)$ will be said to have *T(n) property* if for any two ordered n-tuples $x_1 < \dots < x_n$ and $y_1 < \dots < y_n$ there is $f \in G(M)$ such that $f(x_i) = y_i$ in the first case, and if for any two ordered $(n-1)$-tuples $x_1 < \dots < x_{n-1}$ and $y_1 < \dots < y_{n-1}$ there is $f \in G(M)$ such that $f(x) = x$ and $f(x_i) = y_i$ in the second case.

It is well known that $Diff^r(M)$ and the "classical" groups of diffeomorphisms as well as their identity components have this property (see, e.g., [12]).

Axiom 3. $G(M)_0$ acts $T(3)$ on M.

The theorems of Whittaker type are "integral" counterparts of Pursell-Shanks type theorems. A theorem of Pursell-Shanks states that the Lie algebra of vector fields of a manifold M determines completely the smooth structure of M itself. Several generalizations followed, eg.[1],[10]. Our next axiom will appeal to these theorems (cf.[2]). The reason is that Lie algebras of vector fields are more manageable than diffeomorphism groups.

Definition. A Lie algebra of compactly supported infinitesimal automorphisms of a geometric structure (M, α) satisfies *(PS) property* if it determines uniquely (M, α). Moreover, if $\mathcal{X}(M_1, \alpha_1), \mathcal{X}(M_2, \alpha_2)$ are isomorphic by Φ then the unique diffeomorphism $\phi : M_1 \to M_2$ fulfils $\Phi = \phi_*$ and exchanges the structures in question.

Axiom 4. There is $\mathcal{X}$, a Lie algebra of vector fields satisfying (PS) property, such that all elements of the one parameter transformation group of any element of $\mathcal{X}$ belong to G.

The above axioms are sufficient for the transitive case [14]. Now we consider the nontransitive case. It is easily seen that the identity component of a locally contractible group of diffeomorphisms constitutes a set of arrows (cf.[17]) so that it defines uniquely a generalized foliation, say $\mathcal{F}$. We impose further axioms connected with the nontransitivity.

Axiom 3'. $G(M)$ acts T(3) on any leaf $L \in \mathcal{F}$.

Axiom 5. $\mathcal{F}$ has no leaves of dimension 0, that is $G(M)_0$ fixes no points.

Axiom 6. $G(M)$ preserves the leaves of $\mathcal{F}$.

Axiom 7. If $x, y \in L \cap U, L \in \mathcal{F}, U$ being an open ball, then there is $g \in G(M)_0$ with $supp(g) \subset U$ such that $g(x) = y$.

The following two last axioms are connected with the case of noncompact manifolds.

Axiom 8. For any $x \in M$ and for any $g \in G(M)$ such that $g(x) = x$ there exists $h \in G(M)_c$ such that $h = g$ on a neighbourhood of x.

This is verified for the group of all (possibly leaf preserving) diffeomorphisms (cf.[9]). In the foliated case the proof is essentially the same but one should apply a distinguished chart for a generalized foliation. The existence of such charts has been proved in [5].

Axiom 9. If $\{U_i\}$ is a pairwise disjoint locally finite family of open balls and $g_i \in G(M)$ with $supp(g_i) \subset U_i$, then $g = \prod g_i \in G(M)$, where $\prod g_i = g_i$ on U_i for any i, and $\prod g_i = id$ on $M - \bigcup U_i$.

Note that this is an integral analogue of a condition in the definition of a quasi-foliation in [1].

Now we can formulate

Main Theorem. *Let (M_i, α_i), $i = 1, 2$, be a geometric structure such that its group of automorphisms $G(M_i, \alpha_i)$ satisfies either Axioms 1,2,3 and 4, or Axioms 1,2,3',4,5,6 and 7, and M_i is compact, or Axioms 1,2,3',4,5,6,7,8 and 9. Then if there is a group isomorphism $\Phi : G(M_1, \alpha_1) \to G(M_2, \alpha_2)$ then there is a unique C^∞-diffeomorphism $\phi : M_1 \to M_2$ preserving α_i and such that $\Phi(f) = \phi f \phi^{-1}$ for each $f \in G(M_1, \alpha_1)$.*

In particular, any automorphism of $G(M, \alpha)$ of the above type is inner or coming from a foliation preserving diffeomorphism.

Remark. For C^r diffeomorphisms, r finite, the proof of Main Theorem still works but we get ϕ to be a homeomorphism only. To obtain the C^r-smoothness one should apply the theorem of Bochner-Montgomerry as in [8].

Let us give examples of admissible groups.

Examples: the case transitive. (1) The group of all homeomorphisms of a topological manifold (see [19], our proof is much simpler).

(2) The group $Diff^r(M)$, $1 \le r \le \infty$. The original method [8] of the proof depends heavely on a theorem of Epstein [7] which states that the commutator subgroup is simple. Our pattern [14] is independent of this theorem and is much shorter itself.

(3) The group $G(M,\alpha)$ where α is either a volume form, or a symplectic form, or a contact form [2,3,14]. Again this is a consequence of Main Theorem with some modification in first and second cases as Axiom 1 is satisfied by the kernel of the flux homomorphism. Roughly, this modification consists in passing to the kernels and observing that they verify Axioms 1,2,3 and 4.

(4) The group of cosymplectic diffeomorphisms. The proof will be detailed elsewhere.

Nontransitive geometric structures plays an increasing role in the geometry. On the other hand little is known of "perfectness" theorems, and this reveals the significance of Main Theorem.

Examples: the case nontransitive. (5) The group of all diffeomorphisms of a manifold with boundary as well as some of its subgroups [15].

(6) The group of all leaf preserving diffeomorphisms of a foliation [16].

(7) Let N be a topological manifold of M with $dim(N) > 0$. Then N can be viewed as a geometric structure. It is a consequence of Main Theorem that $G(M,N)$ is admissible.

(8) The group of automorphisms of a regular Poisson manifold. The proof will be detailed elsewhere.

Remark. It is known that all the above groups are locally contractible, so that it is not hard to see that their identity components are still admissible.

3. Proof of the Main Theorem

The proof follows [16]. From now on for simplicity we denote $G^i = G(M_i, \alpha_i), i = 1,2$. Let $\mathcal{F}_i$ be the generalized foliation corresponding to G^i_0. By S_xG^i we denote the isotropy subgroup of G^i at $x \in M_i$. Next we let

$$F^1_y = \Phi^{-1}(S_yG^2), \quad F^2_x = \Phi(S_xG^1).$$

Lemma 3.1. *For any G^i satisfying Axiom 3 or 3' all isotropy subgroups are maximal.*

For the proof see [19], [8], or [16].

Lemma 3.2. *Let G^i satisfy Axiom 3 or Axioms 3',6 and 7. Let $y \in M_2$ and let C be a closed nonempty subset of M_1 satisfying $C \cap L \neq L$ for any leaf $L \in \mathcal{F}_1$ and*

$$f(C) = C \quad \forall f \in F^1_y.$$

Then $C = \{x\}$ and $F^1_y = S_xG^1$.

The proof follows closely that in [19] or [8].

The Main Theorem can be reduced to the following

Theorem 3.3. *For any $y \in M_2$ there exists a unique $x \in M_1$ such that $\Phi(S_xG^1) = S_yG^2$.*

In fact, Theorem 3.3 determines uniquely a bijection $\phi : M_1 \to M_2$ verifying $\Phi(S_xG^1) = S_{\phi(x)}G^2$. It will follow from the proof that the ϕ satisfies

$$\Phi(f) = \phi f \phi^{-1} \quad \forall f \in G^1.$$

That ϕ is a homeomorphism is a consequence of this equality. Now as in [2] or [16] we prove by using Axiom 4 that ϕ is a diffeomorphism preserving geometric structures.

The proof of Theorem 3.3 consists of several propositions. First we introduce the following notation. For $y \in M_2$, we denote by $\mathcal{C}_y$ the totality of open balls U of M_1 satisfying

$$G^1(U)_0 \subset F^1_y = \Phi^{-1}(S_yG^2).$$

Similarly we define a family $\mathcal{D}_x$, $x \in M_1$, as the set of all open balls V of M_2 such that

$$G^2(V)_0 \subset F^2_x = \Phi(S_xG^1).$$

Let $C_y = M_1 - \bigcup \mathcal{C}_y$, $D_x = M_2 - \bigcup \mathcal{D}_x$. By making use of Lemma 3.2 we will show that both C_y and D_x consist of one element.

Proposition 3.4. *The subsets C_y and D_x are preserved by elements of F^1_y and F^2_x, respectively.*

Proof. Let $U \in \mathcal{C}_y$ and $f \in F^1_y$. If $V = f(U)$ then $G^1(V)_0 = fG^1(U)_0f^{-1} \subset fF^1_yf^{-1} = F^1_y$. It follows that $f(C_y) \subset C_y$. Similarly $g(D_x) \subset D_x$ for any $g \in F^2_x$.

Lemma 3.5. *Let $y \in M_2$, and let L be a leaf of $\mathcal{F}_1$. Suppose that for any open ball $U \subset M_1$ with $U \cap L \neq \emptyset$ there is $f_1 \in G^1(U)_0$ with $f_2(y) \neq y$, where $f_2 = \Phi(f_1)$. Then there are open sets $V, W \subset M_1$ such that*

$$\bar{V} \cap \bar{W} = \emptyset, \quad L - \overline{V \cup W} \neq \emptyset,$$

and there are $\tilde{f}_1, \bar{f}_1 \in G^1(V)_0$, $\tilde{g}_1, \bar{g}_1 \in G^1(W)_0$ satisfying

$$y \neq \tilde{f}_2(y) \neq \bar{f}_2(y) \neq y, \quad y \neq \tilde{g}_2(y) \neq \bar{g}_2(y) \neq y,$$

where $\tilde{f}_2 = \Phi(\tilde{f}_1), \bar{f}_2 = \Phi(\bar{f}_1), \tilde{g}_2 = \Phi(\tilde{g}_1), \bar{g}_2 = \Phi(\bar{g}_2)$.

The proof is the same as in [16].

The following is a clue part of the proof.

Proposition 3.6. *For any $y \in M_2$ and for any leaf $L \in \mathcal{F}_1$ there is an open ball U on M_1 such that $U \cap L \neq \emptyset$ and $G^1(U)_0 \subset F^1_y$.*

Proof. Let $y \in M_2$. For any open ball V on M_1 satisfying $V \cap L \neq \emptyset$ we may assume the existence of $f_1 \in G^1(V)_0$ such that $f_2(y) \neq y$ where $f_2 = \Phi(f_1)$; otherwise we are done. Take then two open sets V, W and $\tilde{f}_1, \bar{f}_1 \in G^1(V)_0$, $\tilde{g}_1, \bar{g}_1 \in G^1(W)_0$ as in Lemma 3.5. Then $\bar{V} \cap \bar{W} = \emptyset$ and $G = L - \overline{V \cup W} \neq \emptyset$.

By Axiom 2 we have $h_2 \in G_0^2$ such that $Fix(h_2) = (M_2 - B) \cup \{y\}$ for some open ball B at y so small that the equalities

$$B \cap \tilde{f}_2(B) = \emptyset, \ B \cap \bar{f}_2(B) = \emptyset, \ B \cap \tilde{g}_2(B) = \emptyset, \ B \cap \bar{g}_2(B) = \emptyset$$

are satisfied. Let $h_1 = \Phi^{-1}(h_2)$. Then the two following cases are possible: either (1) $\overline{V \cup h_1(V)} \not\supset L$ or (2) $\overline{V \cup h_1(V)} \supset L$.

In case (1) take an open ball U such that $\overline{U} \cap \overline{(V \cup h_1(V))} = \emptyset$, and $U \cap L \neq \emptyset$. For any $k_1 \in G(U)_0$ one has $[k_1, [\tilde{f}_1, h_1]] = id$ and $[k_1, [\bar{f}_1, h_1]] = id$ as $supp([\tilde{f}_1, h_1])$ and $supp([\bar{f}_1, h_1])$ are contained in $V \cup h_1(V)$. Hence for $k_2 = \Phi(k_1)$ we have

$$[k_2, [\tilde{f}_2, h_2]] = id, \quad [k_2, [\bar{f}_2, h_2]] = id.$$

By definition we get

$$Fix([\tilde{f}_2, h_2]) = M_2 - (B \cup \tilde{f}_2(B)) \cup \{y, \tilde{f}_2(y)\}$$

and similarly for $Fix([\bar{f}_2, h_2])$ with $\bar{f}_2$ instead of $\tilde{f}_2$. Then one gets either $k_2(y) = y$ or $k_2(y) = \tilde{f}_2(y)$ by the above equality, and either $k_2(y) = y$ or $k_2(y) = \bar{f}_2(y)$ by the analogous one. This follows from the equality $f(Fix(g)) = Fix(g)$ whenever f, g commute. Consequently we have $k_2(y) = y$ as $\tilde{f}_2(y) \neq \bar{f}_2(y)$. Thus $G^1(U)_0 \subset F_y^1$, as required.

In case (2) W plays the role of V by assumption. This completes the proof.

Proposition 3.7. *Let $y \in L \in \mathcal{F}_2$. If there is $f_1 \in G_c^1 - F_y^1$ then $\Phi(G_c^1)(y) = L$.*

Proof. It is visible that G_c^1 and consequently $\Phi(G_c^1)$ are normal subgroups. Let $f_2(y) = y_1 \neq y$ where $f_2 = \Phi(f_1)$. For an arbitrary $y_2 \in L$, we wish to choose $g_2 \in \Phi(G_c^1)$ such that $g_2(y) = y_2$. If $y = y_2$ we are done; for otherwise choose an open ball U with $y_1, y_2 \in U, y \notin U$. Axiom 7 yields $h_2 \in G^2(U)_c$ such that $h_2(y_1) = y_2$ (if $dim(L) = 1$ taking f_2^{-1} instead of f_2 one may assume that y_1, y_2 lie in the same component of $L - \{y\}$). Then $g_2 = h_2 f_2 h_2^{-1}$ satisfies the claim.

Proposition 3.8. *The set C_y is nonempty for any $y \in M_2$.*

Remark. Note that by "traditional" argument this may be shown for some y only. This causes that [3] or [8] are not extendable to the nontransitive case.

Proof. First we show that there is $y_0 \in M_2$ such that $\Phi(S_{x_0} G^1) = S_{y_0} G^2$ for some $x_0 \in M_1$. It suffices to take any y which is not fixed by $\Phi(G_c^1)$. Then C_y is nonempty. Indeed, arguing by contradiction suppose that C_y is empty i.e. $\mathcal{C}_y$ is an open cover of M_1 by balls U_i such that $G^1(U_i)_0 \subset F_y^1$ for each i. Then we have $G_0^1 \subset F_y^1$, since for any $f \in G_0^1$ we can choose a finite subcover $\{U_{i_k}\}$ of $\mathcal{C}_y$ such that $supp(f) \subset \bigcup_k U_{i_k}$, and we apply Axiom 1.

Now the quotient $Diff^r(M_1)_c / Diff^r(M_1)_0$ is countable by the definition of the C^∞ topology, and so is G_c^1/G_0^1. On the other hand, G_c^1/G_0^1 is mapped by a map induced by Φ onto the leaf passing through y in view of Proposition 3.7. This gives a contradiction.

End of the proof. In light of Propositions 3.4 and 3.6 and Lemma 3.2, there is a unique $x \in M_1$ such that $\Phi(S_x G^1) = S_y G^2$. We define nonempty saturated sets

$$N_1 = \{x \in M_1 : \exists y \in M_2 \ \Phi(S_x G^1) = S_y G^2\},$$

$$N_2 = \{y \in M_2 : \exists x \in M_1 \ \Phi(S_x G^1) = S_y G^2\}.$$

We have that N_2 (and similarly N_1) is open. Indeed, if $\Phi(G^1_c) \subset S_y G^2$ then cannot be $\Phi(S_x G^1) = S_y G^2$ as G^1_c is not contained in $S_x G^1$. This implies that $y \in N_2$ iff y is not fixed by $\Phi(G^1_c)$. Hence N_2 is open.

It is not hard to see [16] that we may define a unique leaf preserving homeomorphism $\phi : N_1 \to N_2$ and that Φ and ϕ are related by $\Phi(f) = \phi f \phi^{-1}$ for any $f \in G^1$.

Remark. If M_1 is compact, $G^1_c = G^1$ and $N_1 = M_1$. The proof is complete.

Denote $Z_i = M_i - N_i, i = 1, 2$. Z_i are closed sets. It suffices to prove that ∂Z_2 is empty. Suppose then that $z_2 \in \partial Z_2$ and choose a sequence $\{y_i\}_{i=1}^{\infty} \subset N_2$ tending to z_2. Set $x_i = \phi^{-1}(y_i)$. Then, passing if necessary to a subsequence, we have the following three possibilities: either (a) x_i tends to $x_0 \in N_1$, or (b) x_i tends to $z_1 \in Z_1$, or (c) x_i has no convergent subsequences. In each case we shall obtain a contradiction in the following way.

(a) y_i tends to a point of N_2 as ϕ is a homeomorphism.

(b) We apply Axiom 8. Choose $f_1 \in G^1_c$ such that $f_1(z_1) \neq z_1$. Set $x^*_i = f_1(x_i)$, $y^*_i = \phi(x^*_i)$. Due to the argument from the beginning of the proof, $f_2(z_2) = z_2$ where $f_2 = \Phi(f_1)$, and y^*_i tends to z_2. Observe that we can assume that f_2 preserves the orientation at z_2 because we can take f_1 sufficiently near the identity so that $f_1 = h_1^2$ and $f_2 = h_2^2$. On the other hand, by the lemma, there is $g_2 \in G^2_c$, $g_2 = f_2$ on a neighborhood of z_2. Then for $g_1 = \Phi^{-1}(g_2)$ we get $g_1(x_i) = x^*_i$ and consequently $g_1(z_1) \neq z_1$, which contradicts the definition of Z_1 (the elements of Z_1 must be fixed by $\Phi^{-1}(G^2_c)$).

(c) Choose $f_2 \in G^2$ such that $f_2(z_2) \neq z_2$. Setting $y^*_i = f_2(y_i)$ and $x^*_i = \phi^{-1}(y^*_i)$, we can assume that x^*_i has no convergent subsequences; otherwise (a) or (b) can be applied. Passing if necessary to a subsequence one can define by induction $g_k \in G^1$ such that

$$g_k(x_{2k}) = x^*_{2k} \quad \text{and} \quad x_i, x^*_i \notin \mathit{supp}(g_k) \quad \text{for} \quad i \neq 2k,\ k = 1, 2, \dots .$$

We can arrange so that the family $\{\mathit{supp}(g_k)\}$ is pairwise disjoint and locally finite, and we apply Axiom 9. There is a diffeomorphism g such that $g = g_1 g_2 \cdots$. By the definition we get that $\Phi(g)$ cannot be continuous at z_2. This contradiction terminates the proof.

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