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BAYES UNBIASED ESTIMATORS OF PARAMETERS OF LINEAR TREND WITH AUTOREGRESSIVE ERRORS

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Summary. The method of least squares is usually used in a linear regression model $\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ for estimating unknown parameters $\boldsymbol{\beta}$. The case when $\boldsymbol{\varepsilon}$ is an autoregressive process of the first order and the matrix $\boldsymbol{X}$ corresponds to a linear trend is studied and the Bayes approach is used for estimating the parameters $\boldsymbol{\beta}$. Unbiased Bayes estimators are derived for the case of a small number of observations. These estimators are compared with the locally best unbiased ones and with the usual least squares estimators.

Keywords: autoregressive process, linear trend, Bayes, estimator.

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1. INTRODUCTION

The method of least squares or the maximum likelihood method are usually used for estimating unknown parameters of the mean value function of a time series. Nonetheless in the case when we know the structure of the process, we can use another principle. The Bayes approach is used in this paper under the assumption that the mean value function of the process is a linear one. The errors are assumed to be values of an autoregressive process of the first order. We are looking for unbiased estimators of parameters of the linear trend, minimizing the average value of the dispersion of the estimator. This average value is taken with respect to some a priori probability distribution defined on the interval $(-1, 1)$. This interval represents the parameter space for the parameter ϱ of autoregression of the process. The case of the uniform distribution on a subinterval (A, B) of $(-1, 1)$ is studied for a small number of observations. The estimators obtained by this principle are compared with the locally best unbiased ones (LBUE) and with the usual least sqares estimator (LSE).

2. THE LOCALLY BEST UNBIASED ESTIMATORS

Let us consider the general linear model

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{1}$$

where $\mathbf{Y}$ is an $n \times 1$ random vector – an observation of a time series $\{Y_t\}$; $t \in \mathbf{Z} = \{\ldots, -1, 0, +1, \ldots\}$, $\mathbf{X}$ is a known $n \times k$ matrix and $\boldsymbol{\beta}$ is a $k \times 1$ vector of unknown parameters. We shall assume that $\boldsymbol{\varepsilon}$ – a random $n \times 1$ error vector is a finite part of an autoregressive process $\{\varepsilon_t\}$, $t \in \mathbf{Z}$, of the first order with zero mean value and with an unknown parameter ϱ of autoregression such that $|\varrho| < 1$. Then we have $\varepsilon_t = \varrho\varepsilon_{t-1} + \mathrm{m}_t$ for every $t \in \mathbf{Z}$, where $\{\mathrm{m}_t\}$; $\mathrm{t} \in \mathbf{Z}$, is a white noise process. It is well known that the covariance function of the process $\{Y_t\}$; $t \in \mathbf{Z}$ (see [1]), is $R(t) = ((\sigma^2/(1 - \varrho^2))\, \varrho^{|t|}$; $\mathrm{t} \in \mathbf{Z}$, where σ^2 is the dispersion of the white noise.

The best linear unbiased estimator $\boldsymbol{\beta}^*$ of the vector $\boldsymbol{\beta}$ in the model (1) generally depends on the covariance matrix $\boldsymbol{\Sigma}$ of the random vector $\mathbf{Y}$ and is given by

$$\boldsymbol{\beta}^* = (\mathbf{X}'\boldsymbol{\Sigma}^{-1}\mathbf{X})^{-1}\, \mathbf{X}'\boldsymbol{\Sigma}^{-1}\mathbf{Y}\,. \tag{2}$$

In our case the covariance matrix of the random vector $\mathbf{Y}$ depends on the unknown parameter ϱ only: $\boldsymbol{\Sigma} = \Sigma(\varrho)$. Thus the (locally) best unbiased estimator $\boldsymbol{\beta}^*$ depends on the parameter ϱ as well, $\boldsymbol{\beta}^* == \boldsymbol{\beta}^*(\varrho)$; $|\varrho| < 1$. We remark that $\Sigma(\varrho)^{-1}$ exists for every $\varrho \in (-1, 1)$, and for $n \geqq 3$ we have

$$\Sigma(\varrho)^{-1} = \frac{1}{\sigma^2}\begin{bmatrix} 1 & -\varrho & 0 \ldots & 0 & 0 & 0 \\ -\varrho & 1 + \varrho^2 & -\varrho \ldots & 0 & 0 & 0 \\ \cdot & \cdot & \cdot \ldots & \cdot & \cdot & \cdot \\ 0 & 0 & 0 \ldots & -\varrho & 1 + \varrho^2 & -\varrho \\ 0 & 0 & 0 \ldots & 0 & -\varrho & 1 \end{bmatrix}. \tag{3}$$

It is well known that the covariance matrix of $\boldsymbol{\beta}^*(\varrho)$ is given by

$$\Sigma_{\boldsymbol{\beta}^*(\varrho)} = (\mathbf{X}'\Sigma(\varrho)^{-1}\, \mathbf{X})^{-1}\,. \tag{4}$$

This again depends on the parameter ϱ.

For $\varrho = 0$ we get $\Sigma(0) = \sigma^2$. I and according to (2), $\boldsymbol{\beta}^*(0)$ coincides with the usual least squares estimator.

Now let $\mathbf{X} = (1, \ldots, 1)'$. Then the model (1) corresponds to the case when $Y_t = \beta + \varepsilon_t$; $t = 1, \ldots, n$, is a finite observation of the time series Y with an unknown constant mean value β and with autocorrelated errors. According to (2), (3) and (4) we get the LBUE (at the parameter value ϱ) of the constant mean value β in the form

$$\beta^*(\varrho) = \frac{Y_1 + Y_n + (1 - \varrho)\sum_{t=2}^{n-1} Y_t}{2 + (n-2)(1-\varrho)} \quad \text{for} \quad n \geqq 3; \beta^*(\varrho) = \frac{Y_1 + Y_2}{2} \quad \text{for} \quad n = 2\,. \tag{5}$$

This estimator has the dispersion given by

$$D_\varrho[\beta^*(\varrho)] = \frac{\sigma^2}{(1-\varrho)[2+(n-2)(1-\varrho)]} \quad \text{for every} \quad n \geqq 2\,. \tag{6}$$

If $\boldsymbol{X} = \begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & 2 & \dots & n \end{pmatrix}'$ and $\boldsymbol{\beta} = (\beta_0, \beta_1)'$, then the model (1) corresponds to the case when $Y_t = \beta_0 + \beta_1 \,.\, t + \varepsilon_t$; $t = 1, \dots, n$, is a finite observation of the time series $\{Y_t\}$; $t \in Z$, having some "linear trend" mean value function. The vector $\boldsymbol{\beta}$ is the vector of unknown parameters of this linear trend. We can use (2) and (3) again to calculate the estimator $\boldsymbol{\beta}^*(\varrho)$. For example, we get $\boldsymbol{\beta}^*(\varrho) = (2Y_1 - Y_2, Y_2 - Y_1)'$ for $n = 2$ and every $\varrho \in (-1, 1)$. Thus $\boldsymbol{\beta}^*$ is the unique uniformly best unbiased estimator of β with the covariance matrix

$$\boldsymbol{\Sigma}_{\boldsymbol{\beta}^*} = \frac{\sigma^2}{1-\varrho^2} \begin{pmatrix} 5-4\varrho & 3(\varrho-1) \\ 3(\varrho-1) & 2(1-\varrho) \end{pmatrix}.$$

For $n = 3$, according to (2) and (3) we get

$$\beta_0^*(\varrho) = \frac{1}{3-\varrho}[(4-\varrho)\,.\,Y_1 + (1-\varrho)\,.\,Y_2 - (2-\varrho)\,Y_3]\;\beta_1^*(\varrho) = \tfrac{1}{2}(Y_3 - Y_1) \tag{5'}$$

and

$$\boldsymbol{\Sigma}_{\boldsymbol{\beta}^*(\varrho)} = \frac{\sigma^2}{2(1-\varrho)(3-\varrho)} \begin{pmatrix} 2[2(1-\varrho)(3-\varrho)+1] & -2\,.\,(1-\varrho)(3-\varrho) \\ -2(1-\varrho)(3-\varrho) & (1-\varrho)(3-\varrho) \end{pmatrix}.$$

We can see that the estimator $\beta_1^*(\varrho)$ of the parameter β_1 does not depend on ϱ, and it is the uniformly best unbiased estimator with the dispersion $\sigma^2/2$.

The disadvantage of the locally best unbiased estimators is that they depend on the unknown parameter ϱ of autoregression and in the practical cases we do not know which estimator (if any) from the set $\{\boldsymbol{\beta}^*(\varrho); |\varrho| < 1\}$ should be used. These difficulties can be overcome by using, for example, the Bayes approach.

3. THE BAYES UNBIASED ESTIMATORS

Let us consider a linear statistic $\mathbf{c}'\boldsymbol{Y}$ where $\mathbf{c} = (c_1, \dots, c_n)'$ is any real vector and $\boldsymbol{Y}$ is the random vector from the model (1). Then it is clear that $E_{(\boldsymbol{\beta},\boldsymbol{\Sigma})}[\mathbf{c}'\boldsymbol{Y}] = \mathbf{c}'\boldsymbol{X}\boldsymbol{\beta}$ and the dispersion $D_{(\boldsymbol{\beta},\boldsymbol{\Sigma})}[\mathbf{c}'\boldsymbol{Y}] = \mathbf{c}'\boldsymbol{\Sigma}\mathbf{c}$, which is a quadratic form in $\mathbf{c}$. In our special case of autoregression the dispersion $D[\mathbf{c}'\boldsymbol{Y}] = D_\varrho[\mathbf{c}'\boldsymbol{Y}]$ depends only on ϱ. The linear statistic $\mathbf{c}'\boldsymbol{Y}$ is an unbiased estimator of a constant mean value β iff $\sum_{i=1}^{n} c_i = 1$.

By analogy, the statistic $\mathbf{c}'\boldsymbol{X}$ is an unbiased estimator for the parameter β_j iff $\boldsymbol{X}'\mathbf{c} = \boldsymbol{b}_j$; $j = 0, 1$, where $\boldsymbol{b}_0 = (1, 0)'$ and $\boldsymbol{b}_1 = (0, 1)'$.

According to the Bayes approach the linear unbiased Bayes estimator (LUBE) $\tilde{\beta}_j$ of β_j is the estimator minimizing the function $\tilde{D}(\mathbf{c})$ given by

$$(7) \qquad \tilde{D}(\mathbf{c}) = \int_{-1}^{1} D_\varrho[\mathbf{c}'\mathbf{Y}]\, f(\varrho)\,\mathrm{d}\varrho$$

on the set $C_j = \{\mathbf{c}: \mathbf{X}'\mathbf{c} = \mathbf{b}_j\}$; $j = 0, 1$. Here $f(\cdot)$ is some probability density function defined on $(-1, 1)$. In our case $D_\varrho[\mathbf{c}'\mathbf{Y}] = \mathbf{c}'\Sigma(\varrho)\,\mathbf{c}$ and, according to (7), we get $\tilde{D}(\mathbf{c}) = \mathbf{c}'\bar{\Sigma}\mathbf{c}$ where

$$(8) \qquad \bar{\Sigma}_{ij} = \int_{-1}^{1} \Sigma(\varrho)_{ij}\, f(\varrho)\,\mathrm{d}\varrho\;; \quad i, j = 1, \ldots, n\,.$$

The following lemma gives $\mathbf{c}_j^0$ – the value minimizing $\tilde{D}(\mathbf{c})$ on C_j for $j = 0, 1$.

Lemma. *Let $\bar{\Sigma}$ be a positive definite $n \times n$ matrix, $\mathbf{c}$ – any $n \times 1$ vector, $\mathbf{X}$ – an $n \times k$ matrix, and let $\mathbf{u}$ be a $k \times 1$ vector. Let us denote by $\mathbf{S}^-$ any pseudoinversion of the matrix $\mathbf{X}'\bar{\Sigma}^{-1}\mathbf{X}$. Then $\inf_{\mathbf{X}'\mathbf{c}=\mathbf{u}} \mathbf{c}'\bar{\Sigma}\mathbf{c} = \mathbf{u}'\mathbf{S}\mathbf{u}$. This infimum is achieved at the vector $\mathbf{c}^0 = \bar{\Sigma}^{-1}\mathbf{X}\mathbf{S}^-\mathbf{u}$.*

Proof. See [2].

Corollary. *The* LUBE's $\tilde{\boldsymbol{\beta}}$ of the components of $\boldsymbol{\beta}$ *are given by*

$$(9) \qquad \tilde{\boldsymbol{\beta}} = (\mathbf{X}'\,\bar{\Sigma}^{-1}\mathbf{X})^{-1}\,\mathbf{X}'\bar{\Sigma}^{-1}\mathbf{Y}\,,$$

where $\bar{\Sigma}$ is given by (8).

Proof. The assertion follows from Lemma by setting $\mathbf{u} = \mathbf{b}_j$; $j = 0, 1$.

Remark. It can be seen from (9) and (2) that the LUBE $\tilde{\boldsymbol{\beta}}$ is equal to the best linear unbiased estimator under the assumption that the true covariance matrix is $\bar{\Sigma}$.

From now on we shall study the cases when $f(\cdot)$ is the density of the uniform distribution on some subinterval (A, B) of $(-1, 1)$. In this case the value

$$(10) \qquad \int_A^B D_\varrho[\tilde{\beta}_j]\,\mathrm{d}\varrho\;;$$

$j = 0, 1$ will be minimal. If we compute $\tilde{\beta}_j$; $j = 0, 1$ for $A = -1$, $B = 1$, then we can regard these estimators as the best ones minimizing (10), the value of the global criterion. Knowing that $\varrho > 0$ (or $\varrho < 0$) we can calculate $\tilde{\boldsymbol{\beta}}$ for $A = 0$, $B = 1$ (or $A = -1$, $B = 0$). For $f(\cdot)$ being the density of the uniform distribution on (A, B) we get the following expression for $\bar{\Sigma}$:

$$\bar{\Sigma}_{ij} = \int_{-1}^{1} \Sigma(\varrho)_{ij}\,.\,f(\varrho)\,\mathrm{d}\varrho = \frac{\sigma^2}{B - A}\int_A^B \frac{\varrho^{|i-j|}}{1 - \varrho^2}\,\mathrm{d}\varrho\,.$$

Usually we put $A = -1$ and $B = 1$ if we have no information on ϱ. It is easy to show that

$$\int_A^B \frac{\varrho^i}{1-\varrho^2}\,d\varrho = \begin{cases} (1/2)\ln\dfrac{(1-A)(1+B)}{(1+A)(1-B)} + \displaystyle\sum_{k=0}^{\frac{i-2}{2}} \frac{A^{i-(2k+1)} - B^{i-(2k+1)}}{i-(2k+1)} & \text{if } i \text{ is even,} \\ (1/2)\ln\dfrac{1-A^2}{1-B^2} + \displaystyle\sum_{k=0}^{\frac{i-3}{2}} \frac{A^{i-(2k+1)} - B^{i-(2k+1)}}{i-(2k+1)} & \text{if } i \text{ is odd,} \end{cases}$$

where we set the sums equal to zero for $i = 0, 1$. Thus we can get $\bar{\Sigma}$ easily, but for the matrix $\bar{\Sigma}^{-1}$ we now have no explicit formula like (3) for $\Sigma(\varrho)^{-1}$. In some special cases we are able to compute $\tilde{\boldsymbol{\beta}}$ for small n and compare these estimators with the estimators $\boldsymbol{\beta}^*(\varrho)$; $\varrho \in (-1, 1)$.

Let us begin with $n = 3$. Let us denote

$$a = \tfrac{1}{2}\ln\frac{(1-A)(1+B)}{(1+A)(1-B)}, \quad b = \tfrac{1}{2}\ln\frac{1-A^2}{1-B^2} \quad \text{and} \quad c = A - B\,.$$

Then

$$\bar{\Sigma} = \begin{pmatrix} a & b & a+c \\ b & a & b \\ a+c & b & a \end{pmatrix}$$

and

$$\bar{\Sigma}^{-1} = \begin{pmatrix} a^2 - b^2 & bc & b^2 - a^2 - ac \\ bc & -2ac - c^2 & bc \\ b^2 - a^2 - ac & bc & a^2 - b^2 \end{pmatrix} \frac{1}{\det \bar{\Sigma}}\,.$$

The LUBE $\tilde{\beta}$ of β- the unknown constant mean value, is then, by virtue of Corollary, given by

$$\tilde{\beta} = \frac{Y_1 + \left(2 - \dfrac{c}{b-a}\right) Y_2 + Y_3}{4 - \dfrac{c}{b-a}}\,. \tag{11}$$

Comparing this estimator with the estimators given by (5) we see that $\tilde{\beta} = \beta^*(\varrho)$ for $\varrho = c/(b-a) - 1$. Next, ϱ converges to -1 if A tends to -1 and the estimator $\tilde{\beta} = \frac{1}{4}(Y_1 + 2Y_2 + Y_3)$ is the LUBE if we admit $A = -1$.

Some other cases: 1. For $A = 0$, $B = 1$ (the case when we know that $\varrho > 0$) we get

$$\tilde{\beta} = \frac{Y_1 + 0{\cdot}5573\,Y_2 + Y_3}{2{\cdot}5573} = \beta^*(\varrho) \quad \text{for} \quad \varrho = 0{\cdot}4427\,.$$

2. For $A = -\frac{1}{2}$, $B = \frac{1}{2}$ we have

$$\tilde{\beta} = \frac{Y_1 + 1{\cdot}0897 Y_2 + Y_3}{3{\cdot}0897} = \beta^*(\varrho) \quad \text{for} \quad \varrho = 0{\cdot}0897 \,.$$

Remark. For $n = 3$, $\tilde{\beta}$ differs from $\beta^*(0)$ – the LSE which is commonly used in practise, because $\tilde{\beta} = \beta^*(0)$ iff $c = b - a$, that is $A - B = \ln(1 + A/1 + B)$, which is possible only in the case when $A = B$.

For the components of the vector $\boldsymbol{\beta} = \begin{pmatrix}\beta_0\\ \beta_1\end{pmatrix}$ of parameters of a linear trend we get, using (9), estimators given by

$$(11') \qquad \tilde{\beta}_0 = \frac{1}{4 - \dfrac{c}{b - a}} \left[\left(5 - \frac{c}{b - a}\right) . Y_1 + \left(2 - \frac{c}{b - a}\right) . Y_2 - \left(3 - \frac{c}{b - a}\right) . Y_3\right], \tilde{\beta}_1 = \tfrac{1}{2}(Y_3 - Y_1) \,.$$

Comparing these estimators with the estimators given by (5′) we see that the LUBE of β_0 differs from $\beta_0^*(\varrho)$ for every ϱ, but $\tilde{\beta}_1 = \beta_1^*$ – the uniformly best estimator. In the limit case, when A tends to -1 and $b - a$ tends to $-\infty$, we get from (5′) and (11′) that $\tilde{\beta}_0 = \beta_0^*(\varrho)$ for $\varrho = -1$. The LUBE $\tilde{\beta}_0$ of β_0 differs from the LSE $\beta_0^*(0)$, because $\tilde{\beta}_0 = \beta_0^*(0)$ iff $c = b - a$, which is possible only in the case when $A = B$.

For $A = 0$, $B = 1$ we have using (11′)

$$\tilde{\beta}_0 = 1{\cdot}3910 Y_1 + 0{\cdot}2179 Y_2 - 0{\cdot}6089 Y_3 \,,$$

and for $A = -0{\cdot}5$, $B = 0{\cdot}5$ the LUBE of β_0 is

$$\tilde{\beta}_0 = 1{\cdot}3236 Y_1 + 0{\cdot}3527 Y_2 - 0{\cdot}6763 Y_3 \,.$$

Let us consider the case $n = 4$. If we define (a, b and c in the same way as for $n = 3$ and if we set $d = \frac{1}{2}(A^2 - B^2)$, then it can be checked that $\Sigma^{11} = \Sigma^{44} = c[2(b^2 - a^2) - ac]$; $\Sigma^{12} = \Sigma^{34} = -[d(b^2 - a^2) + c(bc - ad)]$; $\Sigma^{13} = \Sigma^{24} = c[2(a^2 - b^2) + c^2 + 3ac - bd]$; $\Sigma^{14} = -[c^2 b + d(a^2 - b^2)]$; $\Sigma^{22} = \Sigma^{33} = c(2b^2 - 2a^2 + 2bd - ac) - ad^2$ and $\Sigma^{23} = -\{b(c^2 - d^2) + d[(a + c)^2 - b^2]\}$, where $\Sigma^{-1} = \{\Sigma^{ij}\}_{i,j=1}^{4}$. Using these results we get from (9) the LUBE $\tilde{\beta}$ of the constant mean value β:

$$\tilde{\beta} = \frac{Y_1 + Y_4 + \left(1 - \dfrac{A + B}{2}\right)(Y_2 + Y_3)}{2 + 2\left(1 - \dfrac{A + B}{2}\right)} \,.$$

Comparing $\tilde{\beta}$ with the estimators given by (5) we see that $\tilde{\beta} = \beta^*(\varrho)$ for $\varrho = \frac{1}{2}(A + B)$ for any $-1 < A < B < 1$. Consequently, if $A = -B$, $0 < B < 1$, then $\tilde{\beta}$ coincides with the usual LSE $Y = \frac{1}{4}\sum_{i=1}^{4} Y_i$ of β.

The estimator $\tilde{\boldsymbol{\beta}}$ for the parameters of linear trend will be studied only in the case $A = -B$; $0 < B < 1$, when

$$\Sigma = \begin{pmatrix} a & 0 & a + c & 0 \\ 0 & a & 0 & a + c \\ a + c & 0 & a & 0 \\ 0 & a + c & 0 & a \end{pmatrix}$$

and

$$\Sigma^{-1} = \frac{1}{a^2 - (a + c)^2} \begin{pmatrix} a & 0 & -a - c & 0 \\ 0 & a & 0 & -a - c \\ -a - c & 0 & a & 0 \\ 0 & -a - c & 0 & a \end{pmatrix}.$$

This yields the estimators

$$\tilde{\beta}_0 = \frac{1}{16 + \frac{6c}{a}} \left[\left(14 + \frac{4c}{a}\right) Y_1 + \left(14 + \frac{9c}{a}\right) Y_2 - \left(6 + \frac{6c}{a}\right) Y_3 - \left(6 + \frac{c}{a}\right) Y_4\right],$$

$$\tilde{\beta}_1 = \frac{1}{16 + \frac{6c}{a}} \left[\left(4 + \frac{c}{a}\right)(Y_4 - Y_1) + \left(4 + \frac{3c}{a}\right)(Y_3 - Y_2)\right].$$

Some special cases: for $A = -1$, $B = 1$ the globally LUBE's are

$$\tilde{\beta}_0 = \tfrac{1}{8}[7(Y_1 + Y_2) - 3(Y_3 + Y_4)],$$

$$\tilde{\beta}_1 = \tfrac{1}{2} \cdot \left[\frac{Y_3 - Y_1}{2} + \frac{Y_4 - Y_2}{2}\right].$$

In this case $\tilde{\beta}_0 \neq \beta_0^*(-1) = 0{\cdot}79 Y_1 + 0{\cdot}96 Y_2 - 0{\cdot}29 Y_3 - 0{\cdot}46 Y_4$, but $\tilde{\beta}_1 = \beta_1^*(-1)$. Next, $\tilde{\beta}_i \neq \beta_i^*(0)$; $i = 0, 1$, where

$$\beta_0^*(0) = Y_1 + 0{\cdot}5 Y_2 - 0{\cdot}5 Y_4$$

and

$$\beta_1^*(0) = 0{\cdot}3(Y_4 - Y_1) + 0{\cdot}1(Y_3 - Y_2)$$

are the LSE's of $\boldsymbol{\beta}$.

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Súhrn

BAYESOVE NEVYCHÝLENÉ ODHADY PARAMETROV LINEÁRNEHO TRENDU S AUTOREGRESNÝMI CHYBAMI

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Pre odhad vektora parametrov $\boldsymbol{\beta}$ v lineárnom regresnom modeli $\boldsymbol{Y} = \boldsymbol{X\beta} + \varepsilon$ obyčajne používame metódu najmenšieho súčtu štvorcov. V článku je študovaný prípad, kedy ε je pozorovaním autoregresného procesu prvého rádu. V tomto prípade môžeme ku odhadu parametrov $\boldsymbol{\beta}$ použiť Bayesov princíp. Na základe tohto princípu sú odvodené nevychýlené odhady parametrov lineárneho trendu. Tieto odhady sú vypočítané pre malé rozsahy pozorovaní vektora $\boldsymbol{Y}$ a sú porovnané s odhadmi získanými metódou najmenšieho súčtu štvorcov.

Резюме

НЕСМЕЩЕННЫЕ БАЙЕСОВЫ ОЦЕНКИ ПАРАМЕТРОВ ЛИНЕЙНОГО ТРЕНДА ПРИ АВТОРЕГРЕССИОННЫХ ОШИБКАХ

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В линейной регрессионной модели $\boldsymbol{Y} = \boldsymbol{X\beta} + \varepsilon$ обычно исползуем метод наименьших квадратов для оценок параметров $\boldsymbol{\beta}$. Изучается случай когда ε-авторегрессионный процесс, матрица $\boldsymbol{X}$ соответствует линейному тренду и для оценок параметров $\boldsymbol{\beta}$ используется принцип Байеса. Полученные несмещенные оценки Байеса сравнены с локально наилучшими оценками и с оценками по методу наименьших квадратов.

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