Czechoslovak Mathematical Journal

James R. Boone; Frank Siwiec

Sequentially quotient mappings

Czechoslovak Mathematical Journal, Vol. 26 (1976), No. 2, 174–182

Persistent URL: `http://dml.cz/dmlcz/101388`

Czechoslovak Mathematical Journal, 26 (101) 1976, Praha

SEQUENTIALLY QUOTIENT MAPPINGS

JAMES R. BOONE, College Station and FRANK SIWIEC, New York

(Received July 6, 1973)

1. Introduction. A mapping $f : X \to Y$ will be said to be *sequentially quotient* provided that: a set H is sequentially closed (open) in Y if and only if $f^{-1}(H)$ is sequentially closed (open) in X.

The sequentially quotient mappings are the convergent sequence analogs of the bi-quotient mappings of MICHAEL [8]. This is due to the equivalence of the notions of bi-quotient mapping and limit lifting mapping of HÁJEK [6]. The presequential characterizations of sequentially quotient mappings contained in Theorem 4.5 indicate, quite clearly, the analogy between the sequentially quotient mappings and the limit lifting mappings. Professor Hájek defines, in [6], a mapping $f : X \to Y$ to be *limit lifting* if: $y_\alpha \to y$ in Y implies there exists a subnet $\{y_\beta\}$ and $x_\beta \to x$ in X such that $f(x_\beta) = y_\beta$ and $f(x) = y$.

It is the purpose of this paper to investigate this natural analog of a bi-quotient mapping. In Section 2, a related concept is discussed to indicate the origin of this class of mappings. Characterizations of sequentially continuous mappings, in terms of inverse images, are presented in Section 3. The sequentially quotient mappings are characterized and some of the properties of this class are presented in Section 4. In Section 5, the main applications of this class of mappings are given, in the form of functional characterizations of sequential spaces as domains and ranges of certain mappings. There is also some discussion of examples to illustrate the results of this study.

Although the mappings in this paper are not generally assumed to be continuous, they will be surjections. The spaces are considered to be Hausdorff, with the exception of the range in the proof of Theorem 5.2.

2. Preliminaries. The notions sequentially open (closed) subsets of a space, and sequential spaces, introduced by FRANKLIN [4], are fundamental to this study. Franklin defines a subset H of a space X to be *sequentially closed* provided that no sequence in H converges to a point in $X - H$. A *sequentially open* subset U (which is the complement of a sequentially closed set) is one in which every sequence in X

which converges to a point in U is eventually in U. A space X is defined to be *sequential* provided that every sequentially open (sequentially closed) subset of X is open (closed).

The notion of a sequentially quotient mapping is the result of a study of an earlier class of mappings, the presequential mappings [2].

A mapping $f: X \to Y$ is said to be *presequential* provided that for each convergent sequence $\{p_n\}$ in Y, with $p_n \to p$, and $\{p_n\}$ not eventually equal to p, $\bigcup\{f^{-1}(p_n) : n \in N, p_n \neq p\}$ is not sequentially closed.

The definition of a presequential mapping is somewhat cumbersome and misleading with respect to its true significance. A meaningful characterization of presequential mappings, in the class of sequentially continuous mappings, is presented in Theorem 4.2. In particular, a mapping is sequentially quotient if and only if it is sequentially continuous and presequential. The presequential mappings have been used to establish the following mapping theorems for two generalizations of paracompactness.

Theorem 2.1. [2] *The closed continuous presequential image of a space with property* (ω) *has property* (ω).

Theorem 2.2. [2] *The closed continuous presequential image of a normal sequentially mesocompact space is a normal sequentially mesocompact space.*

In [3], an example is presented which shows that even if a mapping is perfect, neither sequential mesocompactness nor property (ω) is preserved. The difficulty arises in that some sequential constraint must be imposed on the inverse image of a convergent sequence in order to assure that certain properties relative to availability of convergent sequences in the domain are manifested in the range of a function. Continuous presequential mappings provide a sufficient sequential constraint. Even continuous perfect mappings fail to be presequential, as one can easily observe by considering [5, p. 695] the space $Y = \{0\} \cup \{1/n : n \in N\}$ with the usual topology as a quotient of βN. In particular, let $f: \beta N \to Y$ be the mapping defined by $f(n) = 1/n$ for each $n \in N$, and $f(p) = 0$ for each $p \in \beta N - N$. Then f is continuous and perfect, but $f^{-1}(\{1/n : n \in N\}) = N$. Since N is sequentially closed in βN, f is not presequential.

3. Sequentially continuous mappings. The notion of a presequential mapping is independent of continuity. Since we do need some form of continuity, we recall that a mapping $f: X \to Y$ is *sequentially continuous* if for each convergent sequence $\{p_n\}$ in X, $p_n \to p$, the sequence $\{f(p_n)\}$ in Y converges to $f(p)$. In the following theorem inverse image characterizations of sequentially continuous mappings are presented.

Theorem 3.1. *The following are equivalent for a mapping* $f: X \to Y$.

(a) *f is sequentially continuous.*

(b) *If U is sequentially open in Y, then $f^{-1}(U)$ is sequentially open in X.*

(c) *If H is sequentially closed in Y, then $f^{-1}(H)$ is sequentially closed in X.*

(d) *If H is countable and sequentially closed in Y, then $f^{-1}(H)$ is sequentially closed.*

Proof. (a) $\Rightarrow$ (c) Let H be any subset of Y such that $f^{-1}(H)$ is not sequentially closed in X. Then there exists a sequence $\{q_n\}$ in $f^{-1}(H)$, such that $q_n \to q$ and $q \notin f^{-1}(H)$. Then $\{f(q_n)\}$ is a sequence in H such that $f(q_n) \to f(q)$ and $f(q) \notin H$. Hence, H is not sequentially closed.

(b) $\Leftrightarrow$ (c) Consider complements.

(c) $\Rightarrow$ (d) This is obvious.

(d) $\Rightarrow$ (a) Suppose f is not sequentially continuous.

Then there exists a convergent sequence $\{x_n\}$ in X, with $x_n \to x$, and $f(x_n) \nrightarrow f(x)$. We may assume that $f(x_n) \neq f(x)$ for all $n \in N$. Suppose that some subsequence $\{f(x_{n_i})\}$ of $\{f(x_n)\}$ converges to a point y different from $f(x)$. Let $H = \{f(x_{n_i}) : n \in N\} \cup \{y\}$. Then H is sequentially closed, so that $f^{-1}(H)$ is also sequentially closed in X. Then x is in $f^{-1}(H)$, so that $f(x)$ is in H. But then $f(x) = f(x_n)$ for some index n, which is contrary to assumption. Now suppose that no subsequence of $\{f(x_n)\}$ converges. Since Y is at least a T_1-space, every sequential limit point y of H, such that $y \notin H$, is the limit of an infinite sequence in H. Every infinite sequence in H is a subsequence of $\{f(x_n)\}$. Since no subsequence of $\{f(x_n)\}$ converges, H contains all of its sequential limit points. Accordingly, H is sequentially closed, and by hypothesis $f^{-1}(H)$ is also sequentially closed. Thus x is in $f^{-1}(H)$, so that $f(x)$ is in H, again contrary to assumption. This completes the proof.

Clearly, for a sequentially continuous mapping, the inverse image of a convergent sequence (with its limit) is sequentially closed. However, a simple example shows that the converse is false. Let $X = \{1/n : n \in N\} \cup \{0\}$ and consider the identity mapping on X with the usual topology onto X with the discrete topology.

4. Sequentially quotient mappings. In this section we combine the notions of presequential and sequentially continuous mappings to obtain the sequentially quotient mappings. Some of the properties and characterizations of sequentially quotient mappings are presented also.

Lemma 4.1. *If $f : X \to Y$ is sequentially continuous, then the following are equivalent.*

(a) *f is presequential.*

(b) *For each non-sequentially open subset U of Y, $f^{-1}(U)$ is a non-sequentially open subset of X.*

(c) *For each non-sequentially closed subset H of Y, $f^{-1}(H)$ is a non-sequentially closed subset of X.*

Proof. It is clear, without the assumption of sequential continuity, that conditions (b) and (c) are equivalent, and that (c) implies (a). For the proof that (a) implies (c), let $f : X \to Y$ be presequential, and sequentially continuous. Letting H be a non-sequentially closed subset of Y, there exists a sequence $\{y_n\}$ in H converging to a point y in $Y - H$. Then y is not equal to any y_n. Since the set $H' = f^{-1}(\{y_1, y_2, \ldots\})$ is not sequentially closed, there exists a sequence $\{x_{n_i}\}$ in H' such that $f(x_{n_i}) = y_{n_i}$ for all $i \in N$ and $\{x_{n_i}\}$ converges to a point x in $X - H'$. Since the sequence $\{y_{n_i}\}$ converges to $f(x) = y$, x is not in $f^{-1}(H)$ and $\{x_{n_i}\}$ is in H', and so in $f^{-1}(H)$. Thus $f^{-1}(H)$ is not sequentially closed in X. The result follows.

The contrapositive of conditions (b) and (c) of the preceeding lemma and conditions (b) and (c) of Theorem 3.1 clearly imply the following theorem.

Theorem 4.2. *A mapping is sequentially quotient if and only if it is both sequentially continuous and presequential.*

A sequentially quotient mapping need not be continuous. This may be easily seen by taking a non-discrete space X in which each convergent sequence is eventually constant (for example, a well-known space of Arens). Then consider the identity mapping of X onto itself with the discrete topology.

The class of sequentially quotient mappings does include many mappings of interest. For example, we shall see (Theorem 5.2) that quotient continuous mappings defined on sequential spaces are sequentially quotient. Also countable-to-one perfect continuous mappings are sequentially quotient. This is because if $\{p_n\}$ is a sequence converging to a point p in Y and $p_n \neq p$ for every n, then the set $F = \bigcup\{f^{-1}(p_n) : : n \in N\} \cup f^{-1}(p)$ is compact metric, thus sequential. The restriction of f to F is a quotient mapping and thus sequentially quotient. The set $\bigcup\{f^{-1}(p_n) : n \in N\}$ is then non-sequentially closed in F, and so also in X. Thus f is presequential so also sequentially quotient. The next theorem shows that the class of sequentially quotient mappings contains all sequentially continuous mappings which are either sequence-covering or km-covering. In [9], a mapping $f : X \to Y$ is defined to be *sequence-covering* if whenever $\{y_n\}$ is a sequence in Y converging to a point y in Y, there exists a sequence of points $x_n \in f^{-1}(y_n)$ for $n \in N$, and $x \in f^{-1}(y)$ such that $x_n \to x$. A mapping $f : X \to Y$ is defined, in [10], to be *km-covering* if for every compact metrizable subspace L of Y, there exists a compact metrizable subspace K of X such that $f(K) = L$.

Theorem 4.3. (a) *Every sequentially continuous sequence-covering mapping is sequentially quotient.*

(b) *Every sequentially continuous km-covering mapping is sequentially quotient.*

Proof. It is easily seen that every sequence-covering mapping satisfies condition (c) of Theorem 4.1 and thus is presequential. With the additional assumption of sequentially continuous, we have that the mapping is sequentially quotient.

For (b), let $f : X \to Y$ be km-covering and also sequentially continuous. Letting H be a non-sequentially closed subset of Y, there exists a sequence $\{y_n\}$ in H such that $y_n \to y \in Y - H$. So $\{y, y_1, y_2, \ldots\}$ is compact metric. So there exists a compact metric subspace K of X such that $f(K) = \{y, y_1, y_2, \ldots\}$. Let $x_n \in f^{-1}(y_n) \cap K$ for each $n \in N$; then $x_n \in f^{-1}(H)$ for each n. The sequence $\{x_n\}$ has a convergent subsequence $\{x_{n_i}\}$. So there exists a point x such that $x_{n_i} \to x$; so also $y_{n_i} = f(x_{n_i}) \to$ $\to f(x)$. Then $f(x) = y$ and $x \notin f^{-1}(H)$. Accordingly, $f^{-1}(H)$ is not sequentially closed and f is sequentially quotient.

The mapping of the real line R onto the quotient space R/N is sequentially quotient but not sequence-covering.

Theorem 4.4. *A one-to-one sequentially continuous mapping is sequentially quotient if and only if it is sequence-covering.*

Proof. By the preceeding theorem, we need only prove the "only if". Let $f : X \to Y$ be a one-to-one and sequentially quotient mapping. Let $\{y_n\}$ be a sequence in Y converging to a point y. We may assume that $\{y_n\}$ consists of distinct points. Let $x_n = f^{-1}(y_n)$ and $x = f^{-1}(y)$. Our claim is that $x_n \to x$. Suppose not. Then there exists an open neighborhood G of x such that G does not contain infinitely many points of the sequence $\{x_n\}$. Let these points be denoted by the sequence $\{p_i\}$. Then $f(p_i) = y_i$, which converges to y. Since f is presequential, $\bigcup\{f^{-1}(y_i) : y_i \neq y\}$ is not sequentially closed. So there exists a subsequence $\{p_{i_j}\}$ of $\bigcup\{f^{-1}(y_i) : y_i \neq y\}$ such that $\{p_{i_j}\}$ converges to some point p in X. Then $f(p_{i_j}) = y_{i_j}$ converges to y and $f(p_{i_j})$ converges to $f(p)$, so that $f(p) = y$ and $p = f^{-1}(y) = x$. But G is an open neighborhood of $x = p$, so $p_{i_j} \in G$ for infinitely many j. Since $\{p_{i_j}\}$ is a subsequence of $\{p_i\}$, $p_i \in G$ for infinitely many i; which is a contradiction.

Our next result will be of particular interest in that part (b) will illustrate the essential difference between the sequentially quotient and the sequence-covering mappings. The surprising fact that sequentially quotient mappings are not only the sequential analogs of quotient mappings but also of pseudo-open mappings will be seen in part (c) of the following result. This is because it is known and easily seen that a mapping f is pseudo-open if and only if for each $p \in \mathrm{cl}\,(H)$, there exists a point q in $f^{-1}(p) \cap \mathrm{cl}\,(f^{-1}(H))$. Part (b) is the direct translation of Hájek's definition of limit lifting mappings to convergent sequences.

Theorem 4.5. *For a sequentially continuous mapping $f : X \to Y$, the following are equivalent.*

(a) *f is sequentially quotient.*

(b) *For each convergent sequence $\{p_n\}$ in Y, with $p_n \to p$, there exists a convergent sequence $\{q_i\}$ in X such that $q_i \in f^{-1}(p_{n_i})$, for some subsequence $\{p_{n_i}\}$, and $\{q_i\}$ converges to some $q \in f^{-1}(p)$.*

(c) *For each p in the sequential closure of a subset H of Y, there exists a point $q \in f^{-1}(p)$ such that q is in the sequential closure of $f^{-1}(H)$.*

Proof. (a) $\Rightarrow$ (b) Let $\{p_n\}$ be a convergent sequence in Y, say $p_n \to p$. If $\{p_n\}$ is equal to p for any cofinal subset, we are finished. Suppose $p_n \neq p$, for each $n \in N$. Then $\{p_n : n \in N\}$ is not sequentially closed. Since f is presequential, $\bigcup\{f^{-1}(p_n) : : n \in N\}$ is not sequentially closed. Since f is sequentially continuous, $\bigcup\{f^{-1}(p_n) : : n \in N\} \cup f^{-1}(p)$ is sequentially closed. Thus, there is a sequence $\{t_k\}$ in $\bigcup\{f^{-1}(p_n) : : n \in N\}$ that converges to some point $q \in f^{-1}(p)$. For each $n \in N$, $f^{-1}(p_n)$ is sequentially closed. Thus for each $n \in N$ there are at most a finite number of points from the sequence $\{t_k\}$ which belong to $f^{-1}(p_n)$. Then there exists an infinite number of indices $n_i \in N$, such that $t_k \in f^{-1}(p_{n_i})$, for some $k \in N$. For each $i \in N$, let q_i be any term from the sequence $\{t_k\}$ that belongs to $f^{-1}(p_{n_i})$. Then $\{q_i\}$ is a subsequence of $\{t_k\}$, and thus $q_i \to q \in f^{-1}(p)$.

(b) $\Rightarrow$ (c) Let p be in the sequential closure of a subset H of Y. If $p \in H$, we are finished. Suppose $p \notin H$. Then there exists a sequence $\{p_n\}$ in H such that $p_n \to p$. Let $\{q_i\}$ be a sequence in X such that $q_i \in f^{-1}(p_{n_i})$ for some subsequence $\{p_{n_i}\}$, and $\{q_i\}$ converges to $q \in f^{-1}(p)$. Since $q_i \in f^{-1}(H)$, for each $i \in N$ and $q_i \to q \in \in f^{-1}(p)$, we have a point $q \in f^{-1}(p)$ such that q is in the sequential closure of $f^{-1}(H)$.

(c) $\Rightarrow$ (a) It suffices to show that the sequentially continuous mapping satisfying (c) is presequential. Suppose f is not presequential. Then there exists a non-sequentially closed subset H of Y such that $f^{-1}(H)$ is sequentially closed in X. Let p be a point in the sequential closure of H such that $p \notin H$. Since $f^{-1}(H)$ is sequentially closed and $f^{-1}(p) \cap f^{-1}(H) = \emptyset$, there does not exist $q \in f^{-1}(p)$ such that q is in the sequential closure of $f^{-1}(H)$. This completes the proof.

Now that we have seen that sequentially quotient mappings have a similarity to pseudo-open mappings, our next result that this class of mappings is hereditary will not come as a surprise. This is because ARHANGELŠKIĬ [1] has shown that the pseudo-open continuous mappings are precisely the hereditarily quotient mappings. Recall that a class of mappings is said to be *hereditary* if whenever $f : X \to Y$ is in the class, then for each subspace H of Y, the restriction of f to $f^{-1}(H)$ is in the class.

Theorem 4.6. *Sequentially quotient mappings are hereditarily sequentially quotient.*

Proof. Let $f : X \to Y$ be a sequentially quotient mapping, and let H be any subspace of Y. Clearly the restriction of f to $f^{-1}(H)$ is sequentially continuous. Let T be any non-sequentially closed (in H) subset of H. Then there exists a sequence $\{p_n\}$ in T such that $p_n \to p$ and $p \in H - T$. Since $\{p_n : n \in N\} \cup \{p\}$ is sequentially closed in Y and f is sequentially continuous, $\bigcup\{f^{-1}(p_n) : n \in N\} \cup f^{-1}(p)$ is sequentially closed in X. Since $\bigcup\{f^{-1}(p_n) : n \in N\}$ is not sequentially closed in X, some sequence in $\bigcup\{f^{-1}(p_n) : n \in N\}$ must converge to some point in $f^{-1}(p)$. Since $f^{-1}(p) \subset \subset f^{-1}(H)$, $\bigcup\{f^{-1}(p_n) : n \in N\}$ is not sequentially closed in $f^{-1}(H)$. Thus, the inverse-image of T under the mapping $f|_{f^{-1}(H)}$ is not sequentially closed in $f^{-1}(H)$. Hence, the restriction of f to $f^{-1}(H)$ is presequential. This completes the proof.

5. Functional characterizations of sequential spaces. The first two results in this section characterize sequential spaces as those which are domains for mappings with certain properties.

Theorem 5.1. [7] *A space X is sequential if and only if each sequentially continuous mapping on X is continuous.*

Theorem 5.2. *A space X is sequential if and only if each quotient*[1]) *mapping on X is sequentially quotient.*

Proof. Let X be sequential, and let $f : X \to Y$ be any quotient mapping. Then Y is sequential [4]. Let U be any non-sequentially open subset of Y. Then U is not open and $f^{-1}(U)$ is not open. Since X is sequential, $f^{-1}(U)$ is not sequentially open in X. Conversely, suppose X is not sequential. Let H be a sequentially closed subset of X, which is not closed. Consider the mapping $f : X \to Y$, where $Y = \{0, 1\}$, defined by $f(x) = 0$ if $x \in H$, and $f(x) = 1$ if $x \in X - H$. Let Y have the quotient topology induced by f. Since $f^{-1}(\{1\}) = X - H$ is not open, $\{1\}$ is not open in Y. Thus, the constant sequence 0 converges to 1. Accordingly, the set $\{0\}$ is not sequentially closed, but $f^{-1}(\{0\})$ is sequentially closed. Hence f is a quotient mapping on X that is not sequentially quotient. This completes the proof.

We do not know whether the above result is true for Hausdorff range spaces.

The remaining theorems in this section characterize sequential, Fréchet [4], and countably bi-sequential spaces as those which are range spaces of mappings with certain properties. The following two concepts may be found in [9]. A *countably bi-sequential* space X may be defined (or characterized) by the property: Whenever $\{A_n : n \in N\}$ is a decreasing sequence of sets in X and x is a point which is in the closure of each A_n, then for each $n \in N$, there exists an $x_n \in A_n$ such that the sequence $x_n \to x$. A mapping $f : X \to Y$ is *countably bi-quotient* if for each y in Y and for each increasing open covering $\{U_n : n \in N\}$ of $f^{-1}(y)$ there exists an n such that $y \in \operatorname{Int}(f(U_n))$. The following results extend some of the theorems contained in [9] and [10]. However, the sequentially quotient mapping concept seems to be particularly nice since it yields both domain and range characterizations, while for sequence-covering and km-covering mappings only range characterizations have been obtained.

Theorem 5.3. *A space Y is sequential if and only if every sequentially quotient continuous mapping onto Y is quotient.*

Theorem 5.4. *A space Y is Fréchet if and only if every sequentially quotient mapping onto Y is pseudo-open.*[2])

Theorem 5.5. *A space Y is conutably bi-sequential if and only if every sequentially quotient mapping onto Y is countably bi-quotient.*[2])

---

[1]) A quotient mapping is understood to be also continuous.

[2]) Continuity is not taken as part of the definition of pseudo-open nor countably bi-quotient.

P r o o f of Theorem 5.3. For the "only if", it is easily verified that for a sequentially quotient (not necessarily continuous) mapping onto a sequential space, the inverse-image of every non-open set in the range is a non-open set in the domain. Assuming also continuity, we then have a quotient mapping. For the converse, let Y be a space for which every sequentially quotient continuous mapping onto Y is quotient. Let f be the identity mapping of $s(Y)$ onto Y, where $s(Y)$ is the discrete union of all convergent sequences (with limit) in Y. Then f is sequentially quotient, since f is in fact sequence-covering. Thus f is quotient, and since $s(Y)$ is a metrizable space, Y is sequential. The proof here is identical to that in [9].

P r o o f of Theorem 5.4. The "if" is similar to the case of Theorem 5.3. We will prove the "only if". Let $f : X \to Y$ be sequentially quotient with Y being a Fréchet space. Let y be a point of Y, and U an open neighborhood of $f^{-1}(y)$ in X. We will show that $y \in \operatorname{Int} f(U)$. If not, then $y \in \operatorname{cl}(Y - f(U))$. So there exists a sequence $\{y_n\}$ in $Y - f(U)$ which converges to y. The sequence $\{y_n\}$ is not sequentially closed. So $f^{-1}(\{y_n\})$ is not sequentially closed. Thus there is a sequence $\{x_{n_i}\}$ in $f^{-1}(\{y_n\})$ such that $f(x_{n_i}) = y_{n_i}$ and $x_{n_i} \to x$, where $x \notin f^{-1}(\{y_n\})$. So $y_{n_i} = f(x_{n_i}) \to f(x) = y$. So x is in $f^{-1}(y) \subset U$. So there exists an index i such that x_{n_i} is in U. Then $y_{n_i} = f(x_{n_i}) \in f(U)$, which yields a contradiction.

P r o o f of Theorem 5.5. Let $f : X \to Y$ be sequentially quotient with Y being countably bi-sequential. Let y be a point of Y and let $\{U_n : n \in N\}$ be an increasing open covering of $f^{-1}(y)$ in X. We will show that there exists an n such that $y \in \operatorname{Int} f(U_n)$. If not, then for each $n \in N$, $y \in \operatorname{cl}(Y - f(U_n))$. Since $\{Y - f(U_n)\}$ is a decreasing sequence of sets, for each n there exists a point y_n in $Y - f(U_n)$ such that $y_n \to y$. Then the sequence $\{y_n\}$ is not sequentially closed, so that $f^{-1}(\{y_n\})$ is not sequentially closed. Continuing as above we again obtain a contradiction.

We have already presented an example of a perfect continuous mapping which is not sequentially quotient. Examples of mappings satisfying various properties may be found in [9], and also may be easily constructed by using the method of Theorem 5.3. For example, a mapping which is sequentially quotient, in fact, sequence-covering, and quotient (and continuous), but not pseudo-open, may be most easily obtained by choosing Y to be any space which is sequential but not Fréchet, and letting f be the natural mapping of $s(Y)$ onto Y.

References

[1] *A. Arhangelskiĭ*, Some types of factor mappings and the relation between classes of topological spaces, Soviet Math. Dokl. *4* (1963), 1726—1729.

[2] *J. R. Boone*, A note on mesocompact and sequentially mesocompact spaces, Pacific J. Math. *44* (1973), 69—75.

[3] *J. R. Boone*, Examples relating to mesocompact and sequentially mesocompact spaces, Fund. Math. *77* (1972), 91—93.

[4] *S. P. Franklin*, Spaces in which sequences suffice, Fund. Math. *57* (1965), 107—115.
[5] *S. P. Franklin* and *K. K. Kohli*, On open extensions of maps, Canad. J. Math. *22* (1970), 691—696.
[6] *O. Hájek*, Notes on quotient maps, Comment. Math. Univ. Carolina *7* (1966), 319—323.
[7] *S. Leader* and *S. Baron*, Sequential topologies, Amer. Math. Monthly *73* (1966), 677—678 (problem #5299).
[8] *E. Michael*, Bi-quotient maps and cartesian products of quotient maps, Ann. Inst. Fourier, Grenoble *18*, (1968), 287—302.
[9] *F. Siwiec*, Sequence-covering and countably bi-quotient mappings, General Topology and Appl. *1* (1971), 143—154.
[10] *F. Siwiec*, KM-spaces and km-covering mappings, to appear.

Authors' addresses: J. R. Boone, Texas A & M University, College Station, Texas 77843, U.S.A. F. Siwiec, City University of New York, John Jay College, New York, New York 10010, U.S.A.