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MULTI-POLARIZED GRAPHS

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1. INTRODUCTION

Zítek in [1] introduced a widely applicable notion of polarized graphs. Zelinka [2]–[8] extended the theory and its applications. The purpose of this remark is to generalize the notion of the polarized graph and to show an example of its application.

The graph $G = (V, H)$ with the vertex set V and the edge set H is called partially polarized if there exists a set $W \subset V$ such that every $w \in W$ has two poles w^+ and w^- (i.e. w is a couple (w^+, w^-)). The graph is called polarized if $W = V$.

If $w \in W$ and $h = (w, v)$ or (v, w), then one of the poles w^+ or w^- with which h is incident is indicated. Therefore we write $h = (w^+, v)$, etc.

On a partially polarized or polarized graph one can define an admissible path $v_0, h_1, v_1, \ldots, v_{n-1}, h_n, v_n$ as a path with the property $v_i \in W \Rightarrow h_{i-1}$ is incident with v_i^+ and h_i with v_i^- or h_{i-1} is incident with v_i^- and h_i with v_i^+.

The graph $G = (V, H)$ is called *multi-polarized* if

1. for every vertex $v \in V$, a pole set $P(v) = \{v^{(1)}, \ldots, v^{(n(v))}\}$ is defined.

2. for every $h \in H$, $h = (v, w)$ a pole from $P(v)$ as well as one from $P(w)$ is indicated with which h is indcident; thus we shall write the edge h in the form $h = (v^{(i)}, w^{(j)})$.

3. for every $v \in V$, $n(v) \geqq 2$, a set of ordered pairs $R(v) \subset P^2(v)$ is given. The set $R(v)$ is called the set of *forbidden transitions* in v.

We do not specify whether G is oriented or not (or mixed), because the following considerations are possible in all the three cases.

The path $v_0, h_1, v_1, \ldots, v_{m-1}, h_m, v_m$ on G is said to br admissible if the condition $(v_s^{(j)}, v_s^{(k)}) \notin R(v_s)$ holds for every $h_s = (v_{s-1}^{(i)}, v_s^{(j)})$, $h_{s+1} = (v_s^{(k)}, v_{s+1}^{(k)})$.

We see that

– for $n(v) \equiv 1$ and $R(v) \equiv \emptyset$ we obtain the case of ordinary graphs

– for $n(v) \equiv 2$ and $R(v) = \{(v^+, v^+), (v^-, v^-)\}$ the case of polarized graphs.

2. WEIGHTED MULTI-POLARIZED GRAPHS

A multi-polarized graph, which may be denoted by $G = (V, H, P, R)$, is called weighted if for every $v \in V$, each $(v^{(i)}, v^{(j)}) \in P^2(v) - R(v)$ is assigned a weight $\gamma(v^{(i)}, v^{(j)}) \in \langle 0, \infty)$ and every $h \in H$ a weight $\gamma(h) \in \langle 0, \infty)$.

For an admissible path $C = (v_0, h_1, \ldots, h_n, v_n)$ where $h_s = (v_{s-1}^{(i_s)}, v_s^{(j_s)})$ we define a weight $\gamma(C)$ as follows:

$$\gamma(C) = \sum_{s=1}^{n} [\gamma(v_s^{(j_s)}, v_s^{(i_{s+1})}) + \gamma(v_{s-1}^{(i_s)}, v_s^{(j_s)})] .$$

An admissible path $C = (v = v_0, h_1, v_1, \ldots, h_n, v_n = w)$ is said to be minimal if $\gamma(C)$ is minimal among all possible weights of admissible paths from v to w.

The problem of finding the minimal admissible path from v to w in a weighted polarized graph is a generalization of that for polrrized graph which can be obtained putting $R(v) = \{(v^+, v^+), (v^-, v^-)\}$, $\gamma \equiv 0$ on $P^2(v) - R(v)$ and $\gamma \equiv 1$ on H.

3. APPLICATION

Let us consider a urban strret network with only the right-of-way signs on all crossings (i.e., we suppose that no other means are used for the control of crossings — no trafic lights, near-hand side rule, or roundabouts).

Let us denote by V the set of crossings, H the set of street segments, $P(v)$ the set of entries to a crossing v and $R(v)$ the set of all forbidden passages through v. Let γ express the average time necessary for passing through a crossing or through a segment of a street.

Solving a trafic assignment problem for this net one can suppose that a driver chooses the minimal admissible path and thus it is necessary to solve the problem mentioned in 2.

4. SOLUTION

The solution of the minimal admissible path problem is simple. We adjoin to $G = (V, H, P, R, \gamma)$ a new graph $\bar{G} = (\bar{V}, \bar{H})$ as follows:

1. $\bar{V} = \bigcup_{v \in V} P(v)$,
2. $\bar{H} = H \cup \bigcup_{v \in V} [P^2(v) - R(v)]$.

On G we can let the definition of γ without any change and call it the length of edges.

To find the minimal admissible path in G from v to w is the same as to find the path of minimal length in $\bar{G}$ from the set $P(v)$ to the set $P(w)$, i.e., the minimal path from the shortest paths from $v^{(i)}$ to $w^{(j)}$. The algorithm for this problem can be found in almost every monograph on the graph theory.

References

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Súhrn

MULTIPOLARIZOVANÉ GRAFY

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V článku sa zavádza pojem multipolarizovaného grafu, ktorý je zovšeobecnením pojmu polarizovaného grafu, zavedeného Zítkom [1] a študovaného najmä Zelinkom [2–8]. Takýto graf môže mať vrcholy aj s viac, než dvojma pólmi. Cesta na multipolarizovanom grafe môže prechádzať vrcholom len cez „dovolenú" dvojicu pólov.

Ďalej sa študuje problém minimálnej cesty a aplikácia na cestnú dopravu.

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