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(Preliminary communication)

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THE OBSERVATIONAL PREDICATE CALCULUS AND COMPLEXITY OF COMPUTATIONS

(Preliminary communication)

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Abstract: A close connection between the languages nondeterministically recognizable in polynomial time and projectively definable classes of finite structures is shown. A hierarchy of projective classes of structures is introduced and studied.

Key words: Computational complexity, recognizability in polynomial time, projective (pseudoelementary) classes of finite structures.

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Introduction. By the classical observational predicate calculus we mean the predicate calculus with the usual syntax (predicates, function symbols, connectives, classical quantifiers) but with the semantics modified by allowing only finite models. A variety of the type t is a class K of (finite) models closed under isomorphisms. K is projective x) if there is a sentence φ of a richer type such that a model M of the type t is in K iff it can be expanded to a model of φ. We say that φ projectively defines K (cf.[2],[3],[4]).

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x) Sometimes, "pseudoelementary" is used instead of "projective".

In this paper, a close connection between the languages recognizable by the nondeterministic Turing automata working in a polynomial time and the projective varieties of finite models is shown. For this purpose, a hierarchy of projective varieties is introduced. Further, a result of S.A. Cook [1] about the mentioned languages is used to prove that the hierarchy of projective varieties is strictly increasing.

Notation. 1. The complexity of a sentence φ is the number of the quantifiers contained in φ .

2. When we speak about recognizability of a variety K of oriented graphs, we mean the recognizability of the codes of elements of K . The code of an oriented graph $\langle M,R \rangle$ is a word of the length $|M|^2$ in an alphabet A , $|A| = 4$, in which the cardinality of M and the incidence matrix is marked.

3. The code of a word $\alpha \in \{0,1\}^+$ in the variety of all graphs is every graph which is the union of a strict linear ordering on some set M , $|M| = |\alpha|$, and some loops which mark presence of 1 in α . For an $L \subseteq \{0,1\}^+$ we denote by Cod(L) the variety of codes of words contained in L .

Definition. By $\mathcal{T}_N(m^k)$ we denote the set of all languages in $\{0,1\}$ recognizable nondeterministically in the time m^k .

By $\mathcal{NP}_k$ we denote the set of all varieties of graphs recognizable in time m^k .

By $\mathcal{P}_k$ we denote the set of all varieties of graphs projectively definable by a sentence of complexity k .

Theorem 1. For every $k \geq 2$, $L \in \mathcal{T}_N(m^{2k})$ iff $Cod(L) \in \mathcal{NP}_k$.

Theorem (S.A. Cook [1]). For every $1 \leq k < n$, $\mathcal{T}_N(m^k) \subsetneqq \mathcal{T}_N(m^n)$.

Corollary 2. For every $2 \leq k < n$, $\mathcal{NP}_k \subsetneqq \mathcal{NP}_n$.

Theorem 3. For every $k \geq 2$, $\mathcal{P}r_k \subseteq \mathcal{NP}_{3/2.k} \subseteq$
$\subseteq \mathcal{P}r_{6k}$.

Corollary 4. A variety of graphs is projective iff it is recognizable in a polynomial time.

Corollary 5. For every $k \geq 2$, $\mathcal{P}r_k \subsetneqq \mathcal{P}r_{6k+1}$.

Lemma 6. For every $k \geq 2$, $n \geq 1$, $\mathcal{P}r_k = \mathcal{P}r_{k+1}$ implies $\mathcal{P}r_{n.k} = \mathcal{P}r_{n.(k+1)}$.

Lemma 7. The variety of complete graphs is of complexity 2 but not 1 .

Corollary 8. For every $k \geq 1$, $\mathcal{P}r_k \subsetneqq \mathcal{P}r_{k+1}$.

Remark 9. The assertion of Corollary 8 holds for structures of any type t , whenever t contains at least one predicate or function symbol of arity at least 2 .

Corollary 4 is due to L. Lovász, D.S. Johnson and P. Gács [4]. The importance of this theorem is derived from the following consequence of it:

The class of all projective varieties is closed under complements iff the class of all languages recognizable nondeterministically in a polynomial time is closed under complements.

The original proof of this theorem in [4] uses methods different from our ones. Complete proofs are contained in the author's master thesis.

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