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Concerning the fine topology from the band $\mathcal{M}$

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We consider a possible reformulation of this principle using modal logic.
Our main idea is that it seems to us (from point of view of our knowledge) that set universum behaves as if Cantor's comprehension were sound. More formally: we use the modal lower predicate calculus with identity (we adopt the rule of necessitation too) as is formalized in [1] and we prefer to think about " $\Box\varphi$ " as "we know φ" (i.e., epistemic modality). Additional modal axioms are those of system T (see [1]); $\Box\varphi \longrightarrow \varphi$ and $\Box(\varphi \longrightarrow \psi) \longrightarrow (\Box\varphi \longrightarrow \Box\psi)$. The formal theory MST consists (except preceding logical axioms) of these principles:

(i) $(\forall t;\, t \in x \equiv t \in y) \longrightarrow x = y$ extensionality

(ii) $\Diamond x = y \longrightarrow \Box x = y$

(iii) for any formula $\varphi(t)$ (possibly with modality and parameters) holds:

$$\exists y\, \forall t;\, (\Box\varphi(t) \equiv \Box t \in y)\ \&\ (\Box\neg\varphi(t) \equiv \Box t \notin y)$$

So the last principle is formalization of our main idea. Theory MST interprets, for example, arithmetic with bounded induction $I\Delta_0$ thus it contains some nontrivial mathematics. It also proves existence of an infinite set.
But much more can be done. By strengthening of underlying logic (namely by S4, Brouwer's axiom and Barcan's formula - see [1]) we are able to develop Peano arithmetic with full induction. We are not able to prove consistency of our system. About this problem we have only results of partial relative consistency with respect to ZF (in fact to Gilmore's PST) and to Quine's NF. On the other hand we know that some strong modal axioms (namely S5) make the theory contradictory.

Reference: [1] G.E. Hughes, M.J. Cresswell: An introduction to modal logic, Methuen and Co. Ltd., 1968.

CONCERNING THE FINE TOPOLOGY FROM THE BAND $\mathcal{M}$

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The coarsest topology m making all potentials from the band $\mathcal{M}$ continuous does not generally satisfy the following "quasilindelöf property":

Every family of m-open sets contains a countable subfamily whose union differs from the union of the whole family by a polar (or negligible) set.

The heat equation on $R \times R$ with the cover $\{U_y\}_{y\in R}$ of R^2 where $U_y = \{(x,t): t \neq 0 \text{ or } x=y\}$ serves as a counterexample. The sets U_y are m-open by the following

Theorem. Let X be a $\mathfrak{P}$-harmonic space with a countable base ([1]). If $A \subset X$ is thin at a polar point $z \in X \setminus A$ and $\bar{A} \subset A \cup \{z\}$, then $X \setminus A$ is m-open.

Proof. Let v be a potential on X which is $+\infty$ at z ([1], Ex. 6.2.1) and p be a strictly positive finite continuous potential on X. From [1], Ex. 8.2.2 and C.5.3.2 we deduce $\inf \{R_p^{A\cap V}(z):\ V \text{ is a neighborhood of } z\} = 0$. Hence there is a decreasing sequence $\{V_n\}$ of compact neighborhoods of z and a sequence $\{q_n\}$ of potentials on X such that $\{z\} = \bigcap V_n$, $q_n \leq 2^{-n}v$, $q_n(z) \leq 2^{-n}$ and $q_n = 2p$ on $V_n \cap A$. Let w_n be finite continuous potentials with $w_n \leq q_n$ on X and $w_n > p$ on $(\overline{V_n \setminus V_{n^2}}) \cap A$ ([1], C.2.3.1 and the Dini theorem). Then $w := \sum w_n \in \mathcal{M}$ and

$$w(z) < +\infty = \liminf_{x \to z, x \in A} w(x).$$

Reference: [1] C. Constantinescu, A. Cornea: Potential Theory on Harmonic Spaces, Berlin-Heidelberg-New York: Springer 1972.