Aplikace matematiky

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Aplikace matematiky, Vol. 35 (1990), No. 1, 51–59

Persistent URL: http://dml.cz/dmlcz/104386

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35 (1990) APLIKACE MATEMATIKY No. 1, 51—59

A NOTE ON IMPULSIVE CONTROL OF FELLER PROCESSES WITH COSTLY INFORMATION

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(Received September 26, 1988)

Summary. The paper deals with the optimal inspections and maintenance problem with costly information for a Markov process with positive discount factor. The associated dynamic programming equation is a quasi-variational inequality with first order differential terms. In this paper we study its different formulations: strong, viscosity and evolutionary. The case of impulsive control of purely jump Markov processes is studied as a special case.

Key words: Markov jump processes, Feller process, inspections and maintenance, quasi-variational inequality, viscosity solutions.

AMS Classification: 60G35.

1. INTRODUCTION

In this paper we study problems of impulsive control of a Markov process with costly information available only in inspection and renewal periods. In Section 2 we give auxiliary results concerning different formulations of the first-order variational inequalities: strong, evolutionary and viscosity. In Section 3 we solve a problem of optimal inspections and renewals for a Markov process with positive discount factor. We apply the technique of quasi-variational inequalities. In Section 4 we solve this problem for Feller jump processes. Here the value function is a strong solution of a quasi-variational inequality.

Optimal inspections and maintenance with discounted cost for Wiener processes was studied by Anderson and Friedmann in [1], and for Feller processes by Robin in [9]. This paper generalizes results of Anderson, Friedmann and Robin.

2. AUXILIARY RESULTS FOR FIRST-ORDER VARIATIONAL INEQUALITIES

Let E be a locally compact topological space. Denote by $C_b(E)$ the space of all continuous bounded functions on E. Denote by $\|\cdot\|$ the "e ssup" norm reduced to the norm "sup" on $C_b(E)$. In this section we study the following two formulations of variational inequalities:

Let $x \in E$ and $s, t \in \mathbb{R}_+$.

a) *Evolutionary formulation*:

$$(1)\qquad \begin{cases} \text{(i)} & v(x, s) \leqq v(x, t)\, e^{-\alpha(t-s)} + \int_s^t g(x, u)\, e^{-\alpha(u-s)}\, du\,, \quad t \geqq s\,, \\ \text{(ii)} & v \leqq h\,, \\ \text{(iii)} & \dfrac{\partial}{\partial t} v(x, t) - \alpha\, v(x, t) + g(x, t) = 0 \quad \text{if} \quad v(x, t) < h(x, t)\,. \end{cases}$$

b) *Strong formulation*:

$$(2)\qquad \left(\frac{\partial}{\partial t} v(x, t) - \alpha\, v(x, t) + g(x, t)\right) \wedge (h(x, t) - v(x, t)) = 0\,.$$

We use the notation $a \wedge b = \min\{a, b\}$ and $a \vee b = \max\{a, b\}$. We will say that $v \in C_b(E \times \mathbb{R}_+)$ is an evolutionary solution of (2) if v satisfies (1).

Let us recall the definitions of strong and viscosity solutions of a variational inequality:

Definition 1. [2, 3] 1. *We call $v \in C_b(E \times \mathbb{R}_+)$ a strong solution of (2) if $v(x, \cdot)$ is absolutely continuous on $\mathbb{R}_+$ and (2) holds for almost every $t \in \mathbb{R}_+$ and every $x \in E$.*

2. *We call $v \in C_b(E \times \mathbb{R}_+)$ a viscosity solution of (2) if*

(i) $(p - \alpha\, v(x, t) + g(x, t)) \wedge (h(x, t) - v(x, t)) \leqq 0$ *for any* $x \in E$, $t \in \mathbb{R}_+$ *and* $p \in D^- v(x, t)$, *where the subdifferential* $D^- v(x, t)$ *is given by* $D^- v(x, t) = \{p \in \mathbb{R}: \liminf_{s \to 0} |s|^{-1} \{v(x, t + s) - v(x, t) - ps\} \geqq 0\}$;

(ii) $(p - \alpha\, v(x, t) + g(x, t)) \wedge (h(x, t) - v(x, t)) \geqq 0$ *for any* $x \in E$, $t \in \mathbb{R}_+$ *and* $p \in D^+ v(x, t)$, *where the superdifferential* $D^+ v(x, t)$ *is given by* $D^+ v(x, t) = \{p \in \mathbb{R}: \limsup_{s \to 0} |s|^{-1} \{v(x, t + s) - v(x, t) - ps\} \leqq 0\}$.

Here we consider x only as a parameter.

Remark 1. If v is a viscosity solution of (2) and $v(x, \cdot)$ is absolutely continuous then v is a strong solution of (2).

Proposition 1. *Let $g, h \in C_b(E \times \mathbb{R}_+)$. Then there exists a unique evolutionary solution $v \in C_b(E \times \mathbb{R}_+)$ of the varitational inequality (1), namely*

$$(3)\qquad v(x, s) = \inf_{t \geqq s} e^{\alpha s}\{\textstyle\int_s^t g(x, u)\, e^{-\alpha u}\, du + h(x, t)\, e^{-\alpha t}\}\,.$$

Moreover, v is a viscostty solution of (2).

Remark 2. We put $h(\infty)e^{-\alpha\infty} = 0$ for any bounded function h on $\mathbb{R}_+$.

Continuity of v is implied by the following lemma:

Lemma 1. [9] *Let $g, h \in C_b(E \times \mathbb{R}_+)$ and let v be defined by (3). Then $v \in C_b(E \times \mathbb{R}_+)$ and*

$$(4)\qquad v(x, s) = e^{\alpha s}\{\textstyle\int_s^{t(x,s)} g(x, t)\, e^{-\alpha u}\, du + h(x, t(x, s))\, e^{-\alpha t(x,s)}\}\,,$$

where $t(x, s) = \inf\{t \in \mathbb{R}_+ : t \geqq s, v(x, t) = h(x, t)\}$ $(t(x, s) = +\infty$ *if* $v(x, t) < h(x, t)$ *for any* $t \geqq s)$. □

Proof of Proposition 1. *Uniqueness*. By (1) (i) and (1) (ii), $v(x, s) \leqq h(x, t)\, e^{-\alpha(t-s)} + \int_s^t g(x, u)\, e^{-\alpha(u-s)}\, du$ for any $t \geqq s$.

Let $V = \{(x, t) \in E \times \mathbb{R}_+ : v(x, t) < h(x, t)\}$. Notice that V is open. By (1) (iii), for any $t < t(x, s)$ we have

(5) $$v(x, s) = v(x, t)\, e^{-\alpha(t-s)} + \int_s^t g(x, u)\, e^{-\alpha(u-s)}\, du\,.$$

Since v is continuous,

$$v(x, s) = e^{\alpha s}\{\int_s^{t(x,s)} g(x, u)\, e^{-\alpha u}\, du + h(x, t(x, s))\, e^{-\alpha t(x,s)}\}\,.$$

Hence $v(x, s) = \inf_{t \geqq s} e^{\alpha s}\{\int_s^t g(x, u)\, e^{-\alpha u}\, du + h(x, t)\, e^{-\alpha t}\}$ and the solution of (1) is unique.

Existence. Let v be given by (3). Obviously v satisfies (1) (ii). Besides,

$$v(x, s) = \inf_{t \geqq s} e^{\alpha s}\{\int_s^t g(x, u)\, e^{-\alpha u}\, du + h(x, t)\, e^{-\alpha t}\} \leqq$$
$$\leqq \inf_{t \geqq r} e^{\alpha s}\{\int_s^t g(x, u)\, e^{-\alpha u}\, du + h(x, t)\, e^{-\alpha t}\} = v(x, r)\, e^{-\alpha(r-s)} +$$
$$+ \int_s^r g(x, u)\, e^{-\alpha(u-s)}\, du\,, \quad \text{where} \quad r \geqq s\,.$$

Therefore v satisfies (1) (i).

Let (x, s) be any point from V. There exists an $\varepsilon > 0$ such that $\{(x, u) \in E \times \mathbb{R}_+ : u \in (s - \varepsilon, s + \varepsilon)\} \subseteq V$. Then

$$v(x, s) = v(x, t)\, e^{-\alpha(t-s)} + \int_s^t g(x, u)\, e^{-\alpha(u-s)}\, du$$

for any $s \leqq t \leqq s + \varepsilon$ and

$$v(x, t) = v(x, s)\, e^{-\alpha(s-t)} + \int_t^s g(x, u)\, e^{-\alpha(u-t)}\, du$$

for any $s - \varepsilon \leqq t \leqq s$.

Letting t tend to s and dividing by $|t - s|$ we get (1) (iii). To prove that v is a viscosity solution of (2) notice that either $v(x, t) = h(x, t)$ or v is differentiable at the point (x, t) and $D^- v(x, t) = \left\{\frac{\partial}{\partial t} v(x, t).\right\}$ Hence and by (1), $(p - \alpha v + g) \wedge (h - v) \leqq 0$ for any $p \in D^- v(x, t)$.

To prove that $(p - \alpha v + g) \wedge (h - v) \geqq 0$ for any $p \in D^+ v(x, t)$ suppose that $p < \alpha\, v(x, t) - g(x, t)$. By (1) (i),

$$p < \liminf_{s \to 0+} s^{-1}\{v(x, t + s) - v(x, t)\}\,.$$

Hence

(6) $$0 < \liminf_{s \to 0+} s^{-1}\{v(x, t + s) - v(x, t) - ps\} \leqq$$
$$\leqq \limsup_{s \to 0} |s|^{-1}\{v(x, t + s) - v(x, t) - ps\}\,.$$

Therefore $p \notin D^+ v(x, t)$ and $(p - \alpha v + g) \wedge (h - v) \geqq 0$ for any $p \in D^+ v(x, t)$.

The aim of this section is to solve an optimal inspections and renewals problem, i.e. to characterize the value function and to find the optimal impulsive policy (if it exists). Controller's interventions will be described as follows: Dynamics of a phenomenon is described by a Markov process. The controller does not observe this phenomenon directly, he can only choose certain time periods to make an intervention: to shift the process to a new state or to make an inspection.

After each intervention the controller determines:

(i) when to make the next intervention;

(ii) type of the next intervention (renewal or inspection).

Formal description is the following:

Let $X = (\Omega, \mathcal{F}, \mathcal{F}_t, x_t, P_x)$ be a Markov process on a locally compact state space E describing the dynamics of a phenomenon without controller's interventions.

Denote the transition semigroup of X by Φ, i.e. $\Phi_t f(x) = E_x f(x_t)$ for any $f \in C_b(E)$. Let X be a Feller process, i.e. the semigroup Φ satisfy:

1. $\Phi_t f(x) \to f(x)$ as $t \to 0$ for any $f \in C_b(E)$.
2. $\Phi_t f \in C_b(E)$ for any $f \in C_b(E)$.

The generator of Φ_t in $C_b(E)$ will be denoted by $(A, \mathcal{D}(A))$.

Let $(\Omega^N, \mathcal{F}^{\otimes N}, x_t^\infty)$ be the control reference probability space, where $x_t^\infty(\omega) = (x_t^1(\omega), x_t^2(\omega), \ldots) = (x_t(\omega_1), x_t(\omega_2), \ldots)$ for $\omega = (\omega_1, \omega_2, \omega_3, \ldots) \in \Omega^N$, and let $\mathcal{F}_t^n = \sigma(A_1 \times A_2 \times \ldots : A_i \in \mathcal{F}_t$ for $i \leqq n$ and $A_i \in \{0, \Omega\}$ for $i > n)$. A sequence $\pi = (\tau_n, \xi_n)$ will be called an impulsive control policy, if

(i) $\tau_n \geqq \tau_{n-1}$ and $\tau_n \to \infty$,

(ii) τ_n is a $\sigma(x_0^1, \tau_1, x_{\tau_1}^2, \ldots, \tau_{n-1}, x_{\tau_{n-1}}^n)$-measurable random variable on $\mathbb{R}_+$,

(iii) ξ_n is a $\sigma(x_0^1, \tau_1, x_{\tau_1}^2, \ldots, \tau_{n-1}, x_{\tau_{n-1}}^n)$-measurable random variable on $U^1 = U \cup \{\delta\}$, where $U \subseteq E$ is the set of the admissible starting points after each renewal and $\delta \notin E$, $\xi_n \in U$ means that the n-th intervention is a renewal to the point ξ_n, while $\xi_n = \delta$ means that the n-th intervention is an inspection.

Denote by Π the set of all impulsive control policies. For any impulsive policy $\pi \in \Pi$ there exists a probability measure P_x^π such that (see [8]):

(iv) $(\Omega^N, \mathcal{F}^{\otimes N}, \mathcal{F}_t^1, x_t^1, P_x^\pi)$ is a Feller process with transition operator Φ_t,

(v) $P_x^\pi(x_t^n = \zeta_{n-1}$ for $t \leqq \tau_{n-1}) = 1$ for $n \geqq 2$, where

$$\zeta_n = \begin{cases} \xi_n & \text{if } \xi_n \in U , \\ x_{\tau_n}^n & \text{otherwise.} \end{cases}$$

(vi) $w(x_t^n) - \int_{\tau_{n-1} \wedge t}^t Aw(x_s^n)\,ds$ is an $\mathcal{F}_t^n$-martingale for any $w \in \mathcal{D}(A)$ and $n \geqq 2$ (in other words x_t^n is a Markov process with the generator A and birth time τ_{n-1}).

Let f (holding cost) be a bounded, continuous function on E and let c (inspection or switching cost) be a bounded continuous function on $E \times U^1$. Recall that $U^1 =$

$= U \cup \{\delta\}$, where $U \subseteq E$ and $\delta \notin E$. Define the cost function J by

$$J(s, \pi) = e^{\alpha s}\{\int_s^\infty f(y_u)\, e^{-\alpha u}\, du + \sum_{i=1}^{\infty} c(x^i_{\tau_i}, \xi_i)\, e^{-\alpha \tau_i}\},$$

where

$$y_t = \sum_{i=1}^{\infty} x^i_{\tau_i} I_{\{\tau_{i-1} \leqq t < \tau_i\}}.$$

Our aim is to characterize the value function $w(x, t) = \inf_{\substack{\pi \in \Pi \\ \tau_1 \geqq t}} E^\pi_x J(t, \pi)$ and to find an optimal policy (if it exists). As we know from the theory of impulsive control the value function w yields the complete characterization of the optimal policy. The tool is the following quasi-variational inequality:

$$\text{(7)} \qquad \begin{cases} \text{(i)} & w(x, s) \leqq w(x, t)\, e^{-\alpha(t-s)} + \int_s^t \Phi_u f(x)\, e^{-\alpha(u-s)}\, du, \\ \text{(ii)} & w(x, t) \leqq M\, w(x, t), \\ \text{(iii)} & \partial/\partial t\, w(x, t) - \alpha\, w(x, t) + \Phi_t f(x) = 0 \quad \text{if} \quad w(x, t) < M\, w(x, t), \end{cases}$$

where

$$\text{(8)} \qquad M\, w(x, t) = M_0\, w(x, t) \wedge M_1\, w(x, t),$$

$$\text{(9)} \qquad M_0\, w(x, t) = E_x[c(x_t, \delta) + w(x_t, 0)].$$

$$\text{(10)} \qquad M_1\, w(x, t) = \inf_{y \in U} \{E_x\, c(x_t, y) + w(y, 0)\}.$$

Theorem 1. *Let* A1. $U^1 = U \cup \{\delta\}$, *where* $U \subseteq E$ *is a compact set and* $\delta \notin E$, *and let* α *be a positive number and* f *and* c *given functions, satisfying*

A2. $f \in C_b(E)$ *and* $f \geqq 0$.

A3. $c \in C_b(E \times U^1)$ *and* $c \geqq \gamma > 0$.

Then there exists a unique solution $w \in C_b(E \times \mathbb{R}_+)$ *of the quasi-variational inequaltiy* (7), *namely*

$$\text{(11)} \qquad w(x, t) = \inf_{\substack{\pi \in \Pi \\ \tau_1 \geqq t}} E^\pi_x J(t, \pi).$$

Moreover, there exist two functions $T^*: E \to \mathbb{R}_+$ *and* $\xi: E \to U^1$ *given by*

$$\text{(12)} \qquad T^*(x) = \inf\{s \geqq 0: w(x, s) = M\, w(x, s)\}$$

and

$$\text{(13)} \qquad M\, w(\xi(x), T^*(x)) = E_x\{c(x_{T^*(x)}, \xi(x)) + w(\zeta(x), T^*(x))\},$$

where

$$\text{(14)} \qquad \zeta(x) = \begin{cases} \xi(x) & \text{if} \quad \xi(x) \in U, \\ x_{T^*(x)} & \text{otherwise}, \end{cases}$$

such that

$$w(x, 0) = \inf_{\pi \in \Pi} E^\pi_x J(0, \pi) = E^{\pi^*}_x J(0, \pi^*)$$

where $\pi^* = \{\tau(1), \xi_1^*, \tau(2), \xi_2^*, \ldots\}$,

$$\tau(0) = 0\,, \quad \tau(n+1) = T^*(x^{n+1}_{\tau(n)}) \quad \text{and} \quad \xi^{n+1}_* = \xi(x^{n+1}_{\tau(n)})\,.$$

Remark 3. The assumption $f \geqq 0$ is not restrictive since we can add any constant to f and w without changing the optimal policy.

The proof is based on the following lemma:

Lemma 2. [7, 10] *Let T be a nondecreasing and concave operator on* $C_b(E \times \mathbb{R}_+)_+ = \{g \in C_b(E \times \mathbb{R}_+): g \geqq 0\}$ and let $u \in C_b(E \times \mathbb{R}_+)_+$. *Assume that:*
(i) *there exists a* $\beta \in (0, 1)$ *such that* $\beta u \leqq T(0)$,
(ii) $T(u) \leqq u$.
Then there exists a unique solution $v \in C_b(E \times \mathbb{R}_+)_+$ *of the equation* $T(v) = v$ *and* $T^n(u) \to v$ *uniformly.*

Proof of Theorem 1.

Define an operator T on $C_b(E \times \mathbb{R}_+)_+$ by

$$T(u)(x, s) = \inf_{t \geqq s} e^{\alpha s}\{\textstyle\int_s^t \Phi_r f(x)\, e^{-\alpha r}\, dr + M\, u(x, t)\, e^{-\alpha t}\}. \tag{15}$$

The operator T is concave and nondecreasing as an infimum of affinic functions. Let

$$u(x, s) = e^{\alpha s} \textstyle\int_s^\infty \Phi_u f(x)\, e^{-\alpha u}\, du\,. \tag{16}$$

Notice that

$$u(x, s) = e^{\alpha s}\{\textstyle\int_s^t \Phi_u f(x)\, e^{-\alpha u}\, du + u(x, t)\, e^{-\alpha t}\} \quad \text{for any} \quad s \leqq t \leqq +\infty.$$

Thus $T(u) \leqq u$. Since u is bounded, there exists a $\beta \in (0, 1)$ such that $\beta u \leqq \gamma$ (γ from assumption A3). Obviously $\beta f \leqq f$. Therefore $\beta u \leqq T(0)$.

Hence there exists a unique fixed point w of the equation $T(w) = w$. By the same argument as in [9], for any $\pi \in \Pi$ we obtain

$$w(x, 0) \leqq E_x\, J(0, \pi) \quad \text{and} \quad w(x, 0) = E_x\, J(0, \pi^*)$$

by iterating the operator T.

This completes the proof.

Corollary. *By Proposition* 1, *under the assumptions of Theorem* 1, w *is a viscosity solution of the following quasi-variational inequality:*

$$\left(\frac{\partial}{\partial t} w(x, t) - \alpha\, w(x, t) + \Phi_t f(x)\right) \wedge (M\, w(x, t) - w(x, t)) = 0\,. \tag{17}$$

For certain classes of Markov processes (studied in the next section) this quasi-variational inequality has a strong solution.

4. THE CASE OF FELLER JUMP PROCESSES

Let X be a purely jump Markov process with a finite number of jumps on every bounded interval on E – a closed subset of $\mathbb{R}^n$. A jump Markov process X is determined by a following pair (k, μ), where

1. k is a nonnegative, bounded function on E,
2. $\mu(x, \cdot)$ is a probabilistic measure on $E \setminus \{x\}$ for $x \in E$.

Denote the jump periods of the process by $T_1, T_2, \ldots,$. The process X is a Markov process with the following dynamics:

(i) $x_t = x$ for $t < T_1$ P_x – a.s.,
(ii) $P_x(T_1 > t) = e^{-k(x)t}$,
(iii) $P_x(x_{T_1} \in B) = \mu(x, B)$.

Lemma 3. [5] *Let the following conditions be satisfied:*
B1. $k \in C_b(E)$,
B2. $\int_E w(y)\, \mu(\cdot, dy) \in C_b(E)$ *for any* $w \in C_b(E)$.
Then X is a Feller process. □

Lemma 4. [4, 5] *Let* B1 *and* B2 *be satisfied. Then*

(18) $$(\partial/\partial t)\, \Phi_t\, g(x) = A\Phi_t\, g(x) \quad \textit{for any} \quad g \in C_b(E) \quad \textit{and} \quad t \geqq 0\,,$$

where

(19) $$A\, u(x) = k(x) \int_E (u(y) - u(x))\, \mu(x, \mathrm{d}y)\,.$$ □

Remark 4.

(20) $$\|Ag\| \leqq 2\|k\| \,.\, \|g\| \quad \text{for any} \quad g \in C_b(E)\,.$$

Proposition 2. *Let* A1–A3 *and* B1–B2 *be satisfied. Let w be defined by* (11). *Then $w(x, \cdot) \in W^{1,\infty}(\mathbb{R}_+)$ for any $x \in E$ and $\sup_{x \in E} \|w(x, \cdot)\|_{1,\infty} < \infty$, where $W^{1,\infty}(\mathbb{R}_+)$ is the space of Lipschitz continuous and bounded functions on $\mathbb{R}_+$ and $\|\cdot\|_{1,\infty}$ is the norm in $W^{1,\infty}(\mathbb{R}_+)$.*

Remark 5. We will use the following property of the space $W^{1,\infty}(\mathbb{R}_+)$: Let z be a function $z\colon \mathbb{R}_+ \times K \to \mathbb{R}$, where K is an abstract set. If $\|z(\cdot, k)\|_{1,\infty} \leqq L$ for any $k \in K$ then $\|\inf_{k \in K} z(\cdot, k)\|_{1,\infty} \leqq L$.

The proof of Proposition 2 is based on the following lemma:

Lemma 5. [9] *Let $g \in C_b(\mathbb{R}_+)$ and $h \in W^{1,\infty}(\mathbb{R}_+)$. Let*

$$v(s) = \inf_{t \geqq s} e^{\alpha s}\{\int_s^t g(u)\, e^{-\alpha u}\, \mathrm{d}u + h(t)\, e^{-\alpha t}\}\,.$$

Then $v \in W^{1,\infty}(\mathbb{R}_+)$ and $\|v'\| \leqq 3\|g\| + \|h'\|$.

We give here a new simple proof of Lemma 5.

Proof of Lemma 5. Let $t > 0$. Compute that

(21) $(\partial/\partial s)\{e^{\alpha s}\int_s^{s+t} g(u)\,e^{-\alpha u}\,du\} = \alpha\,e^{\alpha s}\int_s^{s+t} g(u)\,e^{-\alpha u}\,du + g(s+t)\,e^{-\alpha t} - g(s)\,.$

Denote $L(s,t) = \int_s^{s+t} g(u)\,e^{-\alpha(u-s)}\,du + h(s+t)\,e^{-\alpha t}$. By (21), $L(\cdot, t) \in W^{1,\infty}(\mathbb{R}_+)$ and $|(\partial/\partial s)\,L(s,t)| \leqq 3\|g\| + \|h'\|$ for any $s, t > 0$. Further, by continuity of L,

$$v(s) = \inf_{t \geqq 0} L(s,t) = \inf_{t > 0} L(s,t)\,.$$

Hence $v \in W^{1,\infty}(\mathbb{R}_+)$ and $\|v'\| \leqq 3\|g\| + \|h'\|$. □

Proof of Proposition 2. Denote $w_0(x) = w(x, 0)$ and $c_y(x) = c(x, y)$ for $y \in U^1$. Compute that

$$|(\partial/\partial t)\,\Phi_t\,c_y(x)| = |\Phi_t A\,c_y(x)| \leqq \|k\| \,.\, \|c\| \quad \text{for any } y \in U^1 \text{ and } t \geqq 0\,,$$

$$|(\partial/\partial t)\,\Phi_t\,w_0(x)| = |\Phi_t A\,w_0(x)| \leqq \alpha^{-1} \,.\, \|k\| \,.\, \|f\| \quad \text{for any } t \geqq 0\,.$$

Moreover,

(22) $M\,w(x,t) = \min\{\Phi_t\,c_\delta(x) + \Phi_t\,w_0(x), \inf_{y \in U}[\Phi_t\,c_y(x) + w_0(y)]\}\,.$

Thus

$$\|(\partial/\partial t)\,Mw\| \leqq (\alpha^{-1} \,.\, \|f\| + \|c\|) \,.\, \|k\|\,.$$

Since $\|w\| \leqq \alpha^{-1} \,.\, \|f\|$ then $\|Mw\| \leqq \alpha^{-1} \,.\, \|f\| + \|c\|$. Hence $M\,w(x, \cdot) \in W^{1,\infty}(R_+)$ and $\|Mw\|_{1,\infty} \leqq (\alpha^{-1} \,.\, \|f\| + \|c\|)(1 + \|k\|)$. By virtue of the relation

$$w(x,s) = \inf_{t \geqq s} e^{\alpha s}\{\int_s^t \Phi_u(x)\,e^{-\alpha u}\,du + M\,w(x,t)\,e^{-\alpha t}\}$$

and by Lemma 5 we have $w(x, \cdot) \in W^{1,\infty}(\mathbb{R}_+)$ and $\|Mw\|_{1,\infty} \leqq K$, where the constant K is independent *of* x.

Corollary. *By Remark* 1 *w is a strong solution of the variational inequality* (17).

Proposition 2 generalizes the results obtained by Robin in [9] (section 5) for Markov processes with countable state space.

Acknowledgment. I would like to thank the referee for his very helpful comments.

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Souhrn

POZNÁMKA O IMPULZNÍM ŘÍZENÍ FELLEROVÝCH PROCESŮ

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Článek pojednává o problematice optimálního režimu prohlídek a údržby systému, jehož vývoj je popsán markovovským procesem. Příslušná soustava kvazivariačních nerovnic obsahuje diferenciální člen prvního řádu a autor se zabývá třemi typy jejích řešení — silným, viskózním a evolučním. Pozornost je věnována speciálnímu případu, že výše uvedený markovovský proces je po částech konstantní.

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