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ON EQUILIBRIUM POINT IN TOPOLOGICAL VECTOR SPACES

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Abstract: In [7] S. Hahn introduced the notion of σ-admissible subset of a topological vector space and proved, using this notion and Kakutani's fixed point theorem, an interesting and general fixed point theorem for quasicompact multivalued mappings in topological vector spaces, which is a generalization of many fixed point theorems for multivalued mappings. We shall prove in this paper, using Hahn's fixed point theorem, a generalization of Browder's equilibrium point theorem from [1] and of Theorem 1 from Tallos's paper [17].

Key words: Quasicompact multivalued mappings, σ-admissible subset, equilibrium point, topological vector space.

Classification: 47H10

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1. Preliminaries. First, we shall give some notations and definitions from [7] and Hahn's fixed point theorem.

Definition 1. Let E be a topological vector space, Z be a closed subset of E and $\sigma(Z)$ a non-empty system of subsets of Z. The set Z is said to be σ-admissible if for each compact mapping $F: A \to \sigma(Z)$, where A is a topological space, and for each neighborhood V of zero in E there exists a finite dimensional vector subspace E_V of E and a compact mapping $F_V: A \to \sigma(Z)$ such that we have:

(i) $F_V(A) \subseteq E_V$

(ii) For every $x \in A$, $F_V(x) \subseteq F(x) + V$.

If $Z = E$ then E is called σ-admissible topological vector space.

If $\mathcal{G}(Z) = \bigcup_{x \in Z}\{x\}$ then $\mathcal{G}$-admissible subset is admissible in the sense of V. Klee.

The admissibility of many nonlocally convex topological vector space is proved in [14] (for L^p, $0 < p < 1$), in [15] (for the space of measurable functions $S(0,1)$) and for other classes of spaces in [10], [12] and [13]. It is known that every closed and convex subset of a locally convex topological vector space is admissible but for an arbitrary topological vector space this is an open question. There is no example of convex and closed non admissible subset of a topological vector space. If E is a locally convex space, Z is a closed and convex subset of E and $\mathcal{R}(Z) = \{M \mid M \subseteq Z$, M is closed and convex$\}$ then the set Z is $\mathcal{R}$-admissible. In this paper we shall give some examples of $\mathcal{R}$-admissible subsets in non-locally convex topological vector spaces.

In [7] S. Hahn introduced the notion of quasicompact mapping and for such class of mappings proved a very general fixed point theorem.

Definition 2. Let E be a topological vector space, K be a closed, convex and M a closed subset of E with $M \subseteq K$. Let $\mathcal{G}(K)$ be a system of non-empty subsets of K. An upper semicontinuous mapping $F: M \to \mathcal{G}(Z)$ is called quasicompact, if for each $b \in M$ there exists a closed, convex subset $T_0 \subseteq E$ such that we have $b \in T_0$, $F(M \cap T_0) \subseteq T_0 \cap K$ and the set $\overline{F(M \cap T_0)}$ is compact and $K \cap T_0$ is $\mathcal{G}$-admissible.

From Definition 2 it is easy to see that every compact mapping $F: M \to \mathcal{G}(K)$, where K is a closed, convex, $\mathcal{G}$-admissible subset of a topological vector space E, is quasicompact mapping since in this case we can take that $T_0 = K$. In [7] is proved that every Ψ densifying mapping $F:M \to \mathcal{R}(K)$ in locally convex space is a quasicompact mapping. In [8] S. Hahn gave some examples of quasicompact mappings. So the following fixed point theorem generalizes many fixed point theorems for multivalued mappings in localy convex spaces.

Theorem. Let E be a topological vector space which is Hausdorff, W a closed neighborhood of a point $b \in E$, K a closed, convex subset of E such that $b \in K$. Let $F: W \cap K \to \mathcal{R}_0(K)$ be a quasicompact mapping where $\mathcal{R}_0(K)$ is $\mathcal{R}(K)$ or $\bigcup_{x \in K}\{x\}$. If $tx + (1-t)b \notin Fx$ for every $x \in \partial W \cap K$ then there exists a point $x_0 \in W \cap K$ with

$$x_0 \in F(x_0).$$

From the Theorem we obtain the following Corollary.

Corollary. Let E be a Hausdorff topological vector space, K be a closed, convex and $\mathcal{R}$-admissible subset of E and $F: K \to \mathcal{R}(K)$ be a compact mapping (i.e. upper semicontinuous and $\overline{F(K)}$ is compact). Then there exists $x_0 \in K$ so that $x_0 \in F(x_0)$.

2. Two theorems on equilibrium point. First we shall prove two Lemmas. Lemma 1 is a generalization of a result from [13] and Lemma 2 is a generalization of a Browder's result from [1].

Lemma 1. Let E_i $(i \in I)$ be a Hausdorff topological vector space, $K_i \subseteq E_i$ $(i \in I)$, $\sigma_i(K_i)$ a nonempty family of subsets of K_i, $\{a_i\}$ is in $\sigma_i(K_i)$ and K_i be σ_i admissible, for every $i \in I$. If $\sigma(\prod_{i \in I} K_i)$ is a non-empty family of subsets of $\prod_{i \in I} K_i$ such that:

$$A \in \sigma(\prod_{i \in I} K_i) \Longleftrightarrow \mathrm{pr}_{\mathbf{r}}\ A \in \sigma_i(K_i)$$

then $\prod_{i\in I} K_i$ is σ-admissible.

Proof: Let M be an arbitrary topological space and $F: M \to \sigma(\prod_{i\in I} K_i)$ be a compact mapping. We shall denote by $\mathcal{V}_i$ the fundamental system of neighborhoods of zero in E_i and by $\mathcal{V}$ the fundamental system of neighborhoods of zero in $\prod_{i\in I} E_i$ (in the product topology). Let $V \in \mathcal{V}$. We shall show that there exists a compact finite dimensional mapping $\bar{F}: M \to \sigma(\prod_{i\in I} K_i)$ so that for every $x \in M$:

$$(1) \qquad \bar{F}(x) \subseteq F(x) + V.$$

Since V is a neighborhood of zero in $\prod_{i\in I} E_i$ it follows that there exists a finite set $\{i_1, i_2, \dots, i_n\} \subseteq I$ and $V_i \in \mathcal{V}_i$ so that:

$$V = \begin{cases} V_i & i \in \{i_1, i_2, \dots, i_n\} \\ E_i & i \in I \setminus \{i_1, i_2, \dots, i_n\} \end{cases}.$$

Let $F_i(x) = pr_{K_i} F(x)$, for every $i \in I$ and every $x \in M$. Since $F(x) \in \sigma(\prod_{i\in I} K_i)$ and for every $A \subseteq \prod_{i\in I} K_i$, $A \in \sigma(\prod_{i\in I} K_i)$ implies $pr_{K_i} A \in \sigma_i(K_i)$ it follows that for every $x \in M$ and every $i \in I$ $F_i(x) \in \sigma_i(K_i)$ and so $F_i: M \to \sigma_i(K_i)$ is a compact mapping for every $i \in I$. Further for every $i \in I$ the set K_i is σ_i admissible. Let $i \in \{i_1, i_2, \dots, i_n\}$. Since F_i is a compact mapping there exists a finite dimensional subspace $E_{V_i} \subseteq E_i$ and a finite dimensional compact mapping $\bar{F}_i: M \to \sigma_i(K_i)$ so that for every $x \in M$:

$$\overline{\bar{F}_i(M)} \subseteq E_{V_i}, \quad \bar{F}_i(x) \subseteq F_i(x) + V_i.$$

Let $\bar{F}_i(x) = a_i$, for every $x \in M$ and every $i \in I \setminus \{i_1, i_2, \dots, i_n\}$ and for every $x \in M$ we shall define the mapping $\bar{F}$ by $\bar{F}(x) = \prod_{i\in I} \bar{F}_i(x)$. For every $i \in I$ we have that $\bar{F}_i(x) \in \sigma_i(K_i)$.

Further, it is obvious that $\overline{F}$ is an upper semicontinuous mapping which is finite dimensional. Since $\overline{\overline{F}_i(M)}$ is compact for every $i \in I$ it follows that $\overline{F}: M \longrightarrow \mathscr{G}(\prod_{i \in I} K_i)$ is a compact finite dimensional mapping such that (1) is satisfied.

The above Lemma generalizes Satz 1.11 from [13], where $K_i = E_i$, for every $i \in I$ and $\sigma_i(E_i) = \bigcup_{x \in E_i} \{x\}$ and $\mathscr{G}(\prod_{i \in I} E_i) = \bigcup_{x \in \prod_{i \in I} E_i} \{x\}$.

<u>Lemma 2</u>. <u>Let</u> K_i $(i \in I)$ <u>be a non-empty, convex and compact subset of Hausdorff topological vector space</u> E_i $(i \in I)$, $K = \prod_{i \in I} K_i$ <u>and for every</u> $i \in I$, $K_i' = \prod_{j \neq i} K_j$. <u>Further</u> $S_i = \overline{S}_i \subseteq K$, <u>for every</u> $i \in I$ <u>and for every</u> $i \in I$ <u>and every</u> $x \in K$ <u>the set</u> $S_i(x)$ <u>is a nonempty and convex subset of</u> K_i <u>where</u>:

$$S_i(x) = \{ y_i, y_i \in K_i, [y_i, \hat{x}_i] \in S_i \}$$

<u>and for</u> $x = (x_i) \in K$, $\hat{x}_i = pr_{K_i'} x$, $z = [y_i, \hat{x}_i] \in K$ <u>means</u> $z_j = y_i$ <u>for</u> $j = i$, $z_j = x_j$, <u>for</u> $j \neq i$.

<u>If for every</u> $i \in I$, K_i <u>is</u> $\mathfrak{R}$<u>-admissible then</u>

$$\bigcap_{i \in I} S_i \neq \emptyset.$$

Proof: As in [1] let us define the mapping $T: K \longrightarrow \mathfrak{R}(K)$ in the following way:

$$T(x) = \prod_{i \in I} S_i(x), \text{ for every } x \in K.$$

Since $S_i(x) \in \mathfrak{R}(K_i)$ it follows that $T(x)$ is convex and from $S_i(x) = \pi_1(\pi_2^{-1}(\hat{x}_i) \cap S_i)$ (for every $i \in I$ and every $x \in K$) it follows that $T(x)$ is compact, for every $x \in K$, where $\pi_1: K_i \times K_i' \longrightarrow K_i$ and $\pi_2: K_i \times K_i' \longrightarrow K_i'$ are the projection operators. As in [1] it follows that the mapping T is upper semicontinuous and from Lemma 1 it follows that the set $\prod_{i \in I} K_i$ is $\mathfrak{R}$-admissible compact subset of $\prod_{i \in I} E_i$. Applying the Corollary from the Theorem we conclude that there exists $u \in \prod_{i \in I} K_i$ so that $u \in T(u)$ and so $u \in \bigcap_{i \in I} S_i$.

Now, we shall prove the first equilibrium theorem which is a generalization of Browder's theorem from [1].

Theorem 1. Let K_i $(i \in I)$ be a non-empty, compact and convex subset of Hausdorff topological vector space E_i $(i \in I)$, $K = \prod_{i \in I} K_i$, $f_i : K \longrightarrow \mathbb{R}$ $(i \in I)$ be continuous real valued function and $K_i' = \prod_{j \neq i} K_j$ so that each point $x \in K$ can be written uniquely in the form $[x_i, \hat{x}_i]$ $(x_i \in K_i,\ \hat{x}_i \in K_i')$. If for every $i \in I$ and $t \in \mathbb{R}$ the set $\{y_i, y_i \in K_i,\ f_i(y_i, \hat{x}_i) \geq t\}$ is a convex subset of K_i, for every $\hat{x}_i \in K_i'$ and for every $i \in I$ the set K_i is $\mathcal{R}$-admissible, there exists a point $u \in K$ such that:

$$f_i(u) = \max_{y_i \in K_i} f_i(y_i, \hat{u}_i), \text{ for every } i \in I. \tag{2}$$

Proof: As in [1] let:

$S_i = \{u \mid u \in K,\ f_i(u) \geq \max f_i(y_i, \hat{u}_i)\}$, for every $i \in I$.

Then all the conditions of Lemma 2 are satisfied and so there exists $u \in K$ so that $u \in \bigcap_{i \in I} S_i$ which implies that u satisfies (2).

Theorem 2. Let $E_1, E_2, \ldots, E_n$ be metrizable topological vector space and $K_i \subseteq E_i$ a non-empty convex, compact and σ-admissible subset $(i \in 1,2,\ldots,n)$ where $\sigma(K_i) = \bigcup_{x \in K_i} \{x\}$. Moreover let $J_i : \prod_{i=1}^{n} K_i \longrightarrow \mathbb{R}$ be a convex functional so that for every $i \in \{1,2,\ldots,n\}$:

(i) $J_i(\cdot, \hat{x}_i) : K_i \longrightarrow \mathbb{R}$ is lower semicontinuous for every $\hat{x}_i \in K_i' = \prod_{j \neq i} K_j$

(ii) The family $\{J_i(x_i, \cdot) : K_i' \longrightarrow \mathbb{R}\ ,\ x_i \in K_i\}$ is equicontinuous for every $i \in \{1,2,\ldots,n\}$.

<u>Then there exists at least one element</u> $u \in \prod_{i=1}^{n} K_i$ <u>so that</u>:

$$J_i(u) \leq J_i(x_i, \hat{u}_i), \text{ for every } i \in \{1,2,\ldots,n\},\ x_i \in K_i$$

$(\hat{u}_i = pr_{K_i} u)$.

Proof: The proof is very similar to the proof of Theorem 1 from [17] and we shall give only a short proof since the rest of the proof is as in [17]. First, some notations which we need. For every $\hat{x}_i \in K_i'$ $(i \in \{1,2,\ldots,n\})$ and $t > 0$ let:

$$m_i(\hat{x}_i) = \min_{x_i \in K_i} J_i(x_i, \hat{x}_i), \quad \Phi_i^t(\hat{x}_i) = \{x_i \in K_i, J_i(x_i, \hat{x}_i) \leq m_i(\hat{x}_i) + t\}.$$

Let $\hat{x}_i \in K_i'$ and $U(\hat{x}_i)$ be a neighborhood of $\hat{x}_i \in K_i'$ so that:

$$\hat{x}_i' \in U(\hat{x}_i),\ x_i \in K_i \Rightarrow |J_i(x_i, \hat{x}_i) - J_i(x_i, \hat{x}_i')| < \frac{t}{4}.$$

If we suppose that $U(\hat{x}_i) \subset E_i$ is open, since K_i' is compact from the open covering $\{U(\hat{x}_i),\ \hat{x}_i \in K_i'\}$ we can select a finite open subcovering $\{U(\hat{x}_{ij}),\ j = 1,2,\ldots,n_i\}$ of K_i'. If $x_{ij} \in \Phi_i^{t/2}(\hat{x}_{ij})$ $(j \in \{1,2,\ldots,n_i\})$ and $h_j(\hat{x}_i) = x_{ij}$ $(\hat{x}_i \in K_i')$ in [17] is proved that for every $\hat{x}_i \in U(\hat{x}_{ij})$, $h_j(\hat{x}_i) \in \Phi_i^t(\hat{x}_i)$ which means:

$$\hat{x}_i \in U(\hat{x}_{ij}) \Longrightarrow J_i(x_{ij}, \hat{x}_i) \leq m_i(\hat{x}_i) + t.$$

Further, let $\{\theta_{ij},\ j \in \{1,2,\ldots,n_i\}\}$ be a continuous partition of unity subordinated to $\{U(\hat{x}_{ij}),\ j \in \{1,2,\ldots,n_i\}\}$ and $g_i^t = \sum_{j=1}^{n_i} \theta_{ij} h_j$. Then $g_i^t : K_i' \to E_i$ is continuous, maps K_i' into K_i and:

$$g_i^t(\hat{x}_i) \in \Phi_i^t(\hat{x}_i), \text{ for every } \hat{x}_i \in K_i' \ (i \in \{1,2,\ldots,n\}).$$

As in [17] it is enough to prove that the mapping $f^t : \prod_{i \in I} K_i \longrightarrow \prod_{i=1}^{n} K_i$ has a fixed point where:

$f^t = (f_1^t, f_2^t, \ldots, f_n^t)$, $f_i^t(x) = g_i^t(\hat{x}_i)$, for every $x \in \prod_{i=1}^{n} K_i$, $\hat{x}_i = pr_{K_i} x$ and every $i \in \{1,2,\ldots,n\}$. From Lemma 1 it follows that $\prod_{i=1}^{n} K_i$ is admissible and since $\prod_{i=1}^{n} K_i$ is a compact subset of

$\prod_{i=1}^{n} E_i$ it follows that there exists $x^t \in \prod_{i=1}^{n} K_i$ which is a fixed point of the mapping f^t and from the definition of the mapping f^t it follows that $J_i(x^t) \leq J_i(x_i, \hat{x}_i^t) + t$, for every $x_i \in K_i$ and every $i \in \{1,2,\dots,n\}$. The rest of the proof is as in [17].

Now, we shall give an example of $\mathcal{R}$-admissible subset in not necessarily locally convex topological space.

<u>Definition 3.</u> <u>Let</u> E <u>be a topological vector space,</u> $\mathcal{V}$ <u>be the fundamental system of neighborhoods of zero in</u> E <u>and</u> $K \subseteq E$. <u>We say that</u> K <u>is of Zima's type if and only if for every</u> $V \in \mathcal{V}$ <u>there exists</u> $U \in \mathcal{V}$ <u>so that</u>:

(3) $$co(U \cap (K-K)) \subseteq V.$$

Remark: If E is a locally convex topological vector space then we can suppose that U=co U (convex hull of U) and so for U = V (3) is satisfied.

Now, we shall give an example of a subset of Zima s type in a paranormed space [18]. Let E be a linear space over the real or complex number field. The function $\| \|^* : E \to [0,\infty)$ will be called paranorm if and only if:

1. $\|x\|^* = 0 \Longleftrightarrow x = 0$, 2. $\|-x\|^* = \|x\|^*$, for every $x \in E$.

3. $\|x + y\|^* \leq \|x\|^* + \|y\|^*$, for every $x \in E$.

4. If $\|x_n - x_o\|^* \to 0$ and $r_n \to r_o$ then $\|r_n x_n - r_o x_o\|^* \to$ $\to 0$.

Then $(E, \| \|^*)$ is a paranormed space and also a topological vector space if the fundamental system of neighborhoods of zero in E is given by the family $\{V_r\}_{r>0}$, where $V_r = \{x \mid x \in E, \|x\|^* < r\}$.

In [18] there is given an example of $(E, \| \ \|^*)$ and $K \subseteq E$ so that:

(4) $\| tx \|^* \leq t\ C(K) \| x \|^*$, for every $t \in [0,1]$ and every $x \in K-K$.

In [4] we have proved that every subset $K \subseteq E$, where $(E, \| \ \|^*)$ is a paranormed space and (4) is satisfied, is a set of Zima's type in the sense of the Definition 3. Now let us give an example of a nonlocally convex topological vector space E and $K \subseteq E$ so that K is of Zima's type.

Let S(0,1) be the space of measurable finite functions (classes) on the interval [0,1] with the paranorm:

$$\| \hat{x} \|_{S(0,1)} = \int_0^1 \frac{|x(t)|}{1 + |x(t)|} \mu(dt), \ \{x(t)\} \in \hat{x}$$

It is known that S(0,1) is admissible. If $t > 0$ let us define the set K_t in the following way:

$$K_t = \{ \hat{x}, \hat{x} \in S(0,1) \text{ and } |x(u)| \leq t, \text{ for every } u \in [0,1] \}.$$

Then (4) is satisfied with $C(K_t) = 1 + 2t$. Indeed, suppose that $\hat{x}, \hat{y} \in K_t$ and that $\{x(u)\} \in \hat{x}$, $\{y(u)\} \in \hat{y}$. Then $1 + |x(u) - y(u)| \leq \leq 1 + 2t \leq 1 + 2t + (1 + 2t)s\,|x(u) - y(u)|$ and so:

(5) $$\frac{1}{1+s|x(u)-y(u)|} \leq (1+2t) \frac{1}{1+|x(u)-y(u)|} \quad u \in [0,1].$$

From (5) it is easy to see that $C(K_t) = 1 + 2t$.

Now, we shall prove the following Proposition:

Proposition. Let E be a Hausdorff topological vector space and K be a closed and convex subset of Zima's type of E. Then K is $\mathcal{R}$-admissible.

Proof: Let $\mathcal{V}$ be the fundamental system of neighborhoods of zero in E and A be a topological space. If $F: A \longrightarrow \mathcal{R}(K)$ is a compact mapping we have to prove that there exists a finite dimensional compact mapping $F_V: A \to \mathcal{R}(K)$, where $V \in \mathcal{V}$, so that:

(6) $$F_V(x) \subseteq F(x) + V, \text{ for every } x \in A.$$

Since $\overline{F(A)}$ is compact set there exists a finite set $\{x_1, x_2, \ldots, x_n\} \subseteq F(A)$ such that $F(A) \subseteq \bigcup_{i=1}^{m} \{x_i + U\}$, where $\overline{co}(U \cap \cap (K - K)) \subseteq V$. If for every $x \in A$, F_V is defined by:

$$F_V(x) = [F(x) + \overline{co}(U \cap (K - K)] \cap \overline{co} \{x_1, x_2, \ldots, x_n\},$$

then, as in [4] , it follows that F_V is a finite dimensional compact mapping from A into $\mathcal{R}(K)$ so that (6) is satisfied.

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