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SIXTY YEARS OF PROFESSOR VLASTIMIL PTÁK

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Professor Vlastimil Pták was born in Prague on 8 November 1925. He studied Mathematics and Physics at the Faculty of Science of Charles University in Prague in the years 1945–1948. Already as a student he worked as Assistant at the Czech Technical University. After obtaining his doctor's (RNDr.) degree he became research student (aspirant) of Academician M. Katětov. In 1952 je joined the Mathematical Institute of the Czechoslovak Academy of Sciences (then called the Central Mathematical Institute). He received his Candidate of Science degree (CSc.) in 1955 and his Doctor of Science degree (DrSc.) in 1963. In the same year he was appointed Associate Professor and in 1966 Full Professor. He has been head of Department of Functional Analysis at the Institute since 1960.

The scientific activity of Prof. Pták is extraordinary in several aspects: in its depth, its range and the number of original research papers. It includes above all results from various domains of functional analysis, theory of matrices and numerical mathematics.

Prof. Pták obtained his first results in algebra already as a student. His papers which soon followed and in which he dealt with one of the fundamental principles of linear functional analysis – the openness of linear mappings and the continuity of the inverse operator in locally convex spaces – aroused wide interest. His contribution to this field which has been extensively and continuously studied is of fundamental importance. As it is well known, the proof of the classical theorem on the open mapping consists of two steps of different nature. If $\mathscr{E}$, $\mathscr{F}$ are Banach spaces, A a bounded linear operator from $\mathscr{E}$ to $\mathscr{F}$ and $A(\mathscr{E})$ is "sufficiently rich" in $\mathscr{F}$, then we first prove that $\overline{A(\mathscr{U})}$ is a neighbourhood of zero in $\overline{A(\mathscr{E})} = \mathscr{F}$ for an arbitrary neighbourhood $\mathscr{U}$ of zero in $\mathscr{E}$. Then we prove that if $\overline{A(\mathscr{U})}$ is a neighbourhood of zero for every neighbourhood $\mathscr{U}$ of zero in $\mathscr{E}$, then $A(\mathscr{U})$ itself is a neighbourhood of zero. Here the completeness of the space $\mathscr{E}$ plays the crucial role. By analyzing these conditions Prof. Pták succeeded in discovering the very foundation of the classical theorems. He introduced the notion of an almost open mapping, that is, a mapping that – roughly speaking – satisfies the assertion of the second step, and applying an original method using duality he characterized spaces in which every almost open mapping is open. This class of spaces satisfies a completeness condition which is generally stronger than the usual completeness. Similarly he obtained an analogous class of spaces when studying the problem of continuity of the inverse operator. His name has been associated with both these two classes of spaces in mathematical literature, and in 1966 he was granted Klement Gottwald State Prize for the discovery of new methods in this field.

The results mentioned are closely connected with the Closed Graph Theorems and

generally with theorems on continuity of linear mappings, to which other Pták's papers are devoted. He has resumed research in this field several times, dealing with applications to some special spaces and with a quantitative version of the problem. The method of continuous induction, which will be mentioned later, also exhibits certain relations to this field.

Other important results form the contents of a series of papers devoted to one of the fundamental problems of analysis, the problems of interchanging the order of limit processes. For solving problems of this type Prof. Pták suggested the method of convex means which may be applied in the proof of a considerably general theorem on the extension of separately continuous functions. This result covers a number of classical theorems and in literature is often connected with this name. Its basis is a simple combinatorial lemma giving conditions for the existence of certain convex means whose character makes it possible to verify weak compactness on sequences.

Of fundamental importance are also Pták's results in the theory of Hermitean algebras. Here the object of investigation is a Banach algebra $\mathscr{A}$ with a unit and an involution *, that is, an idempotent antilinear mapping of $\mathscr{A}$ into itself without a continuity assumption. The theory is based on the fundamental inequality $|x|_\sigma^2 \leqq |x^*x|_\sigma$ where $x \in \mathscr{A}$ and $|\ |_\sigma$ is the spectral radius. This inequality is an algebraic analogon of the well known C* condition $|x|^2 = |x^*x|$. While the metric C* condition guarantees that the given algebra is isometrically isomorphic to a C* algebra of operators on a Hilbert space, the inequality $|x|_\sigma^2 \leqq |x^*x|_\sigma$ is equivalent to the algebraic condition that selfadjoint elements ($x^* = x$) have a real spectrum. It has turned out that this inequality, also frequently connected with the name of Prof. Pták, is sufficient for developing a very satisfactory theory, which represents an algebraic generalization of the theory of C* algebras.

From the papers devoted to the theory of algebras let us further recall the results concerning factorization and continuity of the spectrum. From algebraic papers, the original application of the duality theory to a surprisingly easy deduction of Jordan's normal form is of special interest.

A number of papers which are important for numerical mathematics are mainly devoted to some properties of nonnegative matrices. Most of them were written together with M. Fiedler. They concern the problem of convergence of iterative processes, in particular the dependence of the rate of convergence on the decomposition of the matrix of the system into a sum of two suitably chosen matrices. These results are applicable to an important problem of numerical analysis, the localization of eigenvalues of matrices. For these results the authors were awarded the Czech National Prize in 1978.

Another field of interest of Prof. Pták, to the development of which he has contributed in several directions, is the theory of iterative processes.

Mathematical formulation of the dependence of the convergence of a process on the initial steps led him to the introduction of the notion of critical exponent. If a process is represented by a matrix (operator) A in a finite dimensional Banach space $\mathscr{H}$, $|A| = 1$, $|A|_\sigma < 1$, then there exists an exponent r $(= r(A))$ such $|A^r| < 1$. The question is whether there is such an exponent q that $|A^q| < 1$ for all $A \in B(\mathscr{H})$, $|A| = 1$, $|A|_\sigma < 1$. It turned out that the existence and magnitude of the least of these exponents, the so called critical exponent, essentially depends on the form of the unit ball in $\mathscr{H}$. Later some deep relations of the solution of the problem with a localized spectrum to the theory of complex functions and the theory of dilatations of contractions in a Hilbert space were established. A cooperation with N. J. Young led in this connection to valuable contributions to the classical problem of determining the number of roots of a polynomial inside, on, and outside the unit circle. These problems represent a contact zone of several branches of mathematics, which is a characteristic feature of the modern functional analysis.

The idea of an open mapping inspired another series of papers devoted to the convergence of iterative processes. Prof. Pták invented a general method, the so called method of nondiscrete induction, which represents a theory applicable to a number of iterative processes, starting with the Newton method. The rate of convergence is replaced by a nonnegative function of real variable (or, as the case may be, a multidimensional variant) which gives a better picture of the problem than the classical concept. The method yields estimates which are sharp not only asymptotically but in each step, optimal initial conditions as well as criteria of convergence. Besides numerous papers, a monograph was published (written with F. A. Potra) devoted wholly to this method.

Even this brief presentation of main topics gives an idea of the scope of problems in which Prof. Pták gained worldwide reputation. Prof. Pták has numerous contacts with foreign mathematicians and receives frequent invitations to conferences and lecture tours. Let us only mention his one-year visits to U.S.A.and to Britain.

Prof. Pták is member of the Editorial Boards of the Czechoslovak Mathematical Journal and the journal Linear Algebra and its Applications. He has held important offices in the organization of Czechoslovak science, for example in committees for scientific degrees and in the board for national projects of basic research.

The seminar in functional analysis, which he has led for many years, has educated a number of research workers. The regular workshops in functional analysis, which he founded and whose organization he has led for sixteen years, represent a source of inspiration not only for the members of his department but for numerous guests as well. For many years he has delivered a special course in functional analysis at Charles University.

We are sure that for many years to come our mathematical community will enjoy the fruits of invention and erudition of Prof. Pták. To him we wish firm health and favourable conditions in his personal life, both being so important for creative work in mathematics.

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