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Vladimir Vladimirovich Uspenskij

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The Mal'tsev operation on countably compact spaces

V.V.USPENSKIJ

Abstract. Let X be a countably compact topological space and $f : X^3 \rightarrow X$ be a continuous mapping such that the identity $f(x, y, y) = f(y, y, x) = x$ holds. Then βX is Dugundji. It follows that compact retracts of topological groups are dyadic.

Keywords: Dugundji space, retract, pseudocompact

Classification: primary 54D30; secondary 22A30

We say that a topological space X is Mal'tsev if there exists a continuous mapping $f : X^3 \rightarrow X$ (called a Mal'tsev operation [M]) such that the identity $f(x, y, y) = f(y, y, x) = x$ holds for all $x, y \in X$. Every topological group G is a Mal'tsev space, since the mapping $(x, y, z) \rightarrow xy^{-1}z$ is a Mal'tsev operation on G . M.G.Tkačenko proved in 1981 that compactly-generated topological groups have the Suslin property [T₁]. This result was extended by the author in [U₁]: every σ -compact Mal'tsev space has the Suslin property. In the present paper we show that compact Mal'tsev spaces are Dugundji (Theorem 1). Moreover, if X is a countably compact Mal'tsev space, then βX is Dugundji (Theorem 2). Clearly any retract of a Mal'tsev space is Mal'tsev. It follows that compact retracts of topological groups are Dugundji. This is an extension of the Ivanovskij - Kuzminov theorem (its proof can be found in [U₃]) which says that compact groups are dyadic.

Let X be a subspace of Y . A topological space Z is said to be injective with respect to the pair (X, Y) iff every continuous function $f : X \rightarrow Z$ has a continuous extension $\bar{f} : Y \rightarrow Z$. For a compact space X the following are equivalent [H]: 1) if Y is a zerodimensional compact space and Z is closed in Y , then X is injective with respect to the pair (Z, Y) ; 2) if X is a subspace of a compact space Y , then every compact convex subset of a locally convex topological vector space is injective with respect to the pair (X, Y) . A compact space X is called Dugundji [P] if one of these conditions holds. Let X be Dugundji. There exist a zero-dimensional compact space Y and an onto mapping $f : Y \rightarrow X$. We may assume that Y is a subspace of a Cantor cube 2^m . Since X is injective with respect to $(Y, 2^m)$, there is an extension $f : 2^m \rightarrow X$ of f . This shows that Dugundji spaces are dyadic [H].

An equivalence relation R on a space X is open if the quotient mapping $X \rightarrow X/R$ is open.

Ščepin's theorem S1 [Š]. A compact space X is Dugundji iff there exists a family Φ of equivalence relations on X such that:

- 1) for every $R \in \Phi$ the quotient space X/R is metrizable
- 2) Φ is closed under countable intersections

- 3) all $R \in \Phi$ are open
 4) Φ separates points of X (that means the intersection $\cap \Phi$ equals the diagonal of X^2).

Let $S = \{X_\alpha, p_\alpha^\beta, A\}$ be an inverse system of compact metric spaces X_α and onto bonding mappings $p_\alpha^\beta: X_\beta \rightarrow X_\alpha$. We call S a Ščepin system if for every countable subset $B \subseteq A$ there exists a least upper bound $\alpha = \sup B \in A$ and the family $\{p_\alpha^\beta: \beta \in B\}$ separates points of X_α . Theorem S1 means that a compact space X is Dugundji iff it is the inverse limit of a Ščepin system $S = \{X_\alpha, p_\alpha^\beta, A\}$ such that the projections $p_\alpha: X \rightarrow X_\alpha$ are open (or, equivalently, the bonding mappings p_α^β are open).

Ščepin's theorem S2 [Š]. Let $S = \{X_\alpha, p_\alpha^\beta, A\}$ and $T = \{Y_\alpha, q_\alpha^\beta, A\}$ be two Ščepin systems (the directed index set A is the same for S and T), $X = \lim S, Y = \lim T$. Let f be a mapping of X to Y . Let A_f be the set of all $\alpha \in A$ with the following property: there exists $f_\alpha: X_\alpha \rightarrow Y_\alpha$ such that $f_\alpha \circ p_\alpha = q_\alpha \circ f$ (here $p_\alpha: X \rightarrow X_\alpha$ and $q_\alpha: Y \rightarrow Y_\alpha$ are the projections). Then A_f is cofinal in A .

Let $f: X^3 \rightarrow X$ be a Mal'tsev operation on a space X . An equivalence relation R on X is called a congruence (or an f -congruence) if $f(x_1, x_2, x_3)$ is R -equivalent to $f(y_1, y_2, y_3)$ whenever x_i is R -equivalent to $y_i, i = 1, 2, 3$.

Mal'tsev's theorem [M]. If f is a Mal'tsev operation on a space X , then every f -congruence R on X is open.

PROOF: Let U be open in X and $V = \{y \in X: (x, y) \in R \text{ for some } x \in U\}$. We have to show that V is open. Let $y \in V$. Choose $x \in U$ so that $(x, y) \in R$. If z is close enough to y , then $f(x, y, z)$ is in U , since $f(x, y, y) = x$ is in U . On the other hand, $f(x, y, z)$ is R -equivalent to $f(x, x, z) = z$. Hence V is a neighbourhood of y . ■

Theorem 1. Every compact Mal'tsev space is Dugundji.

PROOF: Let $f: X^3 \rightarrow X$ be a Mal'tsev operation on a compact space X . Let Φ be the family of all f -congruences R on X such that the quotient space X/R is metrizable. Clearly Φ is closed under countable intersections, so conditions 1) and 2) of Ščepin's theorem S1 are satisfied. Mal'tsev's theorem shows that condition 3) holds, too. It remains to prove that Φ separates points of X . Let $S = \{X_\alpha, p_\alpha^\beta, A\}$ be a Ščepin system whose limit is X . Then X^3 is the limit of Ščepin's system $S^3 = \{(X_\alpha)^3, (p_\alpha^\beta)^3, A\}$. Let $p_\alpha: X \rightarrow X_\alpha$ be the projections of S . Call $\alpha \in A$ nice if there exists $f_\alpha: X_\alpha^3 \rightarrow X_\alpha$ such that $f_\alpha \circ (p_\alpha)^3 = p_\alpha \circ f$. The relation $R_\alpha = \{(x, y) \in X^2: p_\alpha(x) = p_\alpha(y)\}$ is an f -congruence whenever α is nice. Theorem S2 implies that the set A_f of all nice α is cofinal in A . Hence the subfamily $\{R_\alpha: \alpha \in A_f\}$ of Φ separates points of X . ■

If βX is Dugundji, then X is pseudocompact. We give a characterization of spaces X for which βX is Dugundji.

Proposition 1. *Let X be pseudocompact. Then βX is Dugundji if and only if there exists a family Φ of equivalence relations on X such that:*

- 1) *for every $R \in \Phi$ the quotient space X/R is submetrizable*
- 2) *Φ is closed under countable intersections*
- 3) *every $R \in \Phi$ is open*
- 4) *the topology of X is determined by the quotient mappings $f_R : X \rightarrow X/R$, $R \in \Phi$.*

We need some lemmas. Recall that a mapping $p : Y \rightarrow Z$ is d -open if one of the equivalent conditions holds:

- 1) $\text{cl } p^{-1}(U) = p^{-1}(\text{cl } U)$ for every U open in Z
- 2) $p(V) \subseteq \text{int } \text{cl } p(V)$ for every V open in Y .

Lemma 1. *If X is pseudocompact and M is metric, then every d -open mapping $f : X \rightarrow M$ is open.*

PROOF : Let U be open in X and $x \in U$. Choose an open set V so that $x \in V \subseteq \text{cl } V \subseteq U$. Since $\text{cl } V$ is pseudocompact, $f(\text{cl } V)$ is closed in M . It follows that $\text{cl } f(V) = f(\text{cl } V)$, hence $f(x) \in \text{int } \text{cl } f(V) = \text{int } f(\text{cl } V) \subseteq \text{int } f(U)$. This means that $f(U)$ is open. ■

Lemma 2. *Let X be pseudocompact. If R is an open equivalence relation on X such that the quotient space X/R is submetrizable, then X/R is metrizable (and hence compact).*

PROOF : Let $f : X \rightarrow X/R$ be the quotient mapping, $g : X/R \rightarrow K$ be a one-to-one mapping of X/R onto a compact metric space K , and $h = g \circ f$. We have to show that g is a homeomorphism, or, equivalently, that h is open. In virtue of Lemma 1 it suffices to show that h is d -open. Let U be open in K and $F = \text{cl } h^{-1}U$. Since f is open, $F = \text{cl } f^{-1}g^{-1}(U) = f^{-1} \text{cl } g^{-1}U = h^{-1}E$ for some $E \subseteq K$. Now F is pseudocompact, being a regular closed set in X , so $E = h(F)$ is closed in K . It follows that $E = \text{cl } U$ and $\text{cl } h^{-1}U = h^{-1} \text{cl } U$. This means that h is d -open. ■

Lemma 3. *Let X be G_δ -dense in Y (= every non-empty G_δ -subset of Y meets X). If f maps Y to a metric space M and the restriction $f|_X : X \rightarrow M$ is open, then $f : Y \rightarrow M$ is open.*

PROOF : If U is open in Y , then $f(U) = f(U \cap X)$. ■

Lemma 4. *If Y is Dugundji-compact and X is a dense subspace of Y , then the following are equivalent:*

- 1) $Y = \beta X$
- 2) X is pseudocompact
- 3) X is G_δ -dense in Y .

PROOF : Dugundji spaces are perfectly κ -normal (= closures of open sets are zero-sets), and G_δ -dense subspaces of perfectly κ -normal spaces are C -embedded [T2]. This gives 3) \Rightarrow 1). If βX is dyadic, then X is pseudocompact [EP]. Since Dugundji spaces are dyadic, 1) \Rightarrow 2) follows; and 2) \Rightarrow 3) is obvious. ■

PROOF of Proposition 1: The "only if" part follows from theorem S1 and Lemma 1. Conversely, let Φ be such a family as described in Proposition 1. The spaces $X/R, R \in \Phi$, are compact and constitute a Štěpín system S . Let $Y = \lim S$. Then X can be regarded as a G_δ -dense subspace of Y . The open mappings $f_R : X \rightarrow X/R$ are restrictions of the projections of S , so Lemma 3 shows that the projections of S are open. Theorem S1 implies that Y is Dugundji, and Lemma 4 shows that $Y = \beta X$. ■

Suppose G is a subgroup of the product of metrizable groups, H is a closed subgroup of G . Let $X = G/H$ be the quotient space. If X is pseudocompact, then βX is Dugundji. This follows from Proposition 1, combined with Lemma 7 in [U3]. Theorem 1 in [U3] also can be extended to the pseudocompact case: if G is a subgroup of the product of groups with a countable base and G acts transitively on a pseudocompact space X , then βX is Dugundji. If a pseudocompact space X is a retract of a topological group, then βX is Dugundji [U4].

Conjecture 1. If X is pseudocompact and Mal'tsev, then βX is Dugundji.

Conjecture 2. The product of any two pseudocompact Mal'tsev spaces is pseudocompact.

Proposition 2. *Conjectures 1 and 2 are equivalent.*

PROOF: Suppose $\{X_\alpha : \alpha \in A\}$ is a family of pseudocompact spaces such that βX_α is Dugundji for every $\alpha \in A$. Then $Q = \Pi \beta X_\alpha$ is Dugundji and $P = \Pi X_\alpha$ is G_δ -dense in Q . Lemma 4 implies that P is pseudocompact. Hence if Conjecture 1 is true, then the product of any family of pseudocompact Mal'tsev spaces is pseudocompact. Conversely, let $f : X^3 \rightarrow X$ be a Mal'tsev operation on a pseudocompact space X . If Conjecture 2 is true, then X^3 is pseudocompact. By virtue of Glicksberg's theorem [G], [E, problem 3.12.20], $\beta(X^3) = (\beta X)^3$, hence f has an extension $\bar{f} : (\beta X)^3 \rightarrow \beta X$. Clearly \bar{f} is a Mal'tsev operation on βX , so Theorem 1 shows that βX is Dugundji. ■

We shall prove that Conjecture 1 is true for countably compact spaces (Theorem 2).

Rezničenko's theorem R1. *Suppose X_1, X_2, X_3 are countably compact and have the Suslin property, M has a countable base, $f : X_1 \times X_2 \times X_3 \rightarrow M$ is separately continuous (i.e., if x_1, x_2, x_3 are fixed, $x_i \in X_i$, then the functions $f(x_1, x_2, \cdot), f(x_1, \cdot, x_3), f(\cdot, x_2, x_3)$ are continuous on X_3, X_2, X_1 , respectively). Then there exist compact metric spaces Y_1, Y_2, Y_3 , continuous onto mappings $p_i : X_i \rightarrow Y_i, i = 1, 2, 3$, and a separately continuous function $g : Y_1 \times Y_2 \times Y_3 \rightarrow M$ such that $f = g \circ (p_1 \times p_2 \times p_3)$.*

Rezničenko's theorem R2. *Suppose X_1 and X_2 are pseudocompact, M has a countable base, $f : X_1 \times X_2 \rightarrow M$ is (jointly) continuous. Then there exist Eberlein-compact spaces Y_1, Y_2 , continuous onto mappings $p_i : X_i \rightarrow Y_i$, and a separately continuous function $g : Y_1 \times Y_2 \rightarrow M$ such that $f = g \circ (p_1 \times p_2)$. If X_1 and X_2 have the Suslin property, then Y_1 and Y_2 are metrizable.*

Recall that a compact space X is Eberlein iff X embeds in the function space $C_p(Y)$ for some compact Y .

Proposition 3. *Suppose X is a Tikhonov countably compact space having the Suslin property. If there exists a separately continuous Mal'tsev operation $f : X^3 \rightarrow X$, then βX is Dugundji.*

PROOF : Let $\Psi = \{R : R \text{ is an equivalence relation on } X \text{ and } X/R \text{ is submetrizable}\}$ and $\Phi = \{R \in \Psi : R \text{ is an } f\text{-congruence}\}$. Mal'tsev's theorem shows that every $R \in \Phi$ is open. Clearly Φ is closed under countable intersections. To check the condition 4) of Proposition 1, it suffices to prove that for every $T \in \Psi$ there exists $R \in \Phi$ such that $R \subseteq T$. Let $T \in \Psi$. Using theorem R1, construct a sequence $T_0 = T, T_1, T_2 \dots$ such that every T_n is in Ψ and the conditions $(x_i, y_i) \in T_{n+1}, i = 1, 2, 3$, imply that $f(x_1, x_2, x_3)$ and $f(y_1, y_2, y_3)$ are T_n -equivalent, $n = 0, 1, \dots$. Then $R \cap T_n$ is a congruence, so $R \in \Phi$. It follows that Φ satisfies the conditions of Proposition 1. Hence βX is Dugundji. ■

Definition 1. A space X has the property (T) iff for every family $\{x_\alpha : \alpha < \omega_1\}$ of points of X and every family $\{\gamma_\alpha : \alpha < \omega_1\}$ of open covers of X there exist $\alpha, \beta < \omega_1$ such that $\alpha \neq \beta$ and $St(x_\alpha, \gamma_\beta)$ meets $St(x_\beta, \gamma_\alpha)$ (where $St(x, \gamma) = \cup\{U \in \gamma : x \in U\}$).

This property was considered by M. Tkačenko [T1] who proved that every compact space has the property (T). Moreover, every Lindelöf Σ -space has the property (T) [U1], [U2]. A space X is a Σ -space [N] iff there are two covers F and C of X such that F is σ -locally finite, C consists of countably compact sets and for every $K \in C$ and every neighbourhood U of K there exists $E \in F$ such that $K \subseteq E \subseteq U$. Now we prove that countably compact spaces have the property (T).

Proposition 4. *If X is a Σ -space in which every closed discrete subset is countable, then X has the property (T).*

PROOF : Let $\{x_\alpha : \alpha < \omega_1\}$ be a family of points of X and $\{\gamma_\alpha : \alpha < \omega_1\}$ be a family of open covers of X . For $\alpha < \omega_1$ put $X_\alpha = \{x_\beta : \beta < \alpha\}$ and $\lambda_\alpha = \{X_\alpha \cap St(x_\alpha, \gamma_\beta) : \beta < \alpha\}$. For $E \subseteq X$ let $S(E)$ be the set of all $\alpha < \omega_1$ such that $x_\alpha \in E$ and the family $\lambda_\alpha|E = \{L \cap E : L \in \lambda_\alpha\}$ does not have the finite intersection property. We assert that for every $E \subseteq X$ the set $S(E)$ is non-stationary (i.e. some closed unbounded subset of ω_1 does not meet $S(E)$). For every finite set $A \subseteq \omega_1$ fix an open cover γ_A of X which refines γ_α for every $\alpha \in A$. If $\alpha \in S(E)$, there is a finite set $A = f(\alpha) \subseteq \alpha$ such that $X_\alpha \cap E \cap St(x_\alpha, \gamma_A) = \emptyset$. Suppose $S(E)$ is stationary. Fodor's Lemma [K] implies there is an uncountable subset $B \subseteq S(E)$ and a finite set $A \subseteq \omega_1$ such that $f(\alpha) = A$ for every $\alpha \in B$. If $\alpha, \beta \in B$ and $\beta < \alpha$, then $x_\beta \in X_\alpha \cap E \subseteq X \setminus St(x_\alpha, \gamma_A)$. It follows that the set $\{x_\alpha : \alpha \in B\}$ is closed discrete in X , since every element of γ_A contains at most one point of this set. This contradiction proves that $S(E)$ is non-stationary.

Let F and C be two covers of X witnessing the Σ -property. Then F is countable. Since the union of countably many non-stationary subsets of ω_1 is non-stationary [K], there exists $\alpha \in \omega_1 \setminus \{0\}$ such that $\alpha \notin S(E)$ for every $E \in F$. Let K be an element of C containing x_α . If $E \in F$ and $K \subseteq E$, then the family $\lambda_\alpha|E$ has finite intersection property. It follows that the family $\mu = \{K \cap \bar{L} : L \in \lambda_\alpha\}$ also has the finite intersection property. Since K is countably compact and μ is countable,

there exists a point $z \in \cap \mu$. Then $z \in \overline{X_\alpha}$ and $z \in \overline{St(x_\alpha, \gamma_\beta)}$ for every $\beta < \alpha$. Let U be an element of γ_α containing z . Since $z \in \overline{X_\alpha}$, U meets X_α , so there is $\beta < \alpha$ such that $x_\beta \in U$. Since $z \in \overline{St(x_\alpha, \gamma_\beta)}$ and $z \in U \subseteq St(x_\beta, \gamma_\alpha)$, the intersection $St(x_\alpha, \gamma_\beta) \cap St(x_\beta, \gamma_\alpha)$ is non-empty. The proof is complete. ■

Proposition 5. *If a Mal'tsev space X has the property (T) then X has the Suslin property.*

PROOF : Let $\{O_\alpha : \alpha < \omega_1\}$ be a family of non-empty open sets in X . We have to show that this family is not disjoint. For every $\alpha < \omega_1$ pick $x_\alpha \in O_\alpha$. Let $f : X^3 \rightarrow X$ be a Mal'tsev operation. Call a set $U \subseteq X$ α -small if $f(x_\alpha, y, z) \in O_\alpha$ and $f(y, z, x_\alpha) \in O_\alpha$ whenever $y, z \in U$. Since $f(x_\alpha, y, y) = f(y, y, x_\alpha) = x_\alpha \in O_\alpha$ for every $y \in X$, the collection γ_α of all α -small open sets covers X . The property (T) implies there are distinct $\alpha, \beta < \omega_1$ and a point $z \in X$ such that $z \in St(x_\alpha, \gamma_\beta) \cap St(x_\beta, \gamma_\alpha)$. Then $f(x_\alpha, z, x_\beta) \in O_\alpha \cap O_\beta \neq \emptyset$. ■

Propositions 4 and 5 imply

Proposition 6. *If X is a Mal'tsev Σ -space in which every closed discrete subset is countable, then X has the Suslin property.*

Let us say that a space X is G_δ -cellular if for every family λ of G_δ -sets in X there exists a countable subfamily $\mu \subseteq \lambda$ whose union is dense in the union of λ . It can be shown that under the assumptions of Proposition 6, X is G_δ -cellular. For Mal'tsev Lindelöf Σ -spaces this fact is proved in [U2]. In particular, σ -compact topological groups are G_δ -cellular.

Propositions 3 and 6 imply the main result of the paper:

Theorem 2. *If X is a countably compact Mal'tsev space, then βX is Dugundji.*

Now we introduce a new class of spaces which is contained in the class of Mal'tsev spaces. For $a \in X$ define subsets X_a and X^a of X^2 as follows: $X_a = \{(a, x) : x \in X\}$ and $X^a = \{(x, a) : x \in X\}$. Let $\Delta_X = \{(x, x) : x \in X\}$ be the diagonal of X^2 .

Definition 2. A space X has a rectifiable diagonal iff there is a homeomorphism $f : X^2 \rightarrow X^2$ of X^2 onto itself such that:

- 1) $f(X_x) = X_x$ for every $x \in X$;
- 2) $f(\Delta_X) = X^a$ for some $a \in X$.

Every topological group has a rectifiable diagonal, and every space with a rectifiable diagonal is homogeneous.

Proposition 7 (M.M.Čoban). *A space X has a rectifiable diagonal if and only if there exist mappings $p : X^2 \rightarrow X$ and $q : X^2 \rightarrow X$ satisfying the identities $p(x, q(x, y)) = q(x, p(x, y)) = y$ and $p(x, x) = p(y, y)$.*

PROOF : If p and q are as above, let $f(x, y) = (x, p(x, y))$. Then $f^{-1}(x, y) = (x, q(x, y))$, so f is a homeomorphism of X^2 onto itself. If $p(x, x) = a$ for all $x \in X$, then $f(\Delta_X) = X^a$. Conversely, if f is as in definition 2, the same formulas $f(x, y) = (x, p(x, y))$ and $f^{-1}(x, y) = (x, q(x, y))$ define p and q . ■

Corollary. *Every space with a rectifiable diagonal is Mal'tsev.*

PROOF : If p and q are as in Proposition 7, then $(x, y, z) \rightarrow q(x, p(y, z))$ is a Mal'tsev operation. ■

Theorem 3. *Let X be a pseudocompact space with a rectifiable diagonal. If X has the Suslin property, then βX is Dugundji.*

PROOF : Let p and q be as in Proposition 7. We say that an equivalence relation R on X is a congruence if the following conditions holds: if x_i is R -equivalent to $y_i, i = 1, 2$, then $p(x_1, x_2)$ is R -equivalent to $p(y_1, y_2)$ and $q(x_1, x_2)$ is R -equivalent to $q(y_1, y_2)$. Let $\Phi = \{R : R \text{ is a congruence and } X/R \text{ is submetrizable}\}$. Then Φ satisfies the conditions of Proposition 1. If R is a congruence, then R is also an m -congruence, where $m(x, y, z) = q(x, p(y, z))$ is a Mal'tsev operation, so the Mal'tsev theorem shows that R is open. It remains to show that the quotient mappings $X \rightarrow X/R, R \in \Phi$, determine the topology of X . The argument is essentially the same as in the proof of Proposition 3, only Reznichenko's theorem R2 should be used instead of R1. ■

Questions. Is it true that every pseudocompact space with a rectifiable diagonal has the Suslin property? Is it true that every space with a rectifiable diagonal is a retract of a topological group?

O.V.Sipačeva has recently proved that every compact Mal'tsev space is a retract of a topological group. It can be shown that the same is true for countably compact Mal'tsev spaces.

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Inst.of Physics of Earth, Bolshaja Gruzinskaja 10, Moscow 242, USSR

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