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TO THE SIXTIETH ANNIVERSARY OF BIRTHDAY OF PROFESSOR OTTO VEJVODA

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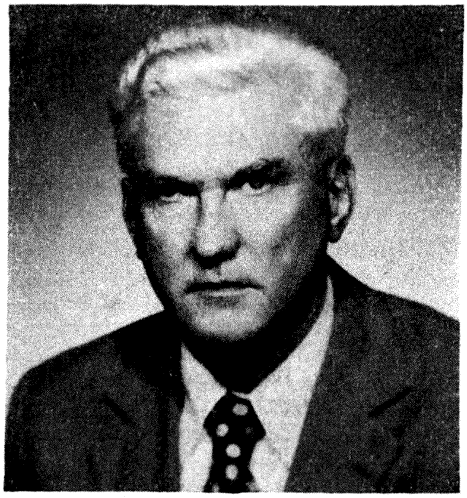
The anniversary of birthday of Professor RNDr. Otto Vejvoda, DrSc., is a welcome opportunity for making the reading public of this Journal acquainted in more detail with some facts from the life and, in particular, with the life-long successful scientific activity of this outstanding research worker in mathematical analysis.

Otto Vejvoda was born on 4 June, 1922, in Záboří nad Labem and since 1927 has



lived permanently in Prague. He finished his grammar school studies in 1941, that is, at the time when the Czech universities were closed by the Nazi occupants. Already during this forced interruption of his studies he demonstrated his interests by engaging himself in teaching Mathematics. After the liberation of our country Vejvoda entered the Faculty of Science, Charles University, starting to study Mathematics and Physics. He graduated in 1949 and received his degree of Doctor of (Natural) Sciences (RNDr.) on the basis of his dissertation from projective differential geometry in 1950. During the years 1950–1953 he was aspirant (postdoctoral student) in the Central Mathematical Institute and later, after its foundation in 1952, in the Mathematical Institute of the Czechoslovak Academy of Sciences. After completing his scientific training as aspirant he became scientific worker in the same Institute and has stayed with the Institute ever since, at present as Head of Department of Evolutionary Differential Equations.

The scientific interest soon led Otto Vejvoda to a deep study of mathematical analysis. In early fifties he started to investigate a wide field of problems from the theory of nonlinear ordinary differential equations, in particular the stability theory, numerical methods and perturbation properties of boundary value problems. Stability of solutions of ordinary differential equations in the complex domain was also the subject of his dissertation from 1956, on the basis of which he received the CSc. (Candidate of Science) degree. In all the directions mentioned above he published original scientific results, the most significant and comprehensive from them being those concerning the perturbation properties of boundary value problems. The climax of his many-years work in this direction was the paper [9], where Vejvoda found sufficient conditions for the existence and uniqueness of a solution $x(t, \varepsilon)$ of the general nonlinear equation (depending on a parameter ε)

$$\dot{x} = f(t, x) + \varepsilon g(t, x, \varepsilon) \tag{1}$$

with the general nonlinear boundary value condition (also depending on the parameter ε)

$$u(x(a, \varepsilon), x(b, \varepsilon)) + \varepsilon v(x(a, \varepsilon), x(b, \varepsilon), \varepsilon) = 0 \tag{2}$$

for small ε; he discussed the case when the so-called "shortened" problem

$$\dot{y} = f(t, y)\,, \tag{3}$$

$$u(y(a), y(b)) = 0 \tag{4}$$

has a single solution, as well as that when the shortened problem has solutions depending on k parameters. If the condition (2) has a special form

$$x(a, \varepsilon) - x(b, \varepsilon) = 0 \tag{5}$$

and if the functions f and g are periodic in the argument t with the period $b - a$, then the solution of the problem (1), (5) can be extended to periodic solutions with the period $b - a$. A special attention is paid to the situation when the equation (1) is autonomous. The results include the case when the autonomous equation (1) is investigated together with the condition (5); also in this case we can easily pass to periodic solutions of the equation (1). Since in the autonomous case the period of its periodic solution is not apriori given, the value of a is in this case chosen fixed while b is sought as a function of the variable ε, so that the period $b(\varepsilon) - a$ depends on ε.

In the late fifties, Otto Vejvoda gradually came to the conclusion that many oscillation phenomena are hidden in the boundary value problems for partial differential equations. He decided to exploit his experience in nonlinear boundary value problems for ordinary differential equations for an investigation of boundary value problems for partial differential equations of evolution, in particular for the investigation of existence and other properties of periodic solutions. This meant to enter an area with much more difficult problems, which since then had been only poorly explored.

It was already in one of the first papers on this subject [11] that O. Vejvoda successfully solved the problem of existence of periodic solutions of a weakly nonlinear wave equation

$$u_{tt} - u_{xx} = \varepsilon f(t, x, u, u_t, u_x, \varepsilon) \tag{6}$$

with the boundary conditions

$$u(t, 0) = u(t, \pi) = 0 \quad (t \in R). \tag{7}$$

Among other, he found conditions on the function f guaranteeing that the problem (6), (7) for sufficiently small values of the parameter ε has 2π-periodic solutions continuously depending pn ε and proved that these conditions are fulfilled by the function $f = \alpha u + \beta u^3 + h(t, x)$ for $\beta \neq 0$ with α/β either nonnegative or sufficiently small, while the function h satisfies certain not too restrictive conditions.

These results (as well as many others) attracted deep interest and it is certainly not out of place to offer here at least a partial explanation of why it was so. The problem of finding periodic solutions of the problem (6), (7) can be in an abstract setting formulated as the problem of finding solutions (continuously depending on the parameter ε) of the equation

$$N(u) = \varepsilon F(u, \varepsilon) \tag{8}$$

in a certain Banach space B (of periodic functions), where F is a nonlinear operator and N a linear operator with a nontrivial (even infinitely dimensional) null space. The paper [11] was one of the first in which a problem of this type was successfully solved.

In the subsequent years Otto Vejvoda systematically treated problems of periodic solutions of (nonlinear) equations of mathematical physics, as the already mentioned wave equation, the telegraph equation, the heat equation, the equation of a beam etc., with various types of (nonlinear) boundary conditions. When solving these problems, a detailed investigation of existence of periodic solutions of linear equations, of existence and properties of solutions of initial-boundary value problems as well as a suitable choice of Banach spaces etc., play an important part. As an example let us introduce here several results from an extensive paper [15] (in the equivalent form from [29]), concerning 2π-periodic solutions of the linear wave equation

$$u_{tt} - u_{xx} = g(t, x) \tag{9}$$

with the boundary conditions

$$u_x(t, 0) + \alpha_0(t)\, u(t, 0) = h_0(t), \tag{10}$$

$$u_x(t, \pi) + \alpha_1(t)\, u(t, \pi) = h_1(t) \quad (t \in R),$$

(the given functions being of course 2π-periodic):

(a) Let $\alpha_1(t + \pi) - \alpha_0(t) = 0$ for $t \in R$. Then (9), (10) has a 2π-periodic solution if and only if there exists a constant K such that

$$K \alpha_0(t) + 2h_1(t + \pi) - 2h_0(t) + \int_0^\pi (g(t - x, x) - g(t + x, x))\, dx +$$

$$+ \alpha_0(t) \int_0^\pi \int_{t+x}^{t+2\pi-x} g(\tau, x)\, d\tau\, dx = 0 \quad \text{for} \quad t \in R .$$

(b) Let $\alpha_1(t + \pi) - \alpha_0(t) \neq 0$ for $t \in R$ and

$$\int_{-\pi}^{\pi} \alpha_1(t + \pi)\, \alpha_0(t)\, (\alpha_1(t + \pi) - \alpha_0(t))^{-1}\, dt \neq 2 .$$

Then the problem (9), (10) has a 2π-periodic solution for every function g.

(c) Let $\alpha_1(t + \pi) - \alpha_0(t) \neq 0$ for $t \in R$ and

$$\int_{-\pi}^{\pi} \alpha_1(t + \pi)\, \alpha_0(t)\, (\alpha_1(t + \pi) - \alpha_0(t))^{-1}\, dt = 2 .$$

Then the problem (9), (10) has a 2π-periodic solution if and only if

$$\int_{-\pi}^{\pi} H_1(t)\, \alpha_0(t)\, (\alpha_1(t + \pi) - \alpha_0(t))^{-1}\, dt = 0 ,$$

where

$$H_1(t) = 2h_1(t + \pi) - 2h_0(t) - \alpha_1(t + \pi) \int_0^{2\pi} h_0(t)\, dt +$$

$$+ \int_0^\pi (g(t - x, x) + g(t + x, x))\, dx + \alpha_1(t + \pi) \int_0^\pi \int_{t+x}^{t+2\pi-x} g(\tau, x)\, d\tau\, dx .$$

In [23] the author introduced new spaces of "piecewise regular" functions and proved the existence of periodic solutions of the weakly nonlinear autonomous wave equation, belonging to these spaces.

Otto Vejvoda also devoted himself to the study of periodic solutions of abstract differential equations. So for example, in [21] he formulated for a wide class of nonlinear operators F sufficient (and also necessary) conditions for existence of periodic solutions of abstract first-order equations

$$(11) \qquad u_t + (A + \gamma I)\, u = \varepsilon F(t, u)$$

in Banach spaces (where A is the generator of a holomorphic semigroup of operators) and of second order equations

$$(12) \qquad u_{tt} + 2(\alpha I + \beta A)\, u_t + (A + \gamma I)\, u = \varepsilon F(t, u)$$

in Hilbert spaces (where A is a strictly positive selfadjoint operator).

The results of many years of research of periodic solutions in the field of both partial and abstract differential equations, which had been obtained by Otto Vejvoda and his collaborators, were collected and also substantically extended in the monograph [29].

Otto Vejvoda has not disregarded boundary value problems for ordinary differential equations even later. Together with M. Tvrdý he studied the linear problem with the "boundary" condition depending not only on the values of the solution at the endpoints of the interval considered but also on the values of the solution at the inner points of the interval, that is, condition expressed by a general linear operator. They found the adjoint problem, proved Fredholm's type necessary and sufficient conditions of solvability and constructed the Green function. Later, they replaced here the linear differential equation by the integro-differential one.

Otto Vejvoda also initiated the monograph [26]. This monograph contains the theory of linear boundary value problems for differential and integro-differential operators with one variable. In the last chapter the perturbation theory of nonlinear boundary value problems is developed (the equation (1) is investigated together with the condition

$$S(x) + \varepsilon R(x, \varepsilon) = 0 , \tag{13}$$

where S is an operator associating a continuous vector function $x : [a, b] \to R^n$ with a vector in R^n, and R is also an operator associating a couple x, ε with a vector in R^n).

Otto Vejvoda is the author or co-author of more than thirty original scientific works, in which he reached substantial and ample results. They are based on combining mathematical invention with patient work, detailed analysis of formulas and painstaking computations as well as on extensive knowledge of many results and procedures. The collection of works on the nonlinear wave equation formed the basis of O. Vejvoda's doctoral (DrSc.) thesis, which was defended in 1973.

There is nothing incidental in the fact that all problems in which O. Vejvoda engaged himself were motivated by technological problems and are of considerable importance for applications. Otto Vejvoda has always believed that one of extraordinarily important and ever lasting tasks of mathematics in general and mathematical analysis in particular is to explain the properties of those mathematical models that are used in physics as well as in technical and other sciences. Otto Vejvoda is still being involved in deepening the interrelations between the theory of evolutionary differential equations and a number of branches in which the evolutionary equations can be applied.

O. Vejvoda has intensively engaged himself in mathematical education. He was Assistent Professor in the Mathematical Institute of the School of Civil Engineering already in 1947–1949 and at the Faculty of Science, Charles University, in 1949 to 1950. Since 1953 he led a number of seminars and advanced courses and was supervisor of many diploma theses and doctoral dissertations. In 1966 he was appointed

Associated Professor (Dozent) for Mathematics. For many years he led the scientific seminar on the theory of evolutionary differential equations, in which many later research workers in both fundamental and applied mathematics started their research work (often already during their university studies) and, thanks to the guidance and valuable advice by O. Vejvoda, prepared numerous research papers and dissertations. This concerns among others all collaborators of O. Vejvoda from the Department of Evolutionary Differential Equations.

However, even the rich research work and manysided educational activity does not exhaust the contribution of O. Vejvoda to the development of Czechoslovak Mathematics. Let us only mention his membership in the Scientific Board for Mathematics of the Czechoslovak Academy of Sciences during 1976—1981. For many years supervised the library of the Mathematical Institute, taking care of the completeness and continual modernization of its book funds. He has always considered the cooperation and exchange of information essential for mathematical research. Therefore the took essential part in organizing big international meetings of mathematicians, as for instance the conferences EQUADIFF and the Czechoslovak-Soviet meetings on applications of the theory of functions and of functional analysis in problems of mathematical physics. He has also contributed to the development of good relations with the mathematicians of various countries, as for example Romania, Italy etc.

The active approach of Otto Vejvoda to all problems concerning the development of Mathematics, as well as his exceptional ability of optimal scheduling and exploitation of his working time, may serve as an ideal for every research worker. On the other hand, Vejvoda has always been able to find time for frequent visits of Prague concert halls as well as for excursions on foot or by boat, and has always been a pleasant and amusing companion. All those who have met him appreciate his unceasing optimism and his extraordinary readiness to offer advice and help to anybody regardless of whether a scientific or a personal problem is concerned. We all wish Otto Vejvoda many years of health, success and happiness.

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