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MODIFICATIONS OF CLOSURE COLLECTIONS

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Z. FROLÍK has introduced the notion of projective topologization of a presheaf ([1] p. 59). In this paper these modifications are studied.

Let $\mathscr{S} = \{(S_U, \tau_U) | \varrho_{UV} | X\}$ be a presheaf of closure spaces (τ_U is a closure in S_U and $\varrho_{UV} : (S_U \tau_U) \to (S_V \tau_V)$ is continuous), $\mu = \{\tau_U\}$ its closure collection. If U is open and $\mathscr{V}$ is an open cover of U, we have a set $\Delta_{\mathscr{V}} = \{\varrho_{UV} \mid \varrho_{UV} : S_U \to (S_V \tau_V), V \in \mathscr{V}\}$ of maps from S_U into the closure spaces $(S_V \tau_V)$, $V \in \mathscr{V}$ (the closure τ_U in S_U is not considered now). Let $\tau_{U\mathscr{V}}$ be the closure in S_U defined by the maps from $\Delta_{\mathscr{V}}$ projectively. The closure collection μ is called projective if $\tau_U = \tau_{U\mathscr{V}}$ for every U and every open cover $\mathscr{V}$ of U.

In (1.1.6) we prove that for every μ there exists the finest projective collection μ' coarser than μ. (This assertion is without proof also in [1] p. 59). The main result is Theorem 1.1.37 which shows how we can get the projective modification μ' of μ in case of locally compact X and finitely projective collection μ. From this follows a methode of construction of the modification μ' for an arbitrary μ and moreover the characterization of projective collections (see 1.1.43, 45).

In 1.2.19 we show that for every presheaf $\mathscr{S}$ over a locally compact X with a projective closure collection there exists (under certain reasonable assumptions) a natural cofiltration. In 1.1.21–26 we get a method of construction of various projective collections.

1. Projective modifications.

1.1.1. Definitions, notations. For a presheaf $\mathscr{S} = \{(S_U \tau_U); \varrho_{UV}; X\}$ of closure spaces let us set

$$\mu = \{\tau_U; U\} \tag{1.1.2}$$

or briefly $\mu = \{\tau_U\}$. The collection μ is called *closure collection of* $\mathscr{S}$, or briefly *collection*.

A. The set of all open subsets of a topological space X is denoted by $\mathscr{B}(X)$.

B. Let t, t' be two closures in a set Y. If t is finer than t' we write $t \leqq t'$. If $\mu = \{\tau_U\}$, $\nu = \{\tau'_U\}$ are two closure collections of $\mathscr{S}$ we write $\mu \leqq \nu$ if $\tau_U \leqq \tau'_U$ for every $U \in \mathscr{B}(X)$. Let $\mathscr{M}$ be a nonempty set of closures in Y. Then the finest closure in Y coarser than every $t \in \mathscr{M}$ is denoted by $\varinjlim_{t \in \mathscr{M}} t$-briefly $\varinjlim \mathscr{M}$. Similarly the lower bound of $\mathscr{M}$ in the set of closures in Y is denoted by $\varprojlim_{t \in \mathscr{M}} t$. If Ω is a nonempty family of collections of thepresheaf $\mathscr{S}$ then by $\varprojlim \Omega$ resp. $\varinjlim \Omega$ is denoted the closure collection $\mu^1 = \{\varprojlim_{\mu \in \Omega} \tau_U^\mu\}$ resp. $\mu^2 = \{\varinjlim_{\mu \in \Omega} \tau_U^\mu\}$. μ^1 and μ^2 are again closure collections. It follows from the commutative diagram

$$(1.1.3) \qquad \begin{array}{ccc} (S_U, \varprojlim_\mu \tau_U^\mu) & \xrightarrow{\varrho^1_{UV}} & (S_V, \varprojlim_\mu \tau_V^\mu) \\ \downarrow i_U & & \downarrow i_V \\ (S_U, \tau_U^\mu) & \xrightarrow{\varrho_{UV}} & (S_V, \tau_V^\mu) \\ \downarrow j_U & & \downarrow j_V \\ (S_U, \varinjlim_\mu \tau_U^\mu) & \xrightarrow{\varrho^2_{UV}} & (S_V, \varinjlim_\mu \tau_V^\mu) . \end{array}$$

ϱ^1_{UV} is continuous iff for every $\mu \in \Omega$ the map $i_V \circ \varrho^1_{UV}$ is. But $i_V \circ \varrho^1_{UV} = \varrho_{UV} i_U$, where both components on the right are continuous. Similarly one can to prove the continuity of ϱ^2_{UV}.

C. Let $\{(X_\alpha, \tau_\alpha) \mid \alpha \in A\}$ be a nonempty family of closure spaces, X a set, and for every $\alpha \in A$ let φ_α be a map $\varphi_\alpha : (X_\alpha, \tau_\alpha) \to X$ resp. $X \to (X_\alpha, \tau_\alpha)$. Then the closure defined in X by the maps $\varphi_\alpha, \alpha \in A$ projectively (inductively) will be denoted by $\varprojlim_\alpha \tau_\alpha)$ $(\varinjlim_\alpha \tau_\alpha)$.

D. Let $U \in \mathscr{B}(X)$, $\alpha \in U$ and let $\mathscr{F}_\alpha$ be the stalk over α in the covering space of $\mathscr{S}$. Then there exists a natural map $\xi_{U\alpha} : S_U \to \mathscr{F}_\alpha$ such that $a \in S_U : \xi_{U_\alpha}(a) =$ *germ of a over* α. Then if $A \subset U$ is an arbitrary subset, we may put $\xi_{UA}(a) = \bigcup_{\alpha \in U} \xi_{U\alpha}(a)$, and more generaly, if $M \subset S_U$ is an arbitrary subset $\xi_{UA}(M) = \bigcup_{a \in M} \xi_{UA}(a)$. Thus for example $\xi^{-1}_{UA}(M) = \{a \mid a \in S_U, \xi_{U\alpha}(a) \in M, \alpha \in A\}$.

E. We say $\mathscr{S} = \{S_U, \tau_U); \varrho_{UV}; X\}$ is projective if the following condition holds: If $U = \bigcup_\alpha V_\alpha$, $U, V_\alpha \in \mathscr{B}(X)$ and if there exist the elements $a_\alpha \in S_{V_\alpha}$ such that for $V_\alpha \cap V_\beta$ we have $\varrho_{V_\alpha V_\alpha \cap V_\beta}(a_\alpha) = \varrho_{V_\beta V_\alpha \cap V_\beta}(a_\beta)$, then there exists $a \in S_U$ such that $\varrho_{UV_\alpha}(a) = a_\alpha$ for all α.

F. We say $\mathscr{S}$ is a presheaf with the unique continuation if the following conditions are satisfied:

1. X is locally connected,
2. if $U \in \mathscr{B}(X)$ is connected, $a, b \in S_U$, $\zeta_{U\alpha}(a) = \zeta_{U\alpha}(b)$ for some $\alpha \in U$, then $a = b$.

G. When speaking about a compact subspace in a topological space X we suppose that X is Hausdorff space.

H. Let $U \in \mathscr{B}(X)$. The set of all open coverings (finite open coverings) of U will be denoted by $\Pi_U(\Pi_U^0)$.

I. For a set Y let us denote by d, (h) the discrette, (acrette) topology in Y.

J. Let (X, t) be a closure space, M its subset. Then every filter base of t-neighborhoods of M will be denoted by $\Delta(M; t)$.

1.1.4. Definition, notation. We say, that $\mu = \{\tau_U\}$ is projective if for any $U \in \mathscr{B}(X)$ and any covering $\mathscr{V} \in \Pi_U$ we have

$$\tau_U = \varprojlim_{V \in \mathscr{V}} \tau_V \quad \text{(see 1.1.1.C)}. \tag{1.1.5}$$

Let us denote $\mu_h = \{\tau_U; \tau_U = h,\ U \in \mathscr{B}(X)\}$ resp. $\mu_d = \{\tau_U; \tau_U = d,\ U \in \mathscr{B}(X)\}$ coarse resp. fine collection (*see* (1.1.1.I)). If μ is a collection, let us denote $\Omega(\mu)$ the set of all projective collections coarser than μ. There is $\Omega(\mu) \neq \emptyset$, for $\mu_h \in \Omega(\mu)$.

1.1.6. Proposition. $\mu' = \varprojlim \Omega(\mu) \in \Omega(\mu)$.

Proof. Because μ' is again a collection, it suffices to verify (1.1.5). Let $U \in \mathscr{B}(X)$, $a \in S_U$, $\tau'_U \in \mu'_U$. Then any τ'_U-neighborhood W of a is of the form $W = \bigcap_{i=1}^{n} W_i$, for some $W_i \in \Delta(a; \tau^i_U)$, where $\tau^i_U \in \mu^i \in \Omega(\mu)$; $i = 1, \ldots, n$. Let $\mathscr{V} \in \Pi_U$. Any W_i is the intersection of a finite number of sets of the form $\varrho^{-1}_{UV}(W^V)$ for $V \in \mathscr{V}$, where $W^V \in \Delta(\varrho_{UV}(a); \tau_V)$, because $\mu_1, \ldots, \mu_n$ are projective. But any τ^i_V-neighborhood of $\varrho_{UV}(a)$ of is also a τ'_V-neighborhood of $\varrho_{UV}(a)$. Therefore W is a finite intersection of sets of the form $\varrho^{-1}_{UV}(W^V)$, $V \in \mathscr{V}$ where $W^V \in \Delta(\varrho_{UV}(a); \tau'_V)$.

1.1.7. Definition. The collection $\varprojlim \Omega(\mu)$ will be called *projective modification of* μ and will be denoted by μ'.

We can see that to every μ there exists its projective modification (see also [1]).

1.1.8. Notation. Let $\mu = \{\tau_U\}$ be a collection. For any $U \in \mathscr{B}(X)$ let us set

$$\tau_{U,\mathscr{V}} = \varprojlim_{V \in \mathscr{V}} \tau_V \quad \text{for} \quad \mathscr{V} \in \Pi_U, \tag{1.1.9}$$

$$\tau^*_U = \varinjlim_{\mathscr{V} \in \Pi_U} \tau_{U,\mathscr{V}}; \quad \mu^* = \{\tau^*_U; U \in \mathscr{B}(X)\} \quad \text{(see 1.1.1.C)}. \tag{1.1.10}$$

1.1.11. Definition. Let $U, V \in \mathscr{B}(X)$, $V \subset U$, $\mathscr{V}_1 \in \Pi_U$, $\mathscr{V}_2 \in \Pi_V$. We say, that $\mathscr{V}_2$ *refines* $\mathscr{V}_1$, if to any $V_2 \in \mathscr{V}_2$ there is $V_1 \in \mathscr{V}_1$ such that $V_2 \subset V_1$.

1.1.12. Notation. Let $U, V \in \mathscr{B}(X)$, $V \subset U$, $\mathscr{V}_1 \in \Pi_U$. Let us set

$$\mathscr{V}_1' = \{V \cap V_1;\, V_1 \in \mathscr{V}_1\} = \operatorname{ind}_V \mathscr{V}_1\,, \tag{1.1.13}$$

$$\operatorname{mod}_V \mathscr{V}_1 = \mathscr{V}_1 \cup \operatorname{ind}_V \mathscr{V}_1\,. \tag{1.1.14}$$

Clearly there is $\operatorname{ind}_V \mathscr{V}_1 \in \Pi_V$, $\operatorname{mod}_V \mathscr{V}_1 \in \Pi_U$ and they both refine $\mathscr{V}_1$.

1.1.15. Proposition. *Let $U, V \in \mathscr{B}(X)$, $V \subset U$, $\mathscr{V}_1 \in \Pi_U$, $\mathscr{V}_2 \in \Pi_V$ and $\mathscr{V}_2$ refines $\mathscr{V}_1$. Then the map $\varrho_{UV}\colon (S_U, \tau_{U,\mathscr{V}_1}) \to (S_V, \tau_{V,\mathscr{V}_2})$ is continuous.*

Proof. Let us consider a commutative diagram with $V_1 \in \mathscr{V}_1$, $V_2 \in \mathscr{V}_2$ and $V_2 \subset V_1$:

$$\begin{array}{ccc} (S_U, \tau_{U,\mathscr{V}_1}) & \xrightarrow{\varrho_{UV_1}} & (S_{V_1}, \tau_{V_1}) \\ \Big\downarrow \varrho_{UV} & & \Big\downarrow \varrho_{V_1V_2} \\ (S_V, \tau_{V,\mathscr{V}_2}) & \xrightarrow{\varrho_{VV_2}} & (S_{V_2}, \tau_{V_2})\,. \end{array} \tag{1.1.16}$$

According to (1.1.9) ϱ_{UV} is continuous and thus the assertion (1.1.15) holds iff for any $V_2 \in \mathscr{V}_2 \varrho_{VV_2} \circ \varrho_{UV}$ is continuous. But with respect to (1.1.16) this map coincides with $\varrho_{V_1V_2} \circ \varrho_{UV_1}$, where both components are continuous (see (1.1.9)).

Now let us notice, how the τ_U^*-neighborhoods of a look like. Let $U \in \mathscr{B}(X)$, $a \in S_U$. For any $\mathscr{V} \in \Pi_U$ let

$$W(\mathscr{V}) = \{V_1, \ldots, V_n\} \subset \mathscr{V} \tag{1.1.17}$$

be a finite choice of sets from $\mathscr{V}$. To any $V_i \in W(\mathscr{V})$ let us assign the uniquely determined $W^{V_i} \in \Delta(\varrho_{UV_i}(a); \tau_{V_i})$ and let us denote for such chosen W^{V_i}

$$\mathscr{R}(W(\mathscr{V})) = \{W^{V_1}, \ldots, W^{V_n}\}\,, \tag{1.1.18}$$

$$\mathscr{P} = \mathscr{P}(\mathscr{R}(W(\mathscr{V})), W(\mathscr{V})) = \bigcap_{i=1}^{n} \varrho_{UV_i}^{-1}(W^{V_i})\,. \tag{1.1.19}$$

Then $\mathscr{P} \in \Delta(a, \tau_{U,\mathscr{V}})$. To every $\mathscr{V} \in \Pi_U$ let us construct some $\mathscr{P}(\mathscr{R}(W(\mathscr{V})), W(\mathscr{V}))$ and let us form

$$Q(\mathscr{R}(W(\mathscr{V})), W(\mathscr{V})) = \bigcup_{\mathscr{V} \in \Pi_U} \mathscr{P}(\mathscr{R}(W(\mathscr{V})), W(\mathscr{V}))\,. \tag{1.1.20}$$

1.1.21. Proposition. *Let $\mu = \{\tau_U\}$ be a collection.*

A. *For all $U \in \mathscr{B}(X)$ there is $\tau_U \leqq \tau_U^*$.*

B. *The maps $\varrho_{UV}^*\colon (S_U, \tau_U^*) \to (S_V, \tau_V^*)$ are all continuous and therefore μ^* is a collection.*

Proof. Every $\mathscr{V} \in \Pi_U$ refines $\mathscr{V}^+ = \{U\} \in \Pi_U$. By (1.1.15) every $\tau_{U,\mathscr{V}}$ is coarser than τ_U, which with (1.1.10) proves A. To prove B let us notice commutative diagram for $U, V \in \mathscr{B}(X)$, $V \subset U$, $\mathscr{V} \in \Pi_U$:

$$\begin{array}{ccccc} (S_U, \tau_{U,\mathscr{V}}) & \xrightarrow{i_U} & (S_U, \tau_U, \mathrm{mod}_V \mathscr{V}) & \xrightarrow{\varrho_{UV}} & (S_V, \tau_V, \mathrm{ind}_V \mathscr{V}) \\ \downarrow{\scriptstyle i_{U,\mathscr{V}}} & & & \swarrow{\scriptstyle i_V^*} & \\ (S_U, \tau_U^*) & \xrightarrow{\varrho_{UV}^*} & (S_V, \tau_V^*) & & \end{array} \tag{1.1.22}$$

The map ϱ_{UV}^* is continuous and thus the assertion B holds iff for any $\mathscr{V} \in \Pi_U$, $\varrho_{UV}^* i_{U,\mathscr{V}}$ is continuous, for (1.1.1) holds. But every $\varrho_{UV}^* i_{U,\mathscr{V}}$ by (1.1.22) coincides with $i_V^* \varrho_{UV} i_U$. Here all the components are continuous maps according to (1.1.10, 12, 15).

1.1.23. Corollary. *For a collection μ there is $\mu \leqq \mu^* \leqq \mu'$.*

Proof. Let $U \in \mathscr{B}(X)$, $\mathscr{V} \in \Pi_U$, and let us denote $\mu' = \{\tau'_U\}$. Because every τ'_V, $V \in \mathscr{V}$ is coarser than τ_V and μ' is (by (1.1.6)) projective, $\varprojlim_{V \in \mathscr{V}} \tau'_V = \tau'_U$ is coarser than $\varprojlim_{V \in \mathscr{V}} \tau_V = \tau_{U,\mathscr{V}}$, which with (1.1.10) finishes the proof.

1.1.24. Remark. The equality $\mu = \mu'$ holds iff $\mu = \mu^*$. If $\mu = \mu'$, then by (1.1.23) $\mu = \mu^*$. If $\mu = \mu^*$, then for all U we have $\tau_U = \tau_U^*$. By (1.1.15) τ_U is finer than any $\tau_{U,\mathscr{V}}$, which is by (1.1.10) finer than τ_U^*. If $\tau_U^* = \tau_U$, there is $\tau_U = \tau_{U,\mathscr{V}}$ for any $\mathscr{V} \in \Pi_U$ and this is (1.1.5). Therefore μ is projective, i.e. $\mu = \mu'$.

1.1.25. Corollary. *If $(\mu^*)^* = \mu^*$, there is $\mu^* = \mu'$. From the supposed equality it follows by* (1.1.24), *that μ^* is a projective collection. Finally from* (1.1.6,23) *we have $\mu^* = \mu'$.*

1.1.26. Definition. We say, that the collection $\mu = \{\tau_U\}$ is *finitely projective*, if for any $U \in \mathscr{B}(X)$, $\mathscr{V} \in \Pi_U^0$ (see (1.1.1) the following holds:

$$\tau_U = \varprojlim_{V \in \mathscr{V}} \tau_V \quad \text{(see 1.1.1.)C}. \tag{1.1.27}$$

1.1.28. Proposition. *To every collection μ there exists a collection μ^+ such that*

(a) $\mu \leqq \mu^+$,
(b) μ^+ *is finitely projective,*
(c) *if ν is a collection satisfying* (a), (b), *then $\mu^+ \leqq \nu$.*

Proof. Let us denote by $\tilde{\Omega}(\mu)$ the set of all collections satisfying (a), (b). This set is nonempty, because $\mu_h \in \tilde{\Omega}(\mu)$ (see (1.1.4)). Let us set $\mu_1 = \varprojlim \tilde{\Omega}(\mu)$, which is again a collection. The fact, that $\mu_1 \in \tilde{\Omega}(\mu)$ can be proved as in (1.1.6). Therefore $\mu^+ = \mu_1$ is the required collection.

1.1.29. Proposition. *Let μ be a collection, $\mu^+ = \{\tau_U^+\}$. Then for any $U \in \mathscr{B}(X)$ there is*

$$\tau_U^+ = \varinjlim_{\mathscr{V} \in \Pi_U{}^0} \tau_{U,\mathscr{V}} \quad \text{(see 1.1.1.H)}. \tag{1.1.30}$$

Proof. Let us set $\tau_U^- = \varinjlim_{\mathscr{V} \in \Pi_U{}^0} \tau_{U,\mathscr{V}}$. Let $a \in S_U$, $\mathscr{V}_1 \in \Pi_U^0$, $W \in \Delta(a; \tau_U^-)$. Thus we have

$$W = \bigcup_{\mathscr{V} \in \Pi_U{}^0} \bigcap_{V \in \mathscr{V}} \varrho_{UV}^{-1}(W_{\mathscr{V}}^V) \tag{1.1.31}$$

where $W_{\mathscr{V}}^V \in \Delta(\varrho_{UV}(a), \tau_V)$ for $V \in \mathscr{V}$. For the sake of simplicity we can suppose, that $\mathscr{V}_1 = (V_1, V_2)$. If $\mathscr{V}^i \in \Pi_{V_i}^0$, $i = 1, 2$, then $\mathscr{V}^{12} = \mathscr{V}^1 \cup \mathscr{V}^2 \in \Pi_U^0$. For $V \in \mathscr{V}^i$ let us put $\tilde{W}_{\mathscr{V}^i}^V = W_{\mathscr{V}^{12}}^V$. For any pair $(\mathscr{V}^1, \mathscr{V}^2)$ (where $\mathscr{V}^i \in \Pi_{V_i}^0$, $i = 1, 2$) let us form $\mathscr{V}^{12} = \mathscr{V}^1 \cup \mathscr{V}^2$ and for $V \in \mathscr{V}^{12}$ let us form $\tilde{W}_{\mathscr{V}^i}^V$, $i = 1, 2$, in the just described way. Then we have

$$\varrho_{UV_1}^{-1} \bigcup_{\mathscr{V}^1 \in \Pi_{V_1}^0} \bigcap_{V \in \mathscr{V}^1} \varrho_{V_1 V}^{-1}(\tilde{W}_{\mathscr{V}^1}^V) \cap \varrho_{UV_2}^{-1} \bigcup_{\mathscr{V}^2 \in \Pi_{V_2}^0} \bigcap_{V \in \mathscr{V}^2} \varrho_{V_2 V}^{-1}(\tilde{W}_{\mathscr{V}^2}^V) \subset$$

$$\subset \bigcup_{\mathscr{V}^1 \in \Pi_{V_1}^0, \mathscr{V}^2 \in \Pi_{V_2}^0} \bigcap_{V \in \mathscr{V}^1 \cup \mathscr{V}^2} \varrho_{UV}^{-1}(\tilde{W}_{\mathscr{V}^1 \cup \mathscr{V}^2}^V) \subset \bigcup_{\mathscr{V} \in \Pi_U^0} \bigcap_{V \in \mathscr{V}} \varrho_{UV}^{-1}(W_{\mathscr{V}}^V) = W.$$

Here the sets $M_i = \bigcup_{\mathscr{V}^i \in \Pi_{V_i}^0} \bigcap_{V \in \mathscr{V}^i} \varrho_{V_i V}^{-1}(\tilde{W}_{\mathscr{V}^i}^V)$ are $\tau_{V_i}^-$-neighborhoods of $\varrho_{UV_i}(a)$, $i = 1, 2$. Thus

$$\varrho_{UV_1}^{-1}(M_1) \cap \varrho_{UV_2}^{-1}(M_2) \subset W. \tag{1.1.32}$$

In the same way as in (1.1.21B) we can prove the continuity of all maps $\varrho_{UV} : (S_U, \tau_U^-) \to (S_V, \tau_V^-)$. This with (1.1.32) proves, that $\mu^- = \{\tau_U^-\}$ is finitely projective. Because $\tau_U \leqq \tau_U^- \leqq \tau_U^+$ for every $U \in \mathscr{B}(X)$ (which follows easily from the definition of τ_U^-), it is necessarily $\tau_U^+ = \tau_U^-$ for all $U \in \mathscr{B}(X)$.

1.1.33. Definition. Let μ be a collection. Then the collection μ^+ is called *finite projective modification of μ.*

1.1.34. Remark. The assertions (1.1.28,29) show, how μ^+ looks like, whereas (1.1.28) gives only the existence, but no so good picture. The assignment $\mu \to \mu^+$ is a map of the set of all collections into itself. Its fixed points are precisely all finitely projective collections.

1.1.35. Notation. For $U \in \mathscr{B}(X)$, $a \in S_U$ let us denote

$$\mathscr{B}(a) = \{\varrho_{UV}^{-1}(W^V);\ V \in \mathscr{B}(U),\ \bar{V} \subset U \ \text{is compact},\ W^V \in \Delta(\varrho_{UV}(a); \tau_V)\}. \tag{1.1.36}$$

It is clear that $\mathscr{B}(a)$ is a base of the filter round a in S_U. These bases form there a closure, which we denote by $\tilde{\tau}_U$. The set $\{\tilde{\tau}_U; U \in \mathscr{B}(X)\} = \tilde{\mu}$ is clearly a collection, coarser than μ.

1.1.37. Theorem. *Let X be locally compact, $\mathscr{S} = \{(S_U, \tau_U), \varrho_{UV}; X\}$ a presheaf over X and $\mu = \{\tau_U\}$ its closure collection. If $\mu = \mu^+$, then $\mu' = \mu^* = \tilde{\mu}$.*

Proof. We shall prove, that $\tilde{\mu}$ is projective, and finer than μ^*. Then (1.1.23,6) imply $\mu' = \tilde{\mu} = \mu^*$. Let $U \in \mathscr{B}(X)$, $a \in S_U$, and let

$$(1.1.38) \qquad W = \bigcup_{\mathscr{V} \in \Pi_U} \bigcap_{V \in W(\mathscr{V}) \subset \mathscr{V}} \varrho_{UV}^{-1}(W_{\mathscr{V}}^V) \in \Delta(a; \tau_U^*) - (\text{see } (1.1.20)) .$$

A local compactness of X and (1.1.15) allows us to restrict ourselves in (1.1.38) only to the union over those $\mathscr{V} \in \Pi_U$, which consists of relatively compact sets in U. Let us choose such relatively compact covering $\mathscr{V} \in \Pi_U$ and let us take a component in the union (1.1.38), which corresponds to it. That is

$$(1.1.39) \qquad W^{\mathscr{V}} = \bigcap_{i=1}^{n} \varrho_{UV_i}^{-1}(W_{\mathscr{V}}^{V_i}) .$$

Then $V = V_1 \cup \ldots \cup V_n$ is in U relatively compact, further $\tilde{W}^V = \bigcap_{i=1}^{n} \varrho_{VV_i}^{-1}(W_{\mathscr{V}}^{V_i}) \in$ $\in \Delta(\varrho_{UV}(a); \tau_V)$, $\varrho_{UV}^{-1}(\tilde{W}^V) \subset W^{\mathscr{V}} \subset W$ and $\varrho_{UV}^{-1}(\tilde{W}^V) \in \mathscr{B}(a)$. Therefore $\mathscr{B}(a) \leqq$ $\leqq \Delta(a; \tau_U^*)$. Let $\varrho_{UV}^{-1}(W^V) \in \mathscr{B}(a)$, $\mathscr{V} \in \Pi_U$. There exist $V_1, \ldots, V_n \in \mathscr{V}$ which cover $\bar{V}$. From the local compactness of X follows the existence of open sets $R_1, \ldots, R_n$, such that $\bar{R}_i \subset V_i$, $\bar{R}_i$ is compact, $i = 1, \ldots, n$, and $R_1 \cup \ldots \cup R_n = V$. Because μ is finitely projective, τ_V-neighborhood W^V of $\varrho_{UV}(a)$ is of the form

$$(1.1.40) \qquad W^V = \bigcap_{i=1}^{n} \varrho_{VR_i}^{-1}(W^{R_i}) ,$$

for some $W^{R_i} \in \Delta(\varrho_{UR_i}(a); \tau_{R_i})$, $i = 1, \ldots, n$. The sets $B_i = \varrho_{V_iR_i}^{-1}(W^{R_i})$ belong to $\mathscr{B}(\varrho_{UV_i}(a))$, $i = 1, \ldots, n$. Hence $\varrho_{UV}^{-1}(W^V) = \bigcap_{i=1}^{n} \varrho_{UV_i}^{-1}(B_i)$. Therefore we have proved that for the closure $\tilde{\tau}_U$ there is $\tilde{\tau}_U = \varinjlim_{V \in \mathscr{V}} \tilde{\tau}_V$ for all $\mathscr{V} \in \Pi_U$. This finishes the proof.

1.1.41. Corollary. *Let X be locally compact, μ a collection. Then $(\mu^+)^* = \mu'$. For $\mathscr{V} \in \Pi_U^0$ there is*

$$(1.1.42) \qquad \tau_U \leqq \tau_{U,\mathscr{V}} \leqq \tau_U^+ \leqq \tau_U^* \leqq \tau_U' ,$$

which follows immediately from the definitions of $\tau_{U,V}$ and τ_U^* in (1.1.9,10). *Therefore $(\tau_U^+)' = \tau_U'$, and because of $(\tau_U^+)^* = (\tau_U^+)'$, we have $\tau_U' = (\tau_U^+)^*$.*

1.1.43. Remark. If X is locally compact, then the collection μ can be projectively modified in two steps. First we do the finite projective modification μ^+ following

(1.1.29), and then the modification $(\mu^+)^*$ of μ^+. But it we need not do in the complicated and for the further progress unconvenient way described in (1.1.17–20), but in the more clear and easy to surwey way with help of bases $\mathscr{B}(a)$ from (1.1.36).

1.1.44. Corollary. *If X is locally compact and $\mu = \mu'$, then for $U \in \mathscr{B}(X)$ and $a \in S_U$ the bases $\mathscr{B}(a)$ and $\Delta(a; \tau_U)$ are equivalent.*

1.1.45. Remark. We get the following description of the projective collections μ for locally compact X. μ is projective iff it is finitely projective and the bases $\mathscr{B}(a)$ from (1.1.36) are bases of the filter of τ_U-neighborhoods of elements $a \in S_U$, $U \in \mathscr{B}(X)$. This follows from (1.1.44) and (1.1.37).

1.1.46. Definition. We say that a presheaf $\mathscr{S} = \{(S_U, \tau_U); \varrho_{UV}; X\}$ is *full*, if the following holds: If $U \in \mathscr{B}(X)$, $a \in S_U$, $W \in \Delta(a; \tau_U)$, then there exists $W' \in \Delta(a; \tau_U)$ such that

$$\xi_{UU}^{-1}\xi_{UU}(W') \subset W \quad \text{(see (1.1.1.D))}. \tag{1.1.47}$$

1.1.48. Remark. *If $\mathscr{S}$ is a full presheaf over a locally compact space with a projective closure collection, then for $U \in \mathscr{B}(X)$, $a \in S_U$ the set*

$$\mathrm{B} = \{\xi_{UK}^{-1}\xi_{UK}(W)\,;\ \ K \subset U \ \text{ compact}, \ \ W \in \Delta(a; \tau_U)\} \tag{1.1.49}$$

is a filter base of τ_U-neighborhoods of a.

Proof. Obviously there is $\Delta(a; \tau_U) \leqq \mathrm{B}$. Conversely let $W \in \Delta(a; \tau_U)$. By (1.1.44) there exists $V \in \mathscr{B}(X)$ (such that $\bar{V} \subset U$ is compact) and $W' \in \Delta(\varrho_{UV}(a); \tau_V)$ such that $W = \varrho_{UV}^{-1}(W')$. To W' we can find $W'' \in \Delta(\varrho_{UV}(a); \tau_V)$ such that for W' and W'' (1.1.47) holds. For $\tilde{W} = \varrho_{UV}^{-1}(W'') \in \Delta(a; \tau_U)$ there is $\xi_{U\bar{V}}^{-1}\xi_{U\bar{V}}(\tilde{W}) \subset \varrho_{UV}^{-1}\xi_{VV}^{-1}\xi_{VV}(W'') \subset \subset \varrho_{UV}^{-1}(W') = W$.

1.1.50. Examples. (1) Let $\mathscr{S} = \{(S_U, \tau_U); \varrho_{UV}; X\}$, where $X = E_n$, and for $U \in \mathscr{B}(X)$ let S_U be some set of continuous functions on U, τ_U the closure of uniform convergence and $\varrho_{UV} : f \in S_U \to f/V \in S_V$. Then for $\mu = \{\tau_U\}$ one can easily find, that

(a) $\mu^+ = \mu$,
(b) $\mu' = \mu^* = \{\tau'_U\}$,

where τ'_U for $U \in \mathscr{B}(X)$ is the closure of localy uniform convergence.

It is clear, that nothing will change in this example, if we take for X instead of E_n an arbitrary locally compact topological space.

(2) Let $\mathscr{S} = \{(S_U, \tau_U), \varrho_{UV}, X\}$ be a projective presheaf, where $\tau_U = d$ for all $U \in \mathscr{B}(X)$ (see (1.1.1.E,I))). Then

(a) $\mu^+ = \mu$,

(b) by (1.1.43) there is $\mu' = \mu^* = \{\tau'_U\}$, where $\Delta(a; \tau'_U)$ and $\mathscr{B}(a)$ from (1.1.36) – i.e. in this case

(1.1.51) $$\mathscr{B}(a) = \{\varrho_{UV}^{-1}\varrho_{UV}(a);\ V \in \mathscr{B}(U),\ \bar{V} \subset U \text{ is compact}\}$$

are equivalent. If moreover $\mathscr{S}$ is a presheaf with unique continuation (see (1.1.1.F)), and U connected, then $\mathscr{B}(a) = a$. Because $\mathscr{S}$ is projective (see (1.1.1.E)), we get $\tau'_U = d$ for every $U \in \mathscr{B}(X)$, which have finitely many components.

2. Cofiltration.

Let $\mathscr{S} = \{(S_U, \tau_U); \varrho_{UV}; X\}$ be a presheaf, X locally compact $\mu = \{\tau_U\} = \mu'$. If $U \in \mathscr{B}(X)$, $a \in S_U$, then according to (1.1.44) the base of the filter $\mathscr{B}(a)$ from (1.1.36) is a base of the filter of τ_U-neighborhoods of a. Thus if $W \in \mathscr{B}(a)$, then the following condition holds:

(1.2.1) There exists $V \in \mathscr{B}(U)$ such that $\bar{V} \subset U$ is compact, and such that $\varrho_{UV}^{-1}(W^V) \subset$ $\subset W$ for some $W^V \in \Delta(\varrho_{UV}(a); \tau_V)$.

Let us denote by $\mathscr{H}(W)$ the set of all bases of the filters $\mathscr{F}(W)$ in U, for which the following conditions hold:

(1.2.2) 1. $F \in \mathscr{F}(W) \Rightarrow F$ is compact, and there is $F = \bar{V}$ for some $V \in \mathscr{B}(U)$.
2. $\varrho_{UV}^{-1}(W^V) \subset W$ for some $W^V \in \Delta(\varrho_{UV}(a); \tau_V)$.

Let us partially order $\mathscr{H}(W)$ by inclusion. Using the maximality principle, we can easily find, that every $\mathscr{F}(W) \in \mathscr{H}(W)$ can be completed to a maximal $\mathscr{M}(W)$. For every maximal $\mathscr{M}(W) \in \mathscr{H}(W)$ let us set

(1.2.3) $$M(W) = \bigcap_{\mathscr{M}(W)} F,$$

which is a nonempty compact subset in U.

It is clear, that there could exist more sets $M(W)$, if $\mathscr{H}(W)$ has more than one maximal element. If all maximal bases $\mathscr{M}(W) \in \mathscr{H}(W)$ are equivalent, there exists the unique $M(W)$.

If $U' \in \mathscr{B}(U)$ is relatively compact in U, and $M(W) \subset U'$, then there exists (by (1.2.3)) $\tilde{\mathscr{M}}(W) \in \mathscr{H}(W)$ and $F \in \tilde{\mathscr{M}}(W)$, such that $F \subset U'$. Moreover we have $F = \bar{V}$ for some $V \in \mathscr{B}(U)$ and $\varrho_{UV}^{-1}(W^V) \subset W$ for some $W^V \in \Delta(\varrho_{UV}(a); \tau_V)$. If we set $W' =$ $= \varrho_{U'V}^{-1}(W^V)$, then $W' \in \Delta(\varrho_{UU'}(a), \tau_{U'})$ and at the same time $\varrho_{UU'}^{-1}(W') \subset W$. Thus $\bar{U}' \in \tilde{\mathscr{M}}(W)$.

1.2.4. Proposition. *Let $K \subset U$ be compact. Then $M(W) \subset K$ for some $M(W)$, iff the following condition holds: "If $U' \in \mathscr{B}(U)$, $\bar{U}' \subset U$ is compact, and $K \subset U'$, then $\bar{U}' \in \mathscr{M}(W)$ for some $\mathscr{M}(W) \in \mathscr{H}(W)$".*

Proof. If the just mentioned assumptions are satisfied, we have proved that the condition in (1.2.4) holds just before the formulation of (1.2.4). Conversely, let the condition hold. Then the set

$$\{\bar{U}'; U' \in \mathscr{B}(U),\ K \subset U',\ \bar{U}' \subset U \text{ is compact}\} = \mathscr{F}(W) \tag{1.2.5}$$

is a base $\mathscr{F}(W) \in \mathscr{H}(W)$. If we complete it to a maximal $\mathscr{M}(W)$, then $M(W) = \bigcap_{\mathscr{M}(W)} F \subset \bigcap_{\mathscr{F}(W)} F = K$.

1.2.6. Proposition. *Let $W_1, W_2 \in \Delta(a; \tau_U)$. Then to arbitrarily chosen $M(W_1)$, $M(W_2)$ there exists $M(W_1 \cap W_2)$ such that $M(W_1 \cap W_2) \subset M(W_1) \cup M(W_2)$.*

Proof. We use (1.2.4). Let $U' \in \mathscr{B}(U)$, $M(W_1) \cup M(W_2) \subset U'$. Because $M(W_i) \subset U'$, there is $\varrho_{UU'}^{-1}(W_i^{U'}) \subset W_i$ for some $W_i^{U'} \in \Delta(\varrho_{UU'}(a); \tau_{U'})$. Then $W_1 \cap W_2 \supset \varrho_{UU'}^{-1}(W_1^{U'}) \cap \varrho_{UU'}^{-1}(W_2^{U'}) = \varrho_{UU'}^{-1}(W_1^{U'} \cap W_2^{U'})$ and the proof is finished.

1.2.7. Proposition. *Let $W_1, W_2 \in \Delta(a; \tau_U)$, $W_1 \subset W_2$. Then to every $M(W_1)$ there exists $M(W_2)$, such that $M(W_2) \subset M(W_1)$.*

Proof. Let $V \in \mathscr{B}(U)$, $M(W_1) \subset V$. Then $\varrho_{UV}^{-1}(W') \subset W_1$ for some $W' \in \Delta(\varrho_{UV}(a); \tau_V)$. Then for this W', $\varrho_{UV}^{-1}(W') \subset W_2$ also holds, which proves the proposition.

1.2.8. Corollary. *Let $W_1, W_2 \in \Delta(a; \tau_U)$. Then to every $M(W_1 \cap W_2)$ there exists $M(W_1)$ and $M(W_2)$ such that $M(W_1) \cup M(W_2) \subset M(W_1 \cap W_2)$.*

Proof. Because $W_1 \cap W_2 \subset W_i$, there exists (by (1.2.7)) $M(W_i)$ such that $M(W_i) \subset M(W_1 \cap W_2)$, $i = 1, 2$. Thus $M(W_1) \cup M(W_2) \subset M(W_1 \cap W_2)$.

1.2.9. Corollary. *Let $W_1, W_2 \in \Delta(a; \tau_U)$. Then to every $M(W_1)$ and $M(W_2)$ there exists $M(W_1 \cap W_2)$ and $\tilde{M}(W_1), \tilde{M}(W_2)$ such that*

$$\tilde{M}(W_1) \cup \tilde{M}(W_2) \subset M(W_1 \cap W_2) \subset M(W_1) \cup M(W_2)\,. \tag{1.2.10}$$

Proof. It is the combination of (1.2.6,8).

1.2.11. Corollary. *Let $W_1, W_2 \in \Delta(a; \tau_U)$. Then to every $M(W_1 \cap W_2)$ there exist $\tilde{M}(W_1)$ and $\tilde{M}(W_2)$ such that $M(W_1 \cap W_2) = \tilde{M}(W_1) \cup \tilde{M}(W_2)$.*

Proof. Let us choose some $M(W_1 \cap W_2)$. According to (1.2.8) we find $\tilde{M}(W_1)$ and $\tilde{M}(W_2)$ such that $\tilde{M}(W_1) \cup \tilde{M}(W_2) \subset M(W_1 \cap W_2)$. By (1.2.6) we find to $\tilde{M}(W_1)$ and $\tilde{M}(W_2)$ a set $\tilde{M}(W_1 \cap W_2)$ such that

$$\tilde{M}(W_1 \cap W_2) \subset \tilde{M}(W_1) \cup \tilde{M}(W_2) \subset M(W_1 \cap W_2)\,, \tag{1.2.12}$$

and thus $\tilde{M}(W_1 \cap W_2) = M(W_1 \cap W_2)$, and in (1.2.12) holds everywhere the equality.

1.2.13. Assumption. Later we shall suppose:

A. If $U \in \mathscr{B}(X)$, $a \in S_U$, there exists with respect to finite intersections a closed filter base $\Delta(a)$ of the τ_U-neighborhoods of a such that for $W \in \Delta(a)$ any two maximal bases $\mathscr{M}_1(W), \mathscr{M}_2(W) \in \mathscr{H}(W)$ are equivalent.

B. If $U, V \in \mathscr{B}(X)$, $V \subset U$, $a \in S_U$, $W \in \Delta(\varrho_{UV}(a))$, then $\varrho_{UV}^{-1}(W) \in \Delta(a)$.

1.2.14. Corollary. *Let* (1.2.13) *hold. Then to any* $W_1, W_2 \in \Delta(a)$ *there is* $M(W_1 \cap W_2) = M(W_1) \cup M(W_2)$.

Proof. From the assumption about $\mathscr{H}(W)$ for $W \in \Delta(a)$ follows that it has the unique maximal element and thus there exists the unique $M(W)$, which with (1.2.11) finishes the proof.

1.2.15. Definition. A family $\mathscr{K}$ of subsets of some set L will be called *cofilter base* (resp. *cofilter*), if it is nonempty and the following holds:

(1.2.16) $$K_1, K_2 \in \mathscr{K} \Rightarrow K_1 \cup K_2 \subset K_3 \quad \text{for some} \quad K_3 \in \mathscr{K},$$

(resp. $K_1, K_2 \in \mathscr{K} \Rightarrow K_1 \cup K_2 \in \mathscr{K}$).

We say that to a presheaf $\mathscr{S} = \{(S_U, \tau_U), \varrho_{UV}, X\}$ there is given a cofiltration, if to every $U \in \mathscr{B}(X)$ and $a \in S_U$ there is given a base $\mathscr{K}_a^U$ of cofilter in U such that the following holds: „If $U, V \in \mathscr{B}(X)$, $V \subset U$, $a \in S_U$, $K \in \mathscr{K}_{\varrho_{UV}(a)}^V$, then $K \subset L$ for some $L \in \mathscr{K}_a^U$." If to $\mathscr{S}$ there is given a cofiltration $\varkappa = \{K_a^U; U \in \mathscr{B}(X), a \in S_U\}$, we shall say, that $\mathscr{S}$ is a presheaf with the cofiltration $\varkappa$.

1.2.17. Corollary. *Let* $U \in \mathscr{B}(X)$, $a \in S_U$ *and let* (1.2.13) *hold. Then the base* $\Delta(a)$ *generates in* U *a cofilter base* $\mathscr{K}_a^U$.

Proof. Let us set $\mathscr{K}_a^U = \{M(W); W \in \Delta(a)\}$. If $K_1, K_2 \in \mathscr{K}_a^U$, then $K_i = M(W_i)$ for $W_i = \Delta(a)$, $i = 1, 2$. Then $W_1 \cap W_2 \in \Delta(a)$ and $M(W_1) \cup M(W_2) = M(W_1 \cap W_2) = K_3 \in \mathscr{K}_a^U$.

We shall notice the relation betwen $\mathscr{K}_a^U$ and $\mathscr{K}_{\varrho_{UV}(a)}^V$, for $V \in \mathscr{B}(U)$. If $W \in \Delta(\varrho_{UV}(a))$, it can be easily seen, that if $\mathscr{F}(W) \in \mathscr{H}(W)$, then $\mathscr{F}(W) \in \mathscr{H}(\varrho_{UV}^{-1}(W))$. Thus $M(\varrho_{UV}^{-1}(W)) \subset M(W)$. For the proof of the conversed inclusion we need.

1.2.18. Assumption. *Let* $U, V \in \mathscr{B}(X)$, $V \subset U$, $a \in S_U$, $W \in \Delta(\varrho_{UV}(a))$. *If for some* $V' \in \mathscr{B}(V)$ *and some* $W^{V'} \in \Delta(\varrho_{UV'}(a))$ *there is* $\varrho_{UV}^{-1}(W^{V'}) \subset \varrho_{UV}^{-1}(W)$, *then* $\varrho_{VV'}^{-1}(\tilde{W}^{V'}) \subset W$ *for some* $\tilde{W}^{U'} \in \Delta(\varrho_{UV'}(a))$.

If (1.2.18) holds, then $M(\varrho_{UV}^{-1}(W)) = M(W)$. If there were $M(\varrho_{UV}^{-1}(W)) \subsetneqq M(W)$, there would be $M(\varrho_{UV}^{-1}(W)) \subset U'$, $M(W) \not\subset U'$ for some $U' \in \mathscr{B}(U)$. Then for some $W^{U'} \in \Delta(\varrho_{UU'}(a))$ there is $\varrho_{UU'}^{-1}(W^{U'}) \subset \varrho_{UV'}^{-1}(W)$. By (1.2.18) we have $\varrho_{VU'}^{-1}(\tilde{W}^{U'}) \subset W$ for some $\tilde{W}^{U'} \in \Delta(\varrho_{UU'}(a))$ and thus $M(W) \subset U'$-contradiction.

1.2.19. Corollary. *Because* (1.2.13) *holds, we can to every* $a \in S_U$ *assign the cofilter base* $\mathscr{K}^U_a$ *in* U. *If* (1.2.18) *holds, then*

$$K^V_{\varrho_{UV}(a)} \subset \mathscr{K}^U_a . \tag{1.2.20}$$

Thus to a presheaf $\mathscr{S} = \{(S_U, \tau_U), \varrho_{UV}; X\}$ with a projective closure collection, which fulfils (1.2.13,18), there exists a natural cofiltration founded by the bases $\mathscr{K}^U_a$ from (1.2.17). It can be easily seen, that this cofiltrations uniquely depends on the choice of the bases $\Delta(a)$ from (1.2.13). For any other choice we could get other natural cofiltration.

If $U \in \mathscr{B}(X)$, $a \in S_U$, $W \in \Delta(a)$, then by (1.2.13) the set

$$\tilde{\mathscr{F}}_K(a) = \{W';\ W' \in \Delta(a),\ K = M(W') = M(W)\} \tag{1.2.21}$$

is a filter base in S_U round a. Then the set

$$\mathscr{F}_K(a) = \{\xi_{UK}(W);\ W \in \tilde{\mathscr{F}}_K(a)\} \tag{1.2.22}$$

is a filter base round $\mathrm{gr}_K\, a$ in $\psi^{-1}(K)$ (see (0.19,20)).

1.2.23. Proposition. *Let* $U, V \in \mathscr{B}(X)$, $V \subset U$, $a \in S_U$, $K \in \mathscr{K}^V_{\varrho_{UV}(a)}$, $L \in \mathscr{K}^U_a$, $K \subset L$. *Then the filter base* $\mathscr{F}_L(a) \cap \psi^{-1}(K)$ *majorizes the base* $\mathscr{F}_K(a)$.

Proof. Let $F_1 = \xi_{UK}(W_1) \in \mathscr{F}_K(a)$ for some $W_1 \in \tilde{\mathscr{F}}_K(\varrho_{UV}(a))$. Let us choose $F_2 = \xi_{UL}(W_2) \in \mathscr{F}_L(a)$ arbitrarily. By (1.2.13B) there is $W = \varrho_{UV}^{-1}(W_1) \cap W_2 \in \Delta(a)$. From (1.2.18) we have $M(W) = M(\varrho_{UV}^{-1}(W_1)) \cup M(W_2) = K \cup L = L$ and thus $W \in \tilde{\mathscr{F}}_L(a)$. Therefore $F = \xi_{UL}(W) \in \mathscr{F}_L(a)$ and $F \cap \psi^{-1}(K) \subset F_1$.

Conversely from (1.2.19,23) we come to the following: If we assign in every $U \in \mathscr{B}(X)$ to every $a \in S_U$ a cofilter base $\mathscr{K}^U_a$ such that every $K \in \mathscr{K}^U_a$ is compact, and if we define moreover for every $K \in \mathscr{K}^U_a$ a filter base $\mathscr{F}_K(a)$ in $\psi^{-1}(K)$ round the set $\mathrm{gr}_K\, a$, such that (1.2.23) holds, we can set (see (0.19))

$$\mathscr{B}(a) = \{\xi_{UK}^{-1}(F);\ F \in \mathscr{F}_K(a),\ K \in \mathscr{K}^U_a\} . \tag{1.2.24}$$

From (1.2.20,19,23) follows easily, that $\mathscr{B}(a)$ is a filter base round a in S_U. These bases form in every S_U a closure $p_{\{\mathscr{K}_a U\}}$ (briefly $p_{\mathscr{K}U}$). The family $\mu_{\mathscr{K}} = \{p_{\mathscr{K}U}\}$ is a closure collection, because as a result from (1.2.20) the all maps $\varrho_{UV} : (S_U, p_{\mathscr{K}U}) \to (S_V, p_{\mathscr{K}V})$ are continuous.

If we moreover take the cofilters $\mathscr{K}^U_a$ such that

$$K \in \mathscr{K}^U_a , \quad K \subset U_1 \cup \ldots \cup U_n ; \quad U_i \in \mathscr{B}(U) , \tag{1.2.25}$$
$$b_i = \varrho_{UU_i}(a) , \quad i = 1, \ldots, n \Rightarrow \text{there exists } K_i \in \mathscr{K}^{U_i}_{b_i} \text{ such that } K = \bigcup_{i=1}^{n} K_i ,$$

and the bases $\mathscr{F}_K(a)$ such that for $L \in \mathscr{K}^U_a$, $K \in \mathscr{K}^V_{\varrho_{UV}(a)}$, $K \subset L$ the bases $\mathscr{F}_K(a)$, $\mathscr{F}_2(a) \cap \psi^{-1}(K)$ are equivalent, then the collection $\mu_{\mathscr{K}}$ is projective, i.e. $\mu'_{\mathscr{K}} = \mu_{\mathscr{K}}$.

Thus we get methode for constructing of projective collections.

1.2.26. Remark. The cofilter base in (1.1.17) depended on the element a. It can be different for any $a \in S_U$. But if our presheaf $\mathscr{S} = \{(S_U, \tau_U), \varrho_{UV}; X\}$ consists of semiuniformisable spaces (with the semiuniformities η_U) we can do the same instead for neighborhoods of the elements, for neighborhoods of the diagonal, if the collection $\{\eta_U, U \in \mathscr{B}(X)\}$ is projective. Then in every U we get the unique base of cofilter $\mathscr{K}^U$, which do not depend on $a \in S_U$. Through the whole this paragraph we can quite analogicaly study the semiuniformity collections. All results concerning modifications and cofiltrations are analogous and the way, in which we get them, is quite same as the method we used here.

Reference

[1] *Z. Frolík:* Structure projective and structure inductive presheaves. Celebrazioni archimedae del secolo XX Simposio di topologia. 1964.

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