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CONTACT PROBLEMS WITH BOUNDED FRICTION. SEMICOERCIVE CASE

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0. INTRODUCTION

Contact problems with friction describe the behaviour of two bodies $\mathscr{B}_1, \mathscr{B}_2$ being in contact, provided the influence of the friction occuring on the common contact surface is significant. Under the assumption of pointwise validity of the Coulomb law ewerywhere on the contact surface, the contact problems were solved in [5] for the first time (the Signorini case for a strip in R^2). Extensions of the result for sufficiently smooth domains in R^3 for the coercive case were published in [3]. The present paper is the continuation of the latter paper, the methods of proofs being essentially based on it.

The bodies $\mathscr{B}_\iota$, $\iota = 1, 2$, occupy domains Ω_ι, $\iota = 1, 2$, respectively; $\Omega_1 \subset R^3$, $\Omega_2 \subset R^3$ and $\Omega_1 \cap \Omega_2 = \emptyset$. Γ_c is the common contact part of the boundaries Γ^ι, $\iota = 1, 2$. The rests $\Gamma^\iota \setminus \Gamma_c$, $\iota = 1, 2$, are divided into Γ^ι_T and Γ^ι_u, $\iota = 1, 2$, where the stresses and the displacements are prescribed, respectively. Throughout the paper we consider the small strain tensors

$$(e_{ij}(u^\iota))_{ij} = \left(\frac{1}{2}\left(\frac{\partial u^\iota_i}{\partial x_j} + \frac{\partial u^\iota_j}{\partial x_i}\right)\right)_{ij}, \quad \iota = 1, 2,$$

and also the validity of the linear Hook law between $(e_{ij}(u^\iota))$ and the corresponding stress tensors $(\tau_{ij}(u^\iota))$: $\tau_{ij}(u^\iota) = a^\iota_{ijkl}\, e_{kl}(u^\iota)$, $i, j = 1, 2, 3$, on Ω_ι, $\iota = 1, 2$, where $[u^1, u^2]$ is a couple of displacements on $\Omega_1 \times \Omega_2$. In particular, if Ω_ι is homogeneous and isotropic, then the Hook law has the form

$$\tau_{ij}(u^\iota) = \frac{E_\iota}{1 + \sigma_\iota}\, e_{ij}(u^\iota) + \frac{E_\iota \sigma_\iota}{(1 + \sigma_\iota)(1 - 2\sigma_\iota)}\, \delta_{ij}\, e_{kk}(u^\iota) \quad \text{on} \quad \Omega_\iota\,,$$

where the constants satisfy $\sigma_\iota \in (0, \frac{1}{2})$, $E_\iota > 0$, $\iota = 1, 2$, and δ_{ij} is the Kronecker symbol. The summation convention is applied consistently for indices i, j, k, l, but never for ι. The equilibrium conditions

$$(0.1)\qquad -\frac{\partial \tau_{ij}(u^\iota)}{\partial x_j} = f_i^\iota \quad \text{a.e. in } \Omega_\iota\,, \quad i = 1, 2, 3\,, \quad \iota = 1, 2\,,$$

must be satisfied for the given volume forces $(f_i^\iota)_{i=1,2,3}$, $\iota = 1, 2$. On Γ_c the following conditions with the evident physical meaning must be fulfilled:

$$(0.2)\qquad T_n(u) \leqq 0\,, \quad u_n^1 - u_n^2 \leqq 0\,, \quad T_n(u)\,(u_n^1 - u_n^2) = 0\,,$$

$$(0.3)\qquad |T_t(u)| \leqq \mathscr{F}|T_n(u)|\,, \quad (|T_t(u)| - \mathscr{F}|T_n(u)|)\,|u_t^1 - u_t^2| = 0\,, \quad \mathscr{F}T_n(u) < 0 \Rightarrow$$
$$\Rightarrow u_t^1 - u_t^2 = \lambda T_t(u) \quad \text{(the Coulomb law of friction)}\,,$$

where for the couple $[u^1, u^2]$ the stress $T(u)$ is given by the equality $T(u) \equiv T(u^1) = -T(u^2)$ and $T(u^\iota) = (-1)^{\iota-1}(\tau_{ij}(u^\iota)\,n_j)$, $\iota = 1, 2$. The terms with t and n in (0.2) and in all the following expressions are the tangential and normal components of the corresponding vectors, respectively. The normal vector n on Γ_c is chosen as the unit outer normal vector with respect to Ω_1. The only given term in conditions (0.2) and (0.3) is the function $\mathscr{F}$ (the coefficient of friction). Λ will be a suitable non-positive function. Moreover, the equalities

$$(0.4)\qquad u^\iota = u_0^\iota \quad \text{a.e. on } \Gamma_u^\iota\,, \quad \iota = 1, 2\,,$$

$$(0.5)\qquad T(u^\iota) = T_0^\iota \quad \text{a.e. on } \Gamma_T^\iota\,, \quad \iota = 1, 2\,,$$

must hold, where u_0^ι, T_0^ι are given, $\iota = 1, 2$.

The contact problem with friction is to find a couple of displacements $[u^1, u^2]$ on $\Omega_1 \times \Omega_2$ such that the conditions (0.1)–(0.5) hold. It is semicoercive, iff at least one of Γ_u^1, Γ_u^2 is of the zero measure.

The Signorini case is the contact problem with Ω_2 rigid and undeformable. It is semicoercive, iff mes $\Gamma_u^1 = 0$.

1. ASSUMPTIONS. VARIATIONAL FORMULATION OF THE PROBLEM

Throughout the paper we shall suppose that both Γ^1 and Γ^2 are Lipschitzian, the sets Γ_c, Γ_u^ι, Γ_T^ι, $\iota = 1, 2$, possess Lipschitz relative boundaries and are pairwise disjoint. $\mathscr{F}: \Gamma_c \to R_+$ is Lipschitz with a compact support and $0 < \delta_0 \equiv \operatorname{dist}(\operatorname{supp}\mathscr{F}, \partial\Gamma_c)$. Moreover, Γ_c fulfils the following conditions:

(1.1) There exist positive constants $k_0 < 1$, K_0, r and Δ_0 and $\varepsilon_0 \in (0, (1 - k_0)\,\delta_0\rangle$ such that for each $\delta \in (0, \Delta_0)$ there exists a finite covering $\mathfrak{A}_\delta$ of Γ_c with the following properties:

(1.1a) For every $V \in \mathfrak{A}_\delta$ there exists a function $\varphi_V \in C^{0,1}(R^2)$ such that $\varphi_V(0) = \varphi_V'(0) = 0$. Denoting by Ψ_V the map $[x_1, x_2, x_3] \mapsto [x_1, x_2, x_3 -$

$- \varphi_V(x_1, x_2)]$, we suppose that after a suitable rotation and shift, $\Psi_V(V)$ is an open set in $R^2 \times (-r, r)$ containing 0; $\Psi_V(V \cap \Gamma_c) \subset R^2 \times \{0\}$, $\Psi_V(V \cap \cap \Omega_1) \subset R^2 \times (0, r)$ and $\Psi_V(V \cap \Omega_2) \subset R^2 \times (-r, 0)$. Put $B(\eta, r) := := B^2_\eta(0) \times (-r, r)$, $B^+(\eta, r) := B^2_\eta(0) \times (0, r)$, $B^-(\eta, r) := B^2_\eta(0) \times (-r, 0)$ for $\eta > 0$, where for a real positive number η, an integer N and $M \subset R^N$, $B^N_\eta(M) := \{x \in R^N;\ \operatorname{dist}(x, M) < \eta\}$. For each $V \in \mathfrak{A}_\delta$ such that $V \cap \cap \overline{B^3_{k_0\delta_0+\varepsilon_0}(\operatorname{supp} \mathscr{F})} \neq \emptyset$ we suppose, moreover, that $\varphi_V \in C^{2,1}(R^2)$, $\|\varphi_V\|_{C^{2,1}(R^2)} < K_0$, $\Psi_V(V) = B(\frac{1}{2}\delta, r)$, $\Psi_V^{-1}(B^+(\delta, r)) \subset \Omega_1$, $\Psi_V^{-1}(B^-(\delta, r)) \subset \Omega_2$ and $\Psi_V^{-1}(B(\delta, r)) \cap (\Gamma^1 \cup \Gamma^2) \setminus \Gamma_c = \emptyset$.

(1.1b) For each $\delta \in (0, \Delta_0)$ there exists a system of non-negative Lipschitz functions $\mathscr{V} = \{g_V, V \in \mathfrak{A}_\delta\}$ which is of the class $C^{2,1}$ for such V for which $V \cap \cap \overline{B^3_{k_0\delta_0+\varepsilon_0}(\operatorname{supp} \mathscr{F})} \neq \emptyset$. For each $V \in \mathfrak{A}_\delta$, $\operatorname{dist}(\operatorname{supp} g_V, R^3 \setminus V) > 0$ and $\mathscr{V}$ is a partition of unity on Γ_c.

We suppose that all a^ι_{ijkl} are Lipschitz on Ω_ι, $\iota = 1, 2$, fulfil the usual symmetry condition $a^\iota_{ijkl} = a^\iota_{jikl} = a^\iota_{klij}$ on Ω_ι for every $i, j, k, l \in \{1, 2, 3\}$, $\iota = 1, 2$, and

$$(1.2) \qquad 0 < a_{0,\iota} \leqq |\xi|^{-2}\, a^\iota_{ijkl}(x)\, \xi_{ij}\xi_{kl} \leqq A_{0,\iota} < +\infty\,, \quad x \in \Omega_\iota\,, \quad \xi \in R^9\,, \quad \iota = 1, 2\,.$$

Let $f = [f^1, f^2] \in \prod_{\iota=1}^{2} L_2(\Omega_\iota; R^3)$, $T^0 = [T^1_0, T^2_0] \in \prod_{\iota=1}^{2} H^{-1/2}(\Gamma^\iota; R^3)$, $\operatorname{supp} T^\iota_0 \subset \subset \Gamma^\iota_T$, let $[u^1_0, u^2_0] \in \mathscr{H}(\Omega_1, \Omega_2) := \prod_{\iota=1}^{2} H^1(\Omega_\iota; R^3)$ satisfy the equality $u^\iota_0 / \overline{\Gamma^\iota \setminus \Gamma^\iota_u} = 0$, $\iota = 1, 2$.

Define $C^{*-} := \{g \in H^{-1/2}(\Gamma_c);\ \langle g, v\rangle \geqq 0\ \forall v \in H^{1/2}(\Gamma_c),\ v \leqq 0 \text{ a.e. in } \Gamma_c\}$ for $\langle \cdot, \cdot \rangle$ the scalar product in $L_2(\Gamma_c)$. Denote by $(\cdot, \cdot)_\iota$ the scalar product in $L_2(\Omega_\iota; R^3)$, $(\cdot, \cdot)_0 = \sum_{\iota=1}^{2} (\cdot, \cdot)_\iota$ on $\prod_{\iota=1}^{2} L_2(\Omega_\iota; R^3)$, by $[\cdot, \cdot]_\iota$ the scalar product in $L_2(\Gamma^\iota; R^3)$ – – analogously $[\cdot, \cdot]_0 = \sum_{\iota=1}^{2} [\cdot, \cdot]_\iota$. For $u^\iota, v^\iota \in H^1(\Omega_\iota; R^3)$ we define

$$(1.3) \quad a^\iota(u^\iota, v^\iota) = \int_{\Omega_\iota} a^\iota_{ijkl}\, e_{ij}(u^\iota)\, e_{kl}(v^\iota)\, \mathrm{d}x\,, \quad \iota = 1, 2\,, \quad a(u, v) = \sum_{\iota=1}^{2} a^\iota(u^\iota, v^\iota)\,.$$

Let $\mathscr{K} := \{v \in \mathscr{H}(\Omega_1, \Omega_2);\ v^\iota = u^\iota_0 \text{ on } \Gamma^\iota_u,\ v^1_n - v^2_n \leqq 0 \text{ on } \Gamma_c\}$. For a given $g_n \in C^{*-}$ define

Problem $\langle g_n \rangle$. Find $u \in \mathscr{K}$ such that for every $v \in \mathscr{K}$,

$$(1.4) \qquad a(u, v - u) \geqq (f, v - u)_0 + [T^0, v - u]_0 + + \langle \mathscr{F} g_n, |v^1_t - v^2_t| - |u^1_t - u^2_t| \rangle\,.$$

In the Signorini case the terms with the index 2 in (1.3), (1.4) and in the other definitions vanish, $a_{0,2} = A_{0,2} = +\infty$.

Now we define $T_n(u)/\Gamma_c$ by the following formula valid for $\iota = 1, 2$:

$$(1.5) \qquad (-1)^{\iota-1} \langle T_n(u), w_n \rangle = a^\iota(u^\iota, w^\iota) - (f^\iota, w^\iota)_\iota$$
$$\forall w^\iota \in \boldsymbol{H}^1(\Omega_\iota, R^3), \quad w_t^\iota/\Gamma^\iota = 0, \quad w_n^\iota/\Gamma^\iota \setminus \Gamma_c = 0.$$

Definition 1.1. A solution u of Problem $\langle g_n \rangle$ *) is called a *solution of the contact problem with friction*, iff $\mathscr{F} T_n(u) = \mathscr{F} g_n$ for the corresponding $T_n(u)$ defined by (1.5).

An analogous definition for the Signorini case see in [3]. Of course, for a sufficiently regular solution of the contact problem with friction in the sense of Definition 1.1 all the classical conditions are satisfied.

2. PROBLEMS OF FICHERA TYPE

Let $\mathscr{Z}$ be the space of all rotations and shifts in R^3, denote $\mathfrak{R} := \{w \equiv [w^1, w^2] \in \in \mathscr{H}(\Omega_1, \Omega_2),\ \exists z^1, z^2 \in \mathscr{Z},\ w^\iota = z^\iota/\Omega_\iota,\ \iota = 1, 2\}$, $\mathfrak{P} = \mathfrak{R}^\perp$. Let $\mathfrak{R}_1 := \{w \in \mathfrak{R}, z^1 = z^2\}$, $\mathfrak{R}_2 \equiv \mathfrak{R}_1^\perp \cap \mathfrak{R}$. Put $L^\iota(v^\iota) = -(f^\iota, v^\iota)_\iota - [T_0^\iota, v^\iota]_\iota$, $\iota = 1, 2$, $L(v) = = L^1(v^1) + L^2(v^2)$ for $v = [v^1, v^2] \in \mathscr{H}(\Omega_1, \Omega_2)$. In this section we shall solve the problem provided one of the following conditions is fulfilled:

(2.1) mes $\Gamma_u^1 > 0$, mes $\Gamma_u^2 = 0$ and $L^2(w^2) > 0$ for every $w^2 = z^2/\Omega_2$ for some $z^2 \in \mathscr{Z} \setminus \{0\}$ such that $w_n^2 \geqq 0$ on Γ_c;

(2.2) mes $\Gamma_u^1 =$ mes $\Gamma_u^2 = 0$, $L(w) > 0$ for each $w \in \mathscr{K} \cap (\mathfrak{R} \setminus \mathfrak{R}_1)$ and $L(\mathfrak{R}_1) = 0$;

(2.3) in the Signorini case mes $\Gamma_u^1 = 0$, $L(w) \equiv L^1(w^1) > 0$ for each $w^1 \in \boldsymbol{H}^1(\Omega_1; R^3)$ such that there is $z^1 \in \mathscr{Z} \setminus \{0\}$ fulfilling $z^1/\Omega_1 = w^1$ and $w_n^1 \leqq 0$ on Γ_c.

Clearly such assumptions cannot be satisfied for arbitrary contact surfaces (e.g. for a plane contact surface).

We introduce

$$(2.4) \qquad J_{g_n}(v) = \tfrac{1}{2} a(v, v) + L(v) + \langle \mathscr{F} |g_n|, |v_t^1 - v_t^2| \rangle, \quad v \in \mathscr{H}(\Omega_1, \Omega_2),$$

and denote by $\|\cdot\|_{\mathscr{H}}$ the norm in $\mathscr{H}(\Omega_1, \Omega_2)$. If (2.1) holds, then there exist constants $\check{c}_1 > 0$, $\check{c}_2 \geqq 0$ such that

$$(2.5) \qquad J_0(v) \geqq \check{c}_1 \|v\|_{\mathscr{H}} - \check{c}_2 \quad \forall v \in \mathscr{K}.$$

We denote $\mathscr{K}_0 := \mathscr{K} - [u_0^1, 0]$. To prove (2.5), we assume the contrary. Hence we can find a sequence $\{v_k\} \subset \mathscr{K}_0$, $\|v_k\|_{\mathscr{H}} \to +\infty$ such that

$$(2.6) \qquad 0 \geqq \gamma_0 \equiv \lim_{k \to +\infty} \tfrac{1}{2} \|v_k\|_{\mathscr{H}}\, a(w_k, w_k) + a^1(u_0^1, w_k) + L(w_k),$$

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*) In what follows, we shall write "$\langle g_n \rangle$" instead of "Problem $\langle g_n \rangle$".

where $w_k = v_k/\|v_k\|_{\mathscr{H}} \in \mathscr{K}_0$. Let w_0 be the weak limit of $\{w_k\}$ (after passing to a suitable subsequence). For the corresponding orthogonal projection $\Pi_{\mathfrak{P}}$ onto $\mathfrak{P}$ we obtain $a(w_k, w_k) = a(\Pi_{\mathfrak{P}} w_k, \Pi_{\mathfrak{P}} w_k) \to 0$. From (2.6) and the Korn inequality, $w_k^1 \to 0$ and $\Pi_{\mathfrak{P}} w_k \to 0$. Hence $w_k \to w_0 \in \mathscr{K} \cap \mathfrak{R}$, $\|w_0\|_{\mathscr{H}} = 1$, $w_0^1 = 0$ and$(w_0^2)_n \geqq 0$. However, (2.6) yields $L(w_0) \leqq 0$ which contradicts (2.1).

If (2.2) is satisfied, J_{g_n} does not depend on elements from $\mathfrak{R}_1$. Putting $\mathfrak{P}_1 = \mathfrak{R}_1^{\perp}$, we can find $\check{c}_1 > 0$, $\check{c}_2 \geqq 0$ such that

$$J_0(v) \geqq \check{c}_1 \|v\|_{\mathscr{H}} - \check{c}_2 \quad \forall v \in \mathscr{K} \cap \mathfrak{P}_1 . \tag{2.7}$$

The converse assertion yields the existence of a sequence $\{v_k\} \subset \mathscr{K} \cap \mathfrak{P}_1$, $\|v_k\|_{\mathscr{H}} \to$ $\to +\infty$, $v_k/\|v_k\|_{\mathscr{H}} \equiv w_k \rightharpoonup w_0 \in \mathscr{K} \cap \mathfrak{P}_1$ such that (2.6) with $u_0^1 = 0$ holds. Hence $\Pi_{\mathfrak{P}} w_k \to 0$, $w_k \to w_0 \in \mathfrak{R}_2 \cap \mathscr{K}$, $\|w_0\|_{\mathscr{H}} = 1$, $L(w_0) \leqq 0$ which contradicts (2.2). The case (2.3) is analogous, there are $\check{c}_1 > 0$, $\check{c}_2 \geqq 0$ such that (2.8)

$$a(v, v) - (f, v) - [T_0, v] \geqq$$
$$\geqq \check{c}_1 \|v\|_{\mathscr{H}(\Omega; R^3)} - \check{c}_2 \;\; \forall v \in \mathscr{H}(\Omega; R^3), \quad v_n/\Gamma_c \leqq 0 , \tag{2.8}$$

where we have omitted the indices 1 in the corresponding terms. For the above used technique cf. [4].

Analogously as in [3] we are able to prove the existence of a solution of $\langle g_n \rangle$ for every $g_n \in \mathbf{C}^{*-}$ and the continuity of the operator Φ_0: $\mathscr{F} g_n \mapsto \mathscr{F} T_n(u)$, single-valued due to the uniqueness of $\Pi_{\mathfrak{P}}$ of the solution of $\langle g_n \rangle$. The inequalities (2.5), (2.7), (2.8) ensure the uniform boundeness of Φ_0 on $\mathbf{C}^{*-}$. Now we use the technique developed in [3], Secs. 3 and 4, and the Tichonov fixed point theorem to prove the existence of a solution of the contact (or Signorini) problem with friction under one of the assumptions (2.1), (2.2), (2.3) and one of the following conditions:

$$\|\mathscr{F}\|_\infty < \sqrt{\left(\frac{a_{0,1} a_{0,2}}{2A_{0,1} A_{0,2}}\right)} \frac{A_{0,1} + A_{0,2}}{\sqrt{(a_{0,1} A_{0,1})} + \sqrt{(a_{0,2} A_{0,2})}} , \tag{2.9a}$$
$$\text{for} \quad A_{0,1} \leqq A_{0,2} \quad \text{and} \quad \frac{a_{0,2}}{a_{0,1}} \geqq \frac{(A_{0,2} - A_{0,1})^2}{4A_{0,1} A_{0,2}} ;$$

$$\|\mathscr{F}\|_\infty < \sqrt{\left(\frac{2a_{0,1} a_{0,2}}{A_{0,1}}\right)} \frac{1}{\sqrt{(a_{0,2})} + \sqrt{(a_{0,1} + a_{0,2})}} , \tag{2.9b}$$
$$\text{for} \quad A_{0,1} \leqq A_{0,2} \quad \text{and} \quad \frac{a_{0,2}}{a_{0,1}} \leqq \frac{(A_{0,2} - A_{0,1})^2}{4A_{0,1} A_{0,2}} ;$$

$$\|\mathscr{F}\|_\infty < \sqrt{\left(\frac{a_0}{2A_0}\right)} \quad \text{in the Signorini case} \tag{2.10}$$

($\|\cdot\|_\infty$ is the norm in $\boldsymbol{L}_\infty(\Gamma_c)$).

If both $\mathscr{B}_1$ and $\mathscr{B}_2$ are homogeneous and isotropic, the estimate foı $\|\mathscr{F}\|_\infty$ in (2.9), (2.10) can be replaced by (2.11), (2.12), respectively, where σ_ι are the appropriate Poisson ratios, E_ι the corresponding Young moduli of elasticity,

$$s_\iota = \frac{1-\sigma_\iota}{1-2\sigma_\iota}, \quad t_\iota = \frac{E_\iota}{1+\sigma_\iota}, \quad \iota = 1, 2, \quad \mathfrak{v} = \sqrt{\frac{s_1}{s_2}} \quad \text{and} \quad \mathscr{T} = \frac{t_2}{t_1}:$$

(2.11)

$$\|\mathscr{F}\|_\infty < \frac{1}{\sqrt[4]{(2s_1s_2)}} \frac{t_1\sqrt{s_1} + t_2\sqrt{s_2}}{t_1\sqrt[4]{s_1} + t_2\sqrt[4]{s_2}} \quad \text{if} \quad \mathfrak{v} \geqq 4 \;\&\; \mathscr{T} \geqq \mathfrak{v} - 2\sqrt{\mathfrak{v}} \quad \text{or} \quad \mathfrak{v} \in \langle 0, \tfrac{1}{4}\rangle \;\&$$

$$\& \; \mathscr{T} \in \left\langle 0, \frac{\mathfrak{v}}{1-2\sqrt{\mathfrak{v}}}\right\rangle \quad \text{or} \quad \mathfrak{v} \in \langle \tfrac{1}{4}, 4\rangle \;\&\; \mathscr{T} \geqq 0\,,$$

$$\|\mathscr{F}\|_\infty < \frac{1}{\sqrt[4]{(2s_1)}} \cdot \frac{2}{1+\sqrt{(1+1/\mathscr{T})}} \quad \text{if} \quad \mathfrak{v} \in \langle 0, \tfrac{1}{4}\rangle \;\&\; \mathscr{T} \geqq \frac{\mathfrak{v}}{1-2\sqrt{\mathfrak{v}}}\,,$$

$$\|\mathscr{F}\|_\infty < \frac{1}{\sqrt[4]{(2s_2)}} \cdot \frac{2}{1+\sqrt{(1+\mathscr{T})}} \quad \text{if} \quad \mathfrak{v} \geqq 4 \;\&\; \mathscr{T} \in \langle 0, \mathfrak{v} - 2\sqrt{\mathfrak{v}}\rangle\,;$$

(2.12)

$$\|\mathscr{F}\|_\infty < \sqrt[4]{\left(\frac{1-2\sigma}{2-2\sigma}\right)}.$$

Theorem 2.1. *Let all the suppositions of Sec.* 1, (2,1) *or* (2.2) *and, furthermore, one of the conditions* (2.9) *or* (2.11) *hold. Then there exists a solution of the contact problem with friction. If all the suppositions of Sec.* 1 *for the Signorini case,* (2.3) *and* (2.10) *or* (2.12) *hold, then there is at least one solution of the Signorini problem with friction.*

Remark 2.1. As $a_{0,\iota}$, $A_{0,\iota}$ in (2.9), we can take $\lim\limits_{\delta\to 0} \inf\limits_{\substack{x\in B_\delta{}^3(\operatorname{supp}\mathscr{F})\cap\Omega_\iota \\ \xi\in R^9}} |\xi|^{-2}\, a^\iota_{ijkl}(x)\, \xi_{ij}\xi_{kl}$, $\lim\limits_{\delta\to 0} \sup\limits_{\substack{\xi\in B_\delta{}^3(\operatorname{supp}\mathscr{F})\cap\Omega_\iota \\ \xi\in R^9}} |\xi|^{-2}\, a^\iota_{ijkl}(x)\, \xi_{ij}\xi_{kl}$, respectively, instead of the constants from (1.2). The case (2.10) is analogous. The same assertion is true for the coercive case. The reader can easily reformulate (2.9) provided $A_{0,1} \geqq A_{0,2}$.

Remark 2.2. More regular solutions of the contact problem with friction can be found both in the coercive and the semicoercive case. For instance in [2], existence of a solution with $T_n(u) \in L_2(\Gamma_c)$, $u/\Gamma_c \in H^1(\Gamma_c; R^2)$ for Ω being a strip in R^2 is proved.

3. TWO-DIMENSIONAL SIGNORINI PROBLEM WITH A STRAIGHT CONTACT SURFACE

Let $\Omega \equiv \Omega_1 \subset R^2$, $\Gamma_c = \langle x_1, x_2\rangle \times \{0\}$, $x_1, x_2 \in R^1 \cup \{-\infty, +\infty\}$ (after a suitable rotation and shift). We assume that $n/\Gamma_c = [0, 1]$ and $\Gamma_T = \Gamma \setminus \Gamma_c$. Let the appropriate suppositions of Sec. 1 be valid in their two-dimensional modification.

Denote $L(w) = -[T_0, w] - (f, w)$ (the index 1 is omitted), $w \in \boldsymbol{H}^1(\Omega; R^2)$, $\mathfrak{R} := \{w \in \boldsymbol{H}^1(\Omega; R^2), \exists z \in \mathscr{Z}, w/\Omega = z/\Omega\}$, $\mathfrak{P} = \mathfrak{R}^\perp$, $\varphi_1 \equiv ([x, y] \mapsto [1, 0], [x, y] \in R^2)$, $\varphi_2 \equiv ([x, y] \mapsto [0, 1], [x, y] \in R^2)$, $\psi \equiv ([x, y] \mapsto [-y, x], [x, y] \in R^2)$, $\mathfrak{R}' = \operatorname{sp} \varphi_1$, $\mathfrak{R}'' = \operatorname{sp}\{\varphi_2, \psi\}$, where sp denotes the span. Of course $\mathfrak{R}' \subset \mathscr{K} := \{v \in \boldsymbol{H}^1(\Omega; R^2); v_2/\Gamma_c \leqq 0\}$. Our purpose is to prove the following theorem.

Theorem 3.1. *Under the suppositions stated in Sec. 1 let* $L(\varphi_1) = 0$, *let* $v \in \mathscr{K} \cap \mathfrak{R}'' \setminus \{0\}$ *imply* $L(v) > 0$. *Let* $\|\mathscr{F}\|_\infty < \sqrt[4]{((1 - 2\sigma)/(2 - 2\sigma))}$ *in the homogeneous isotropic case,* $\|\mathscr{F}\|_\infty < \sqrt{(a_0/2A_0)}$ *in the general case. Then there exists a solution of the Signorini problem with friction.*

Proof. Without a loss of generality we suppose $x_1 = 0$, $x_2 = 2\pi$, $\varepsilon > 0$, $\operatorname{supp} \mathscr{F} \subset \langle 4\varepsilon, 2\pi - 4\varepsilon\rangle$. Let $\varrho \in \boldsymbol{C}^{2,1}(R^2)$ have $\operatorname{dist}(\operatorname{supp} \varrho, \Gamma_T) > 2\varepsilon$, let $\operatorname{supp} \varrho \subset (0, 2\pi) \times (-1, 1)$. Let $\varrho(R^2) \subset \langle 0, 1\rangle$, $\varrho/\langle 3\varepsilon, 2\pi - 3\varepsilon\rangle \times \{0\} = 1$. Analogously to the procedure used in Sec. 2, for every $g_n \in \boldsymbol{C}^{*-} \setminus \{0\}$ there is a minimum point of the functional $J_{g_n}(v) = \frac{1}{2}a(v, v) + L(v) + \langle \mathscr{F}|g_n|, |v_t|\rangle$ on $\mathscr{K}$. J_0 does not depend on elements of $\mathfrak{R}'$, hence we can find one of its minimum points u on $\mathscr{K} \cap (\mathfrak{P} \oplus \mathfrak{R}'')$, where it is coercive, and for every $r \in R^1$, $u + r\varphi_1$ is a minimum point of J_0 on $\mathscr{K}$. Because of a suitable analogue to (2.8) and of the inequality

$$J_0(u) \leqq J_{g_n}(u) \leqq J_{g_n}(0) = J_0(0) = 0\,, \quad g_n \in \boldsymbol{C}^{*-}\,, \tag{3.1}$$

which is valid for every solution of the problem $\langle g_n\rangle$, there is a constant $\tilde{K}$ independent of g_n and such that $\Pi_{\mathfrak{P}\oplus\mathfrak{R}''} u \leqq \tilde{K}$. The operator $\Phi_0: \mathscr{F} g_n \mapsto \mathscr{F} T_n(u)$ acting from $\boldsymbol{C}^{*-}$ into itself is continuous. Namely, let $\mathscr{F} g_n^m \to \mathscr{F} g_n^0 \neq 0$ in $\boldsymbol{H}^{-1/2}(\Gamma_c)$, then $\langle \mathscr{F}|g_n^m|, (\varphi_1)_t\rangle > k_0$, $m = 0, m_0, m_0 + 1, \ldots$, for a suitable m_0. Hence $(k_0/\sqrt{(2\pi)})\,\|\Pi_{\mathfrak{R}'} u_m\|_{1/2,\Gamma_c} \leqq \langle \mathscr{F}|g_n^m|, |(\Pi_{\mathfrak{R}'} u_m)_t|\rangle \leqq 2\langle \mathscr{F}|g_n^m|, |(\Pi_{\mathfrak{P}} u_m)_t|\rangle \leqq \text{const}$, $m = 0, m_0, m_0 + 1, \ldots$, provided u_m is a solution of $\langle g_n^m\rangle$. Now, there are constants $k', k'' > 0$ such that for every $m \geqq m_0$

$$\begin{aligned} k'\|\Pi_{\mathfrak{P}} u_m - \Pi_{\mathfrak{P}} u_0\|_{1,\Omega}^2 &\leqq a(u_m - u_0, u_m - u_0) \leqq \\ &\leqq \|\mathscr{F} g_n^m - \mathscr{F} g_n^0\|_{-1/2,\Gamma_c}[\|u_0\|_{1,\Omega} + \|u_m\|_{1,\Omega}] \leqq k''\|\mathscr{F} g_n^m - \mathscr{F} g_n^0\|_{-1/2,\Gamma_c}\,. \end{aligned} \tag{3.2}$$

Hence $\mathscr{F}\, T_n(u_m) \to \mathscr{F}\, T_n(u_0)$ in $\boldsymbol{H}^{-1/2}(\Gamma_c)$. If $\mathscr{F} g_n^m \to 0$, then there exists a subsequence of $\{u_m\}$ (denoted by $\{u_m\}$ again) such that $\Pi_{\mathfrak{P}\oplus\mathfrak{R}''} u_m \rightharpoonup \tilde{u} \in \boldsymbol{H}^1(\Omega; R^2)$. For each $v \in \mathscr{K}$,

$$\begin{aligned} J_0(\tilde{u}) \leqq \varliminf_{m\to+\infty} J_0(u_m) \leqq \varlimsup_{m\to+\infty} J_0(u_m) \leqq \varlimsup_{m\to+\infty} J_{g_n^m}(u_m) \leqq \\ \leqq \lim_{m\to+\infty} J_{g_n^m}(v) = J_0(v)\,, \end{aligned} \tag{3.3}$$

hence $\tilde{u}$ solves $\langle 0\rangle$, $J_0(\tilde{u}) = \lim_{m\to+\infty} J_0(u_m)$ (put $v = \tilde{u}$ in (3.3)). In particular, $a(u_m, u_m) \to a(\tilde{u}, \tilde{u})$. This together with the Korn inequality yields $\Pi_{\mathfrak{P}} u_m \to \Pi_{\mathfrak{P}} \tilde{u}$ in $\boldsymbol{H}^1(\Omega; R^2)$ and $T_n(u_m) \to T_n(u_0)$ in $\boldsymbol{H}^{-1/2}(\Gamma_c)$.

The proof of regularity for $\alpha \in (0, \frac{1}{2})$ and the estimates making the use of the Tichonov fixed point theorem possible require a certain modification due to the non-existence of a satisfactory estimate of $\Pi_{\mathfrak{R}'}u$ for u solving $\langle g_n \rangle$. In the variational inequality to $\langle g_n \rangle$ – cf. (1.4) – we put $u + \varrho((\Pi_{\mathfrak{P}\oplus\mathfrak{R}''}u)_{-h} - \Pi_{\mathfrak{P}\oplus\mathfrak{R}''}u)$ for v, in the shifted inequality we put $u_{-h} + \varrho(\Pi_{\mathfrak{P}\oplus\mathfrak{R}''}u - (\Pi_{\mathfrak{P}\oplus\mathfrak{R}''}u)_{-h})$ for v_{-h} $(u_{-h}(x) \equiv u(x+h)$, $h \equiv [h, 0] \in R^1)$. With the exception of the term

$$(3.4) \quad \mathscr{I} \equiv \int_{-\infty}^{+\infty} |h|^{-1-2\alpha} (\langle \mathscr{F}|g_n|, |v_t| - |u_t| \rangle + \langle (\mathscr{F}|g_n|)_{-h}, |v_t|_{-h} - |u_t|_{-h} \rangle) \, dh$$

all the other terms do not depend on $\Pi_{\mathfrak{R}'}u$ and can be estimated as usual (cf. [3]). Denote $\mathscr{u} = \Pi_{\mathfrak{P}\oplus\mathfrak{R}''}u$, let $\Pi_{\mathfrak{R}'}u = q\varphi_1 \equiv q \in R^1$. Because of the suppositions on supp $\mathscr{F}$ and ϱ we can assume that

$$v_t = (\varrho \mathscr{u}_t)_{-h} + q\,, \quad (v_t)_{-h} = \varrho \mathscr{u}_t + q\,, \quad u_t = \varrho \mathscr{u}_t + q\,, \quad (u_t)_{-h} = (\varrho \mathscr{u}_t)_{-h} + q\,,$$

for $|h| \leqq \varepsilon$ and we obtain

$$(3.5) \quad \mathscr{I} = \int_{-\varepsilon}^{\varepsilon} |h|^{-1-2\alpha} \langle (\mathscr{F}|g_n|)_{-h} - \mathscr{F}|g_n|, |(\varrho \mathscr{u}_t)_{-h} + q| - |\varrho \mathscr{u}_t + q| \rangle \, dh +$$
$$+ \int_{|h| \geqq \varepsilon} |h|^{-1-2\alpha} \langle \mathscr{F}|g_n|, |\varrho(\mathscr{u}_t)_{-h} + (1-\varrho)\, \mathscr{u}_t + q| - |\mathscr{u}_t + q| \rangle +$$
$$+ \langle (\mathscr{F}|g_n|, |\varrho \mathscr{u}_t + (1-\varrho)(\mathscr{u}_t)_{-h} + q| - |(\mathscr{u}_t)_{-h} + q| \rangle \, dh\,.$$

The second term in (3.5) can be estimated by

$$\int_{|h| \geqq \varepsilon} 2\|\mathscr{F}\|_\infty \|\varrho g_n\|_{-1/2,R^1} \|\varrho((\mathscr{u}_t)_{-h} - \mathscr{u}_t)\|_{1/2,R^1} |h|^{-1-2\alpha} \, dh \leqq$$
$$\leqq \frac{4}{\alpha \varepsilon^{2\alpha}} \|\mathscr{F}\|_\infty \|\varrho g_n\|_{-1/2,R^1} \|\varrho \mathscr{u}\|_{1/2,R^1} \leqq k_1(\varepsilon, \alpha, f, T_0, \varrho) \|\mathscr{F}\|_\infty \|\varrho g_n\|_{-1/2,R^1}\,.$$

Denote $\tilde{\omega} = |\varrho \mathscr{u}_t + q|$, $\mathfrak{F}^h = \tilde{\omega}_{-h} - \tilde{\omega}$ and the first term in (3.5) by $\mathscr{I}_\varepsilon$. We have

(3.6)
$$\mathscr{I}_\varepsilon \leqq \int_{-\varepsilon}^{\varepsilon} \|(\mathscr{F}g_n)_{-h} - \mathscr{F}g_n\|^2_{-1/2,R^1} |h|^{-1-2\alpha} \, dh\Big)^{1/2} \Big(\int_{-\varepsilon}^{\varepsilon} \|\mathfrak{F}^h\|^2_{1/2,R^1} |h|^{-1-2\alpha} \, dh \Big)^{1/2} \leqq$$
$$\leqq \|\mathscr{F}\|_\infty \left[\|\varrho g_n\|_{-1/2+\alpha,R^1} \sqrt{\left(\frac{c(\alpha)\, c(\frac{1}{2} - \alpha)}{c(\frac{1}{2})} \right)} + K_1(\alpha) \|\varrho g_n\|_{-1/2,R^1} \right] \cdot$$
$$\cdot \left[\left(\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (\tilde{\omega}_{-h} - \tilde{\omega})^2 |h|^{-1-2\alpha} \, dx \, dh \right) + \left(\int_{-\varepsilon}^{\varepsilon} |h|^{-1-2\alpha} \|\mathfrak{F}^h\|'^2_{1/2,R^1} \, dh \right)^{1/2}, \right.$$

where

$$(3.7) \qquad \|w\|'^2_{1/2,R^1} \equiv \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{(w_{-h}(x) - w(x))^2}{|h|^2} \, dx \, dh\,, \quad w \in H^{1/2}(R^1)\,,$$

$c(\alpha)$ will be given in (3.10) and $K_1(\alpha)$ is a suitable constant. Naturally

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(\tilde{\omega}_{-h}-\tilde{\omega})^2\,|h|^{-1-2\alpha}\,\mathrm{d}x\,\mathrm{d}h \leqq$$

$$\leqq \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}((\varrho u_t)_{-h}-\varrho u_t)^2\,.\,|h|^{-1-2\alpha}\,\mathrm{d}x\,\mathrm{d}h \leqq k_2(f, T_0, \varrho)\,,$$

so it remains to estimate the most important term $\int_{-\varepsilon}^{\varepsilon}|h|^{-1-2\alpha}\,\|\tilde{\vartheta}^h\|_{1/2,R^1}'^2\,\mathrm{d}h$. $\operatorname{Supp}\tilde{\vartheta}^h \subset \langle +\varepsilon, 2\pi-\varepsilon\rangle$, hence

$$(3.8)\quad \|\tilde{\vartheta}^h\|_{1/2,R^1}'^2 = \int_0^{2\pi}\int_0^{2\pi}\left(\frac{\tilde{\vartheta}^h(x)-\tilde{\vartheta}^h(y)}{x-y}\right)^2\mathrm{d}x\,\mathrm{d}y + 2\int_{-\infty}^{0}\int_0^{2\pi}\left(\frac{\tilde{\vartheta}^h(x)}{x-y}\right)^2\mathrm{d}x\,\mathrm{d}y + 2\int_{2\pi}^{+\infty}\int_0^{2\pi}\left(\frac{\tilde{\vartheta}^h(x)}{x-y}\right)^2\mathrm{d}x\,\mathrm{d}y\,.$$

The sum of the second and the third term in (3.8) is equal to

$$4\pi\int_0^{2\pi}\frac{(\tilde{\vartheta}^h(x))^2\,\mathrm{d}x}{x(2\pi-x)} \leqq \frac{4\pi}{\varepsilon(2\pi-\varepsilon)}\int_0^{2\pi}((\varrho u_t)_{-h}-\varrho u_t)^2\,\mathrm{d}x\,,\quad \text{and}$$

$$\int_{-\varepsilon}^{\varepsilon}|h|^{-1-2\alpha}\frac{4\pi}{\varepsilon(2\pi-\varepsilon)}\int_0^{2\pi}((\varrho u_t)_{-h}-\varrho u_t)^2\,\mathrm{d}x\,\mathrm{d}h \leqq k_3(\varepsilon, \alpha, f, T_0, \varrho)\,.$$

Let us introduce the 2π-periodical function ω such that $\omega/\langle 0, 2\pi\rangle = \tilde{\omega}/\langle 0, 2\pi\rangle$, $\vartheta^h = \omega_{-h}-\omega$. We have

$$\int_0^{2\pi}\int_0^{2\pi}\left(\frac{\tilde{\vartheta}^h(x)-\tilde{\vartheta}^h(y)}{x-y}\right)^2\mathrm{d}x\,\mathrm{d}y = \int_0^{2\pi}\int_0^{2\pi}\left(\frac{\vartheta^h(x)-\vartheta^h(y)}{x-y}\right)^2\mathrm{d}x\,\mathrm{d}y \leqq$$

$$\leqq \int_{-\infty}^{\infty}|\hbar|^{-2}\int_0^{2\pi}(\vartheta^h(x+\hbar)-\vartheta^h(x))^2\,\mathrm{d}x\,\mathrm{d}\hbar,\ |h|<\varepsilon\,.$$

For a 2π-periodical function F and $\alpha\in(0,1)$ we introduce a seminorm $\|\cdot\|_\alpha^\Delta$ by

$$(3.9)\qquad (\|F\|_\alpha^\Delta)^2 \equiv \int_{-\infty}^{+\infty}\int_0^{2\pi}|h|^{-1-2\alpha}(F(x+h)-F(x))^2\,\mathrm{d}x\,\mathrm{d}h\,.$$

We define

$$(3.10)\qquad c(\alpha) = 2^{2-2\alpha}\int_{-\infty}^{+\infty}\frac{\sin^2 t}{|t|^{1+2\alpha}}\,\mathrm{d}t\,.$$

Lemma 3.1. *For every $\alpha, \beta \in \langle 0, 1\rangle$, $\alpha+\beta<1$, and for every 2π-periodical function F such that one of the parts of (3.11) is finite, we have*

$$(3.11)\qquad \int_{-\infty}^{+\infty}|\delta|^{-1-2\beta}(\|F_{-\delta}-F\|_\alpha^\Delta)^2\,\mathrm{d}\delta = \frac{c(\alpha)\,c(\beta)}{c(\alpha+\beta)}(\|F\|_{\alpha+\beta}^\Delta)^2\,.$$

Proof. Let $F(x) = (2\pi)^{-1/2} \sum_{k=-\infty}^{+\infty} s_k e^{ikx}$ (the Fourier expansion), where $s_k = (1/\sqrt{(2\pi)}) \cdot \int_0^{2\pi} F(y)\, e^{-iky}\, dy$ — the conveigence is in the sense of $L_2(0, 2\pi)$. We have

$$(3.12)\quad (\|F\|_\alpha^\Delta)^2 = \sum_{k=-\infty}^{+\infty} |s_k|^2 \int_{-\infty}^{+\infty} |h|^{-1-2\alpha} \cdot |1 - e^{ikh}|^2\, dh = c(\alpha) \sum_{k=-\infty}^{+\infty} |s_k|^2\, |k|^{2\alpha}.$$

Hence

$$\int_{-\infty}^{+\infty} (\|F_{-\delta} - F\|_\alpha^\Delta)^2\, |\delta|^{-1-2\beta}\, d\delta = c(\alpha) \sum_{k=-\infty}^{+\infty} |s_k|^2\, |k|^{2\alpha} \int_{-\infty}^{+\infty} |\delta|^{-1-2\beta}\, |1 - e^{ik\delta}|^2\, d\delta =$$

$$= \frac{c(\alpha)\, c(\beta)}{c(\alpha + \beta)} (\|F\|_{\alpha+\beta}^\Delta)^2 .$$

Applying Lemma 3.1 to $F = \omega$, we obtain

$$\int_{-\infty}^{+\infty} (\|\omega_{-h} - \omega\|_{1/2}^\Delta)^2\, |h|^{-1-2\alpha}\, dh = \frac{c(\frac{1}{2})\, c(\alpha)}{c(\frac{1}{2} + \alpha)} \int_{-\infty}^{+\infty} \int_0^{2\pi} \frac{(\omega(x + h) - \omega(x))^2}{|h|^{2+2\alpha}}\, dx\, dh \leqq$$

$$\leqq \frac{c(\frac{1}{2})\, c(\alpha)}{c(\frac{1}{2} + \alpha)} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{((\varrho u_t)_{-h} - \varrho u_t)^2}{|h|^{2+2\alpha}}\, dx\, dh + k_4(f, T_0, \varrho, \alpha) .$$

Summing up this estimate, (3.5), (3.6) and (3.8), we get

$$(3.13)\quad \mathcal{I} \leqq \|\mathcal{F}\|_\infty \left[c(\alpha) \sqrt{\left(\frac{c(\frac{1}{2} - \alpha)}{c(\frac{1}{2} + \alpha)} \right)} \|\varrho g_n\|_{-1/2+\alpha, R^1} + K_2(\alpha)\, \|\varrho g_n\|_{-1/2, R^1} \right] \cdot$$

$$\cdot \left[\|u_t \varrho\|_{1/2+\alpha, R^1} + k_5(\varepsilon, \alpha, f, T_0, \varrho) \right] + k_6(f, T_0, \varrho) .$$

Carrying out the appropriate estimations like in Sec. 2 of [3] and using (3.13), we obtain

$$\int_{-\infty}^{+\infty} |h|^{-1-2\alpha}\, a((\varrho u)_{-h} - \varrho u, (\varrho u)_{-h} - \varrho u)\, dh \leqq$$

$$\leqq (1 + \varepsilon')\, \|\mathcal{F}\|_\infty \sqrt{\left(\frac{2c(\alpha)\, c(\frac{1}{2} - \alpha)}{a_0} \right)} \left[\|\varrho g_n\|_{-1/2+\alpha, R^1} + K(\alpha)\, \|\varrho g_n\|_{-1/2, R^1} \right] \cdot$$

$$\cdot \left[\left(\int_{-\infty}^{+\infty} |h|^{-1-2\alpha}\, a((\varrho u)_{-h} - \varrho u, (\varrho u)_{-h} - \varrho u)\, dh \right)^{1/2} + k_7(\varepsilon, \alpha, f, T_0, \varrho) \right] +$$

$$+ k_8(f, T_0, \varrho, \varepsilon') , \quad \varepsilon' > 0 \quad \text{arbitrary} ,$$

hence

$$(3.14)\quad \left(\int_{-\infty}^{+\infty} |h|^{-1-2\alpha}\, a((\varrho u)_{-h} - \varrho u, (\varrho u)_{-h} - \varrho u)\, dh \right)^{1/2} \leqq$$

$$\leqq (1 + \varepsilon')\, \|\mathcal{F}\|_\infty \sqrt{\left(\frac{c(\alpha)\, c(\frac{1}{2} - \alpha)}{a_0} \right)} \left[\|\varrho g_n\|_{-1/2+\alpha, R^1} + K(\alpha)\, \|\varrho g_n\|_{-1/2, R^1} \right] +$$

$$+ k_9(\varepsilon, \alpha, \varepsilon', \varrho, f, T_0) .$$

The use of the uniform boundedness of $\varrho\, T_n(u)$ on $H^{-1/2}(R^1)$ (defined by means of (1.5)) and of the equivalence of the norms $\|\cdot\|_{-1/2+\alpha,R^1}$ and $\|\cdot\|_{-1/2+\alpha,R^1} + K(\alpha)\,.$ $.\,\|\cdot\|_{-1/2,R^1}$ on $H^{-1/2+\alpha}(R^1)$ completes the proof.

Remark 3.1. In the 3-dimensional case, if Γ_c is stiainght in only one direction, an analogous theorem can be proved by the same method. If Γ_c is part of a plane, then existence is still an open problem because of the rotation around the axis perpendicular to the plane.

Remark 3.2. If $L(\mathfrak{R}'') = 0$ then for each $v \in \mathscr{H}$ with $v/\Gamma_c \in L_\infty(\Gamma_c)$ there exists $\varrho_0 \in R^1$ such that $r + \varrho_0\varphi_2 \in \mathscr{K}$. Provided $g_n \in C^{*-} \cap H^{-1/2+\alpha}(\Gamma_c)$, the solution $u(g_n)$ of

Problem $\langle g_n \rangle'$.

$$J_{g_n}(v) \to \min\,, \quad v \in \mathscr{H}\,,$$

has Γ_c-traces in $H^{1/2+\alpha}(\Gamma_c)$, hence in $L_\infty(\Gamma_c)$ (the proof uses analogous arguments as for $\langle g_n \rangle$). For every $v \in \mathscr{H}$ with $v_t/\Gamma_c = 0$, $\langle T_n(u), v_n \rangle = a(u, v) - L(v) = 0$. So $T_n(u) = 0$ for every $u \equiv u(g_n)$, $g_n \in C^{*-} \cap H^{-1/2+\alpha}(\Gamma_c)$, and the set of all solutions of $\langle 0 \rangle'$ is exactly the set of all solutions of the Signorini problem with friction in the described case.

Remark 3.3. The problem with a given normal displacement and with friction in the Coulomb sense (see [1]) can be solved by the same methods as those used for the Signorini problem with friction. The estimations for maximal admissible $\|\mathscr{F}\|_\infty$ for the existence theorem are identical, the other sufficient conditions are very similar.

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