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An application of the method wherein the *phi*- and *chi*- methods are combined for the determination of the grating constant. [II.]

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An application of the method wherein φ- and $\varkappa$-methods are combined for the determination of the grating constant. Part II.

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The method wherein the φ- and the $\varkappa$-methods are combined for the determination of the grating constants of crystals is applied for the precise adjustment of the Bragg's reflecting plane of the crystal in the axis of the spectrograph. The values of the fictive grating constants d_1, d_2 and the real grating constant d_∞ obtained in this case from the glancing angle φ agree with those obtained from the differential glancing angle $\varkappa$. The precision of the obtained values is discussed and they are used for the further application of the differential glancing angle $\varkappa$.

A method is given for the measurement of the wave-lengths of X-radiation, with the help of the differential glancing angle $\varkappa$ of the same wave-length in two different orders and two fictive grating constants, by means of the equation

$$m\lambda = \frac{2d_m \sin \varkappa_{m,n}}{\sqrt{\left(\frac{n}{m}\right)^2 \left(\frac{d_m}{d_n}\right)^2 - 2\frac{n}{m}\frac{d_m}{d_n}\cos\varkappa + 1}}, \quad \text{for } n > m.$$

This method is verified with a zinc sulphide (ZnS) crystal and the precise values of λ for the Kα-lines of gallium are given. These measured values are compared with those obtained by Uhler and Cooksey with the help of Moseley's law as modified by Dolejšek and Pestrecov.

In the preceding part published in this Journal[1]) I have announced my results which give the values of the individual fictive grating constants of the zinc sulphide (ZnS) crystals. The results verify and show the special advantages of the method wherein the φ- and the $\varkappa$-methods are combined for the precise determination of the grating constant of crystals. It has been shown that, by the application of this method it is possible to determine precisely the constant of crystal grating even with imperfect crystals, in as much as the said method eliminates the errors due to the defects of the crystal such as displacement. I have undertaken to determine precisely the individual fictive grating constants of the zinc

[1]) Swami Jnanananda, Časopis **65** (1936), 155.

First series with a displacement of 0,0015 mm.
Crystal No. II.

Table 1a. Cu, φ_1

Plate No	Δ mm	Δ'	α	φ_t	$\Delta\varphi$ for Δt° C	φ 18° C	mean value of φ 18° C
438	0,182	2′ 29,4″	94° 56′ 53,5″	23° 44′ 50,7″	2,83″	23° 44′ 53,5″	23° 44′ 50,2″
439	0,179	2′ 27,3″	94° 56′ 55,5″	23° 44′ 50,7″	3,15″	23° 44′ 53,9″	
440	0,238	3′ 15,4″	95° 2′ 16,5″	23° 44′ 45,0″	3,44″	23° 44′ 48,4″	
441	0,244	3′ 20,3″	95° 2′ 26,5″	23° 44′ 46,6″	3,28″	23° 44′ 50,0″	
442	0,241	3′ 17,8″	95° 2′ 26,5″	23° 44′ 47,2″	2,2″	23° 44′ 49,4″	
443	0,262	3′ 35,0″	95° 2′ 27,0″	23° 44′ 43,0″	2,1″	23° 44′ 45,1″	
445	0,168	2′ 17,9″	94° 56′ 59,0″	23° 44′ 49,0″	2,07″	23° 44′ 51,1″	
446	0,172	2′ 21,2″	94° 56′ 49,0″	23° 44′ 47,6″	2,51″	23° 44′ 50,1″	

Table 1b. Mo, φ_1

Plate No	Δ mm	Δ'	α	φ_t	$\Delta\varphi$ for Δt° C	φ 18° C	mean value of φ 18° C
464	0,197	2′ 41,7″	42° 41′ 37,5″	10° 41′ 4,8″	1,2″	10° 41′ 6,0″	10° 41′ 6,4″
465	0,207	2′ 49,9″	42° 41′ 35,5″	10° 41′ 6,35″	0,9″	10° 41′ 7,3″	
466	0,119	1′ 37,7″	42° 45′ 39,5″	10° 41′ 5,45″	1,0″	10° 41′ 6,5″	
467	0,117	1′ 36,0″	42° 46′ 0,0″	10° 41′ 6,00″	0,9″	10° 41′ 6,9″	
468	0,200	2′ 44,2″	42° 41′ 45,5″	10° 41′ 7,4″	1,1″	10° 41′ 8,5″	
469	0,199	2′ 43,7″	42° 41′ 44,5″	10° 41′ 7,05″	1,2″	10° 41′ 8,3″	
470	0,222	3′ 02,2″	42° 47′ 2,5″	10° 41′ 0,0″	1,2″	10° 41′ 1,2°	
471	0,215	2′ 56,5″	42° 47′ 10,0″	10° 41′ 3,4″	1,1″	10° 41′ 4,5″	

Table 1c.

Displacement of the crystal Δ	=	0,0015 mm
φ_1Cu Kα_1	=	23° 44′ 50,18″
φ_1Mo Kα_1	=	10° 41′ 6,4″
φ_1Cu — φ_1Mo = $\varkappa_1$	=	13° 3′ 43,78″

sulphide crystal, because I use this crystal as a diffraction grating for the work in which I will show that the differential glancing angle $\varkappa_{m,n}$ can be used not only for the determination of the grating constant of crystals, but also for the precise determination of the wave-lengths of X-radiation. Before I proceed to show the possibility of the said precise method of the absolute measurement of the wave-lengths, I will discuss the results of the previous parts with this in view, in the light of the results of my new measurements of the individual fictive grating constants of zinc sulphide crystals. In these new series of measurements, I have, calculating the amount of displacement with the aid of our method, readjusted the grating crystal in such a position that its Bragg's reflecting plane has

Table 2.

Representation of the first series of measurements and their results.
($\varDelta = 0{,}0015$ mm).

$\frac{1}{2}\delta_1 = 1{,}2''$

φ_1Cu meas. 23° 44′ 50,2″	$d_1 =$ 1908,81 XU	φ_1Cu $\sim$ to 1908,867 XU 23° 44′ 49,0″	
mean $\varkappa_1$ 13° 3′ 43,78″	$d_1 =$ 1908,867 XU	corr. 13° 3′ 43,86″	corr. $d_1 =$ **1908,866 XU**
φ_1Mo meas. 10° 41′ 6,4″	$d_1 =$ 1908,82 XU	φ_1Mo $\sim$ to 1908,867 XU 10° 41′ 5,3″	

$\frac{1}{2}\delta_1 = 1{,}1''$

	$\varDelta$ mm	mean $\varDelta$ mm	$\frac{1}{2}\delta$	$\varepsilon\delta\varkappa = -(\frac{1}{2}\delta\text{Cu} - \frac{1}{2}\delta\text{Mo})$
Cu	0,00155	0,0015	1,13″	+ 0,08″
Mo	0,00145		1,21″	

coincided with the axis of the spectrograph as precisely as possible. Such a precision in the adjustment of an imperfect crystal cannot be obtained with the usual optical methods. In the group of measurements of the preceding part I have used different positions of the crystal, which have been away from the axis of the spectrograph from 0,015 mm to 0,05 mm. The corrections of the grating constant in these cases have been from 0,02 to 0,1 X. U. But the differences in the values derived from the glancing angle φ varied from 0,2 to 1 or more X. U. In this work I have adjusted the crystal with the mentioned process in such a way that the displacement of the reflecting plane from the axis is only 0,0015 mm. Such an adjustment can be obtained with the optical methods only in the case of perfect crystals. From the fact which can be seen in the preceding part that the displacement of 0,02 mm has an influence of approximately 0,02 X. U. in the value of the grating constant derived from the differential glancing angle $\varkappa$, we note that the displacement of 0,0015 mm can have only an influence on the third decimal place of a X. U. Such a precision is however under the limits of the precision I could obtain with the spectrograph. With this adjustment of the crystal the application of the method wherein the φ- and the $\varkappa$-methods are combined has no practical value, and the differential glancing angle $\varkappa$ in this case and in the limits of the precision gives the correct value of the grating constant. But from what I have mentioned it becomes obvious that if our spectrograph could have a scale having a precision of about 0,5″ (which we could not obtain due to want of financial means) we could with the application of this method and with this adjustment of the crystal in a way guarantee the accuracy of our measurements to a thousandth part of a X. U. (0,001 X. U.). The series of measurements and the results made with the said precise adjustment of the crystal are as in the previous part given in the Tables No. 1 (a, b, c), 2, 3 (a, b, c) and 4. From the results given in the Table No. 2 it can be seen that the values for d_1 obtained from the directly measured glancing angles φ of copper and molybdenum do not differ in the obtained limits of observation from the value d_1 derived from the differential glancing angle $\varkappa$, the difference between the values being 0,06 X. U. I have of course given the value of the grating constant to the third decimal of a X. U. to show that if the said precision is at our hand the derived value of d_1 from the uncorrected diffential glancing angle $\varkappa$ would be, in this case, with the kind of adjustment that I have, precise enough to the limit of 0,001 X. U.

It will now be shown how the defect only in a certain part of the reflecting plane of the crystal could enter into the value of the grating constant. Fig. 1 shows the spectrogram having two Kα doublets, one of copper and the other of molybdenum. The spectrogram is taken for the measurement of the differential glancing angle $\varkappa$ between Cu Kα_1 and Mo Kα_1 with the help of Siegbahn's precise method. The difference Δ between the Kα_1 line of Cu and that of Mo is so chosen that the Kα_1 of molybdenum comes between the α_1 and α_2 of copper so that the two α_1 lines are near to each other. In this spectrogram the lines are not quite perfect and they show some irregulatiries in some places, which would influence the precision of the measurements of Δ. The said influence of these irregularities can be eliminated if we choose such places where the irregularities are uniform in both the lines for the measurements. In the said present series of measurements, I have tried as much as possible to obtain the reflection from the same position of the reflecting surface to avoid the errors which do not enter uniformly into either one of the expositions of each individual spectrogram.

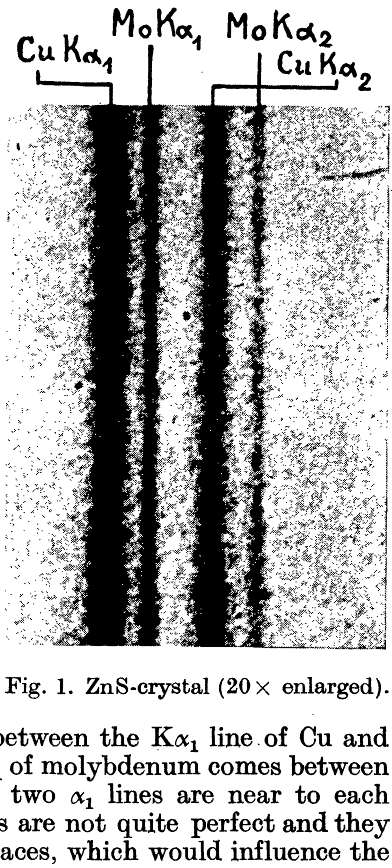

Fig. 1. ZnS-crystal (20× enlarged).

The dispersion obtained with the zinc sulphide crystal has been at the said measurements very great as it can be seen from the spectrogram (Fig. 2a). This figure is an enlargement of the two $K\alpha$ doublets of Cu and Mo. The spectrogram of which this figure is an enlargement by 25 times is taken with zinc sulphide crystal, the distance slit-plate being 50 cm. If we compare the dispersion of our spectrogram (Fig. 2a) with the dispersion obtained by Valasek[2]) at

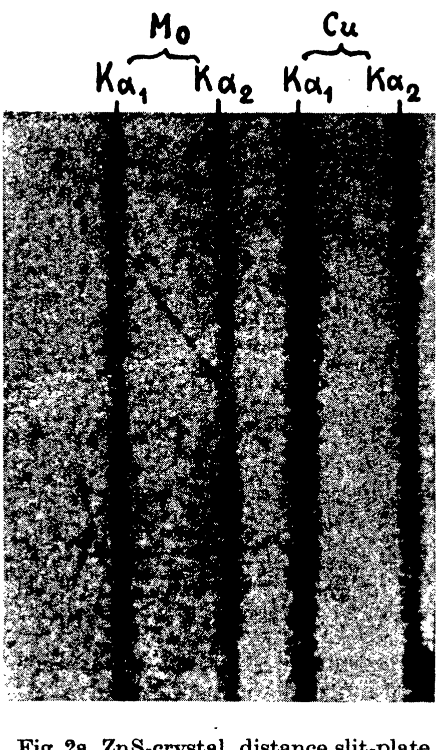

Fig. 2a. ZnS-crystal, distance slit-plate 50 cm (25 × enlarged).

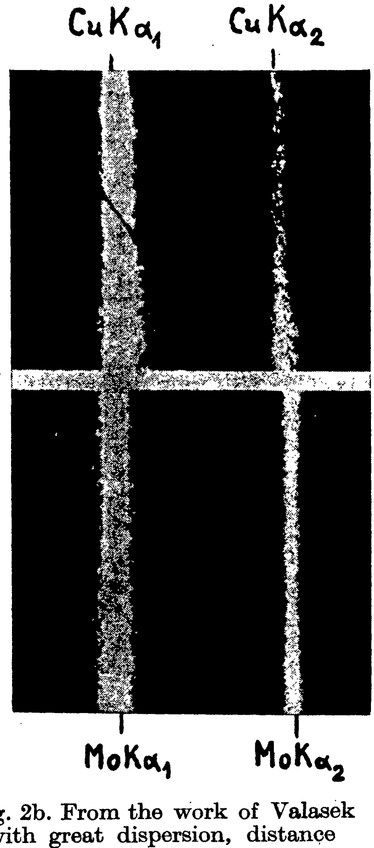

Fig. 2b. From the work of Valasek with great dispersion, distance slitplate 295 cm.

a great distance (slit-plate) of 3 metres (Fig. 2b), we see that for the copper lines the dispersion in our case is only a little less than that of Valasek and the dispersion of molybdenum lines which are sharper than the copper lines is approximately the same as that obtained by Valasek. I will at a later stage show from the spectro-

[2]) J. Valasek, Phys. Rev. 35 (1930), 291.

gram that the zinc sulphide crystal as a diffraction grating is more advantageous for the shorter wave-lengths than for longer ones. Here I should like to point out that as a consequence of such a large dispersion with the zinc sulphide crystal we could obtain the highest possible precision in this direction.

From the above discussion of the possible errors due to the irregularities in the lines and from a comparison of the values of the fictive grating constants d_1 given in the preceding part as well as the results which will be given at a later stage, it becomes evident that the precision of our measurements is limited by the systematic and accidental errors of the scale. If we note the comparison of the values of the fictive grating constant d_1 of the two said zinc sulphide crystals given in the preceding part and compare it with the results given in the Tables 1 and 2, we must say that the influence of the impurities on the values of the grating constants of both the zinc sulphide crystals is even less than what I have mentioned already (0,2—0,3 X. U.). Now in these series of measurements I have obtained for the second crystal $d_1 = 1908{,}87$ X. U., while the mean value of the grating constant d_1 of the first crystal is 1908,93 X. U. These values agree in the limits of the obtained precision.

Before I proceed to give the value of the fictive grating constant d_2 I should mention that I am not in a position to study and discuss the influence of the anomalous dispersion on the values of the grating constant derived from $\varkappa$-method due to the inadequacy of precision of the scale of our spectrograph. The anomalous dispersion of the zinc sulphide crystal might have an influence on our results as zinc K-absorption edge from zinc sulphide crystal lies between the two wave-lengths Cu $K\alpha_1$ and Mo $K\alpha_1$ which I have used for the measurement of the grating constant of the zinc sulphide crystal. It can be concluded from the analogy of the other crystals that the said influence of the anomalous dispersion can cause only a small change in the value of the differential glancing angle $\varkappa$, which we cannot verify with our obtained precision.

In connection with the anomalous dispersion, I will give here the spectrogram (Fig. 3) of the zinc K-absorption edge caused by the zinc sulphide crystal. The absorption edge just cuts through the $L\beta_1$ line of tungsten so that the line is split into two and the absorption edge of zinc appears as a white absorption line. The value of the wave-length of zinc K-absorption edge calculated from the L-tungsten lines amounts to 1280,5 X. U. when the edge of the white line is taken as the mentioned absorption edge. This value agrees with the value of the K-absorption edge of zinc free element $\lambda = 1280{,}5$ X. U. obtained by Dolejšek and Pestrecov,[3]) Kievit

[3]) V. Dolejšek-K. Pestrecov, C. R. 188 (1929), 164.

and Lindsay,[4]) though the absorption edge is obtained from zinc sulphide. If we measure from the middle of the white line, we obtain the value 1280,4 X. U. From the comparison of these values of λ we can say that the difference between the values of the zinc and zinc sulphide K-absorption edges is, within the limits of observation, 0,1 X. U.

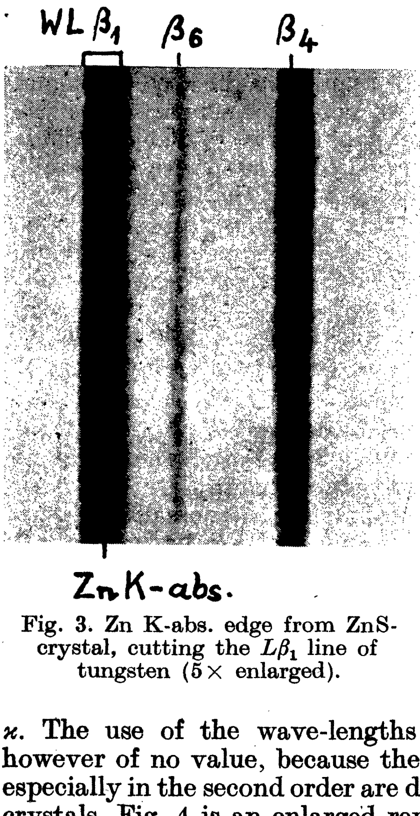

Fig. 3. Zn K-abs. edge from ZnS-crystal, cutting the $L\beta_1$ line of tungsten (5× enlarged).

The value of the fictive grating constant obtained from the new series of measurements which I have given in the Tables 3 (a, b, c) and 4 agrees with the value obtained from the series of measurements of the preceding part. This agreement in the two values of d_2 which is better than the agreement of the values of d_1 can be explained by the fact that the same errors of the scale in the case of greater glancing angles in the second order influence in a lesser degree the value of d than in the case of the smaller glancing angles in the first order. The adjustment of the crystal in this case has been the same as in the case of the measurements in the first order and with this kind of adjustment there is no practical value in the correction of the differential angle $\varkappa$. The use of the wave-lengths longer than those of copper is however of no value, because the lines of the longer wave-lengths especially in the second order are diffused in the case of zinc sulphide crystals. Fig. 4 is an enlarged reproduction of the spectrogram of two Mo Kα doublets in the second order and Fig. 5 is that of two Cu Kα doublets in the second order, both taken with the method of Siegbahn for the measurement of the glancing angle φ_2. If we compare these two spectrograms (Fig. 4 and 5), we can see, that the zinc sulphide crystal is more advantageous for shorter wave-lengths than for longer ones.

I have mentioned in the preceding part that the value δ/λ^2 calculated from the fictive values of d_1 and d_2 is greater than the

[4]) Kievit and Lindsay, Phys. Rev. 36 (1930), 648.

Second series wirth a displacement of 0 mm.

Table 3a. Cu, φ_2

Plate No	Δ mm	Δ'	α	φ_t	$\Delta\varphi$ for Δt° C	φ 18° C	mean value of φ 18° C
444	0,249	3′ 24,16″	214° 36′ 56,0″	53° 38′ 13,0″	9,3″	53° 38′ 22,3″	53° 38′ 19,04″
447	0,500	6′ 50,4″	214° 26′ 4,0″	53° 38′ 13,6″	7,1″	53° 38′ 20,7″	
448	0,500	6′ 50,4″	214° 26′ 4,0″	53° 38′ 13,6″	7,0″	53° 38′ 20,6″	
449	0,396	5′ 25,34″	214° 36′ 55,5″	53° 37′ 52,5″	6,9″	53° 37′ 59,4″	
450	0,555	7′ 31,32″	214° 26′ 7,0″	53° 38′ 24,6″	7,7″	53° 38′ 32,3″	
451	0,597	8′ 09,76″	214° 25′ 50,0″	53° 38′ 27,4″	6,6″	53° 38′ 34,0″	
452	0,345	4′ 43,04″	214° 36′ 38,0″	53° 37′ 58,7″	8,3″	53° 38′ 7,0″	
453	0,232	3′ 10,26″	214° 36′ 37,5″	53° 38′ 21,8″	8,4″	53° 38′ 30,2″	
454	0,522	7′ 08,76″	214° 26′ 41,0″	53° 38′ 27,4″	7,7″	53° 38′ 35,1″	
455	0,450	6′ 09,4″	214° 36′ 56,5″	53° 38′ 41,8″	7,0″	53° 37′ 48,8″	

Table 3b. Mo, φ_2

Plate No	Δ mm	Δ'	α	φ_t	$\Delta\varphi$ for Δt° C	φ 18° C	mean value of φ 18° C
456	0,252	3′ 27,3″	86° 59′ 5,5″	21° 45′ 38,2″	2,6″	21° 45′ 40,8″	21° 45′ 46,04″
457	0,249	3′ 24,4″	86° 59′ 15,0″	21° 45′ 39,9″	2,7″	21° 45′ 42,6″	
458	0,181	2′ 28,6″	87° 5′ 19,0″	21° 45′ 42,6″	2,6″	21° 45′ 45,2″	
459	0,185	2′ 31,8″	87° 5′ 23,0″	21° 45′ 42,8″	2,1″	21° 45′ 44,9″	
460	0,301	4′ 07,1″	86° 59′ 14,5″	21° 45′ 50,4″	2,5″	21° 45′ 52,9″	
461	0,272	3′ 43,3″	86° 59′ 14,5″	21° 45′ 44,45″	1,9″	21° 45′ 46,4″	
462	0,183	2′ 30,6″	87° 5′ 16,5″	21° 45′ 41,475″	2,5″	21° 45′ 44,0″	
463	0,202	2′ 45,8″	87° 5′ 59,0″	21° 45′ 48,8″	2,7″	21° 45′ 51,5″	

Table 3c.

Displacement of the crystal Δ .... = 0 mm (not estimable)
φ_2Cu Kα_1 .... = 53° 38′ 19,04″
φ_2Mo Kα_1 .... = 21° 45′ 46,04″
φ_2Cu — φ_2Mo = $\varkappa_2$ .... = 31° 52′ 33″

Representation of the second seris of measurements and their results (Δ cannot be estimated).

Table 4.

φ_2Cu meas. 53° 38′ 19,0″	$d_2 =$ 1909,11 XU
mean $\varkappa_2$ 31° 52′ 33″	$d_2 =$ 1909,11 XU
φ_2Mo meas. 21° 45′ 46,0″	$d_2 =$ 1909,11 XU

value derived theoretically from the physical data. From these new measurements we obtain from the values of d_1 and d_2 the value of $\delta/\lambda^2 = 11{,}5 \,.\, 10^{-12}$. It is greater than the value of δ/λ^2 derived from the density ($5{,}2 - 5{,}7 \,.\, 10^{-12}$) or that derived from the number of electrons in a cubic centimeter of the crystal ($5{,}3 \,.\, 10^{-12}$). The value of δ/λ^2 obtained from my measurements corresponds to a difference $d_2 - d_1 = 0{,}24$ X. U. This difference is very small and any small change in the values of d_1 and d_2 due to the errors of measurements causes proportionally very great change in the value of δ/λ^2. The errors of the scale manifest themselves in another magnitude in the value of δ/λ^2 if we derive it from the fictive grating constants $d_{m,n}$ with the help of the already mentioned Kunzl - Köppel's equation. For molybdenum we obtain from our measurements $d_{1,2} = 1909{,}32$ X. U. while for copper we have $d_{1,2} = 1909{,}23$ X. U. The difference $(d_{1,2} - d_1)$ between the fictive grating constants $d_{1,2}$ and d_1 is greater than the difference $(d_2 - d_1)$ between the fictive grating constants d_2 and d_1 and depends upon the wave-lengths used. The value of δ/λ^2 can be derived from d_1 and $d_{1,2}$ by means of the Kunzl-Köppel's equation[5]):

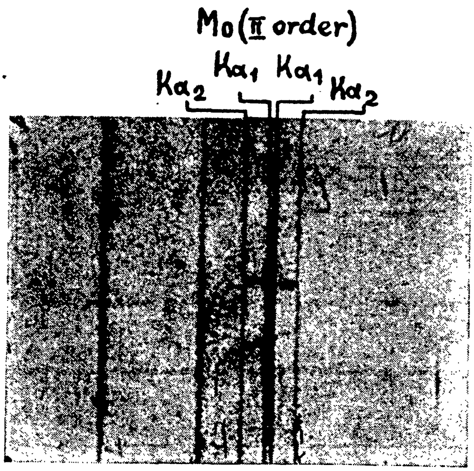

Fig. 4. ZnS-crystal (6× enlarged).

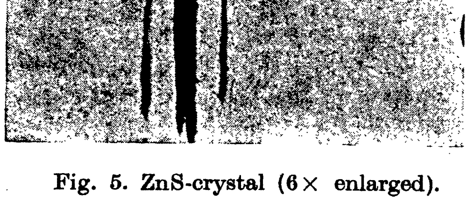

Fig. 5. ZnS-crystal (6× enlarged).

$$\frac{\delta}{\lambda^2} = \frac{d_{1,2} - d_1}{3d^3} \cdot \frac{5 - 4\cos \varkappa_{1,2}}{4 - 2\cos \varkappa_{1,2}},$$

[5]) V. Kunzl-J. Köppel, C. R. **196** (1933), 787; **196** (1933), 940; Časopis **63** (1934), 109; Journ. de Phys. **5** (1934), 145.

from which we obtain for copper $\delta/\lambda^2 = 11{,}7 \,.\, 10^{-12}$ and for molybdenum $\delta/\lambda^2 = 14{,}6 \,.\, 10^{-12}$.

The discussion of the precision of the above mentioned values of δ/λ^2 can be made after having derived the real grating constant with the help of the obtained values of δ/λ^2. From d_1 and with $\delta/\lambda^2 = 11{,}7 \,.\, 10^{-12}$ we have $d_\infty = 1909{,}20$ X. U.; putting $\delta/\lambda^2 = 14{,}6 \,.\, 10^{-12}$ we obtain $d_\infty = 1909{,}28$ X. U. The difference between these two values of d_∞ is 0,08 X. U. This difference can be ascribed not only to the accidental errors but also to the systematic errors of the scale. From these various considerations we come to the conclusion that the systematic errors of the scale have greatly influenced the value of δ/λ^2 obtained from the data of molybdenum and that they have similary influenced the value of δ/λ^2 derived from my other measurements and that therefore the value of δ/λ^2 derived from my measurements differs from the theoretical value. I have tried to measure the fictive grating constant $d_{1,2}$ of the mentioned ZnS crystal with NiKα lines, which after Kunzl-Köppel's equation is expected to agree with the real grating constant, and I have obtained the value 1909,20 X. U. which agrees well with the above mentioned value of the real grating constant d_∞. I cannot however draw any further conclusions from this agreement as the lines of nickel in the second order are very much diffused.

Now when we compare the different measurements of each individual series we see that the error of the angle measured in the same place of the scale is approximately 0,5″. But the systematic error of the scale amounts to some seconds and as an example, I mention that the values of the differential glancing angles measured on the left and on the right sides of the slit have differed by an amount of nearly 4″. Though I tried to avoid measuring with such parts of the scale which would seem to be contributing great errors, the systematic errors of the scale which have entered in my measurements are such that they alone have caused a difference between the theoretical and the measured values δ/λ^2. For a further application of the differential glancing angle $\varkappa$ I have used the measured values of the fictive grating constants. To diminish the influence of the said systematic errors of the scale I have measured the differential glancing angle for the proposed application almost with the same part of the scale with which the angles for the determination of the fictive grating constants in the last group have been measured.

Here I give in the Table No. 5 the values of the fictive grating constants of the zinc sulphide crystal which I have obtained and whose precision can be said to be amounting to 0,04 X. U.

Table 5.

ZnS-crystal.

$$\left.\begin{array}{l} d_1 = 1908{,}87 \text{ X. U.} \\ d_2 = 1909{,}11 \text{ X. U.} \end{array}\right\} d_\infty = 1909{,}20 \text{ X. U.}$$

$$\begin{array}{llll} \text{Ni} & d_{1,2} = d_\infty = 1909{,}20 \text{ X. U.} & & \\ \text{Cu} & d_{1,2} = 1909{,}23 \text{ X.U.} & d_\infty = 1909{,}2 & \text{X. U.} \\ \text{Mo} & d_{1,2} = 1909{,}32 \text{ X.U.} & d_\infty = 1909{,}28 & \text{X. U.} \end{array}$$

In the preceding parts, I have given a method wherein the φ- and the $\varkappa$-methods are combined for the determination of the grating constant of crystals. In this method it could be possible only to use two wave-lengths in the same order as was done by Valouch[6]) and Bouchal-Dolejšek.[7]) In the said method wherein the φ- and the $\varkappa$-methods are combined it is not possible to make use of the same wave-length in two different orders as was done by Kunzl-Köppel[8]) in the manner of Pavelka.[9])

Just as the differential glancing angle $\varkappa$ is used for the precise determination of the constant of crystal grating with special advantages, it can also be used with similar advantages for the determination of the wave-lengths of X-radiation. For the absolute measurement of the said wave-lengths, it is not possible to measure the differential glancing angle $\varkappa$ from two wave-lengths in the same order. This is however possible only for relative measurements in a manner analogous to the method of Lang.[10]) He has precisely measured in this way the chief lines of the K-series of some of the elements relatively to the $K\alpha_1$ of copper. Schrör[11]) has measured with the same method some of the lines of L-series with reference to Cu $K\alpha_1$ which is used as normal.

For the absolute measurement of the wave-lengths of X-radiation it is only possible to measure the differential glancing angle $\varkappa_{m,n}$ of the same wave-length in two different orders m and n ($n < m$). Such a determination of the wave-lengths from $\varkappa_{m,n}$ is, in a way, an analogue of the Kunzl-Köppel's method for the determination of the grating constant of crystals. Kunzl and Köppel in working out their method have shown that there exists a difference between the fictive grating constants derived from the angle φ and those derived from the angle $\varkappa_{m,n}$. It was shown that

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[6]) M. A. Valouch, Bull. de l'Acad. de Sc. de Bohême **28** (1927), 31.

[7]) F. Bouchal-V. Dolejšek, C. R. **199** (1934), 1054; Časopis **65** (1935), 34.

[8]) V. Kunzl-J. Köppel, C. R. **196** (1933), 787; **196** (1933) 940; Časopis **63** (1934), 109; Journ. de Phys. **5** (1934), 145.

[9]) A. Pavelka, Bull. de l'Acad. de Sc. de Bohême **28** (1927), 442.

[10]) K. Lang, Ann. d. Phys. **75** (1924), 489.

[11]) J. Schrör, Ann. d. Phys. **80** (1926), 297.

the fictive grating constants derived from the differential glancing angle $\varkappa$ in two different orders of the same line, where

$$\varkappa = \varphi^*_n - \varphi^*_m \text{ for } n > m, \tag{A'}$$

are not functions of each individual index seperately as it is in the case of

$$m\lambda = 2d_m \sin \varphi_m, \tag{1}$$
$$n\lambda = 2d_n \sin \varphi_n, \tag{2}$$

where

$$\varkappa = \varphi_n - \varphi_m, \tag{A}$$

φ_m and φ_n being directly measured glancing angles, but are the functions of both indices, m and n simultaneously. In this case, therefore, the Bragg's law must be expressed thus:

$$m\lambda = 2d_{m,n} \sin \varphi^*_m, \tag{1'}$$
$$n\lambda = 2d_{m,n} \sin \varphi^*_n. \tag{2'}$$

From the original equation of Bragg, Pavelka[12]) has deduced an equation showing the relation between φ and $\varkappa$ which runs thus:

$$\sin \varphi_m = \frac{m\lambda}{2d} = \frac{\sin \varkappa}{\sqrt{\left(\frac{n}{m}\right)^2 - 2\frac{n}{m}\cos \varkappa + 1}} \tag{3}$$

$$d = \frac{m\lambda}{2 \sin \varphi_m}.$$

This equation, as the original equation of Bragg, does not hold precisely true. Kunzl and Köppel[13]) however having grasped the difference between φ^* and φ, which I have already pointed out, have deduced the following equation which gives the relation between φ^* and $\varkappa$ and from which precise values of the fictive grating constants can be derived:

$$\sin \varphi^*_m = \frac{m\lambda}{2d_{m,n}} = \frac{\sin \varkappa}{\sqrt{\left(\frac{n}{m}\right)^2 - 2\frac{n}{m}\cos \varkappa + 1}} \tag{3'}$$

$$d_{m,n} = \frac{m\lambda}{2 \sin \varphi^*_m}.$$

From the equations (1'), (3') and the Kunzl-Köppel's equation expressing the direct connection between the fictive grating constants $d_{m,n}$, d_m and d_n:

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[12]) A. Pavelka, loc. cit.
[13]) V. Kunzl-J. Köppel, loc. cit.

$$d_{m,n} = d_m \sqrt{\frac{\left(\frac{n}{m}\right)^2 - 2\frac{n}{m}\cos\varkappa + 1}{\left(\frac{n}{m}\right)^2\left(\frac{d_m}{d_n}\right)^2 - 2\frac{n}{m}\frac{d_m}{d_n}\cos\varkappa + 1}},$$

I have derived an equation for the precise determination of the wave-lengths, which needs the measurement of the differential glancing angle $\varkappa$, of the same wave line in two different orders m and n and the fictive grating constants d_m and d_n. It runs:

$$m\lambda = \frac{2d_m \sin\varkappa_{m,n}}{\sqrt{\left(\frac{n}{m}\right)^2\left(\frac{d_m}{d_n}\right)^2 - 2\frac{n}{m}\frac{d_m}{d_n}\cos\varkappa + 1}}.$$

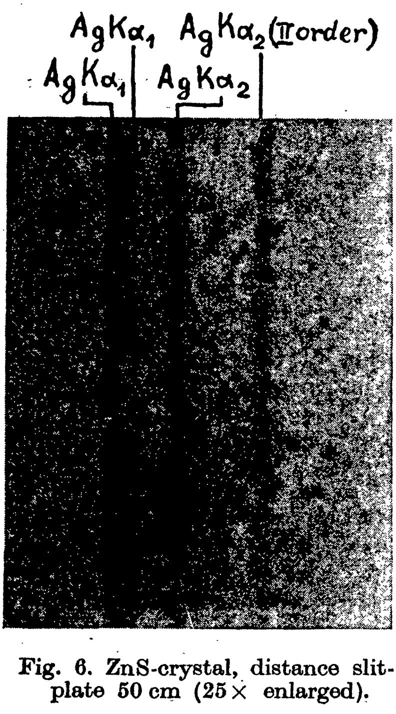

Fig. 6. ZnS-crystal, distance slit-plate 50 cm (25 × enlarged).

Now in order to verify this equation I have measured the wave-length λ of $K\alpha_1$ of Ag which has been measured precisely by different authors. I have also measured the $K\alpha_1$ line of Ga which has been untill now measured only by one author and the measured data are not sufficient. I give here a series of measurements of the differential glancing angle $\varkappa$ of Ag $K\alpha_1$ in the Table No. 6. The value of the wave-length of Ag $K\alpha_1$ 558,11 X. U. is derived from the use of the measured angle $\varkappa$ and the mentioned values of the fictive grating constants d_m and d_n of the zinc sulphide crystal. The silver lines obtained with zinc sulphide have been very sharp and we could with the obtained dispersion and with our method guarantee the precision of our values of λ to the third decimal place of a X. U. From the spectrogram of the Ag $K\alpha_1$ lines in the first and in the second orders for $\varkappa_{1,2}$ measurements given in the Fig. No. 6 it can be seen, that the use of zinc sulphide crystal as a diffraction grating for this region of shorter wave-lengths is very advantageous. But as the differential glancing angle $\varkappa$ is small and as the mentioned errors of the scale influence the value of the measured glancing angle, which is comparatively small, we can only say that the said value of λ of Ag $K\alpha_1$ agrees within the limits of the obtained precision and observation with the values determined by different authors mentioned in the Table No. 6.

Table 6.

A series of measurements of the differential glancing angle $\varkappa_{1,2}$ of silver (Ag $K\alpha_1$).

Plate No	Δ mm	Δ'	α	$\varkappa_{1,2;\,t}$	$\Delta\varkappa$ for $\Delta t°$ C	$\varkappa$ 18° C	mean value of $\varkappa$ 18° C
473	0,138	1′ 53,3″	17° 12′ 50,0″	8° 35′ 28,3″	0,99″	8° 35′ 29,3″	8° 35′ 31,0″
480	0,167	2′ 17,1″	17° 8′ 43,0″	8° 35′ 30,1″	0,83″	8° 35′ 30,9″	
482	0,197	2′ 41,7″	17° 8′ 20,0″	8° 35′ 30,9″	1,17″	8° 35′ 32,1″	
484	0,129	1′ 45,6″	17° 12′ 48,0″	8° 35′ 31,2″	0,61″	8° 35′ 31,8″	

From Ag $K\alpha_1$ differential glancing angle $\varkappa_{1,2} = 8\ 35'\ 31{,}0''$; and ZnS crystal $d_1 = 1908{,}87$ XU, $d_2 = 1909{,}11$ XU .... Ag $K\alpha_1$ λ = 558,11 XU

Lang (rel. to Cu $K\alpha_1$) ... 558,18 XU
Leide .... 558,21 XU
Edlen .... 558,28 XU.

Another verification of this method I give through the measurement of the wave-length of gallium $K\alpha_1$ whose glancing angle φ and the differential glancing angle $\varkappa_{m,n}$ are comparatively greater than those of Ag and the errors of the scale have smaller influence on the wave-length λ of $K\alpha_1$ of gallium than in the case of silver. The obtained gallium lines have been sharp enough for the precise measurements. These measurements and the results are given in the Table No. 7a and 7b. In the Table No. 7a we have a series of measurements of the glancing angle φ_1 and in the Table No. 7b a series of measurements of the differential glancing angle $\varkappa_{m,n}$. From the angle φ I have obtained a value of λ for Ga $K\alpha_1$ = = 1337,19 X. U. and from the differential glancing angle $\varkappa_{m,n}$ a value of $\lambda = 1337{,}35$ X. U. In these measurements with the said adjustment of the crystal the influence of the errors of the scale is greater than the influence of the error due to the displacement of the reflecting plane of the grating crystal. But in the measurements of the differential glancing angle $\varkappa_{m,n}$, though the influence of the errors of the scale is similar to that in the measurement of the glancing angle φ, the errors of the adjustment of the crystal are very much diminished in the value of the wave-length derived from the differential glancing angle $\varkappa_{m,n}$. Therefore the value

A series of measurements of the glancing angle φ_1 of Ga $K\alpha_1$.

Table 7a.

Plate No	Δ mm	Δ'	α	φ_t	$\Delta\varphi$ for $\Delta t°$ C	φ 18° C	mean value of φ 18° C
500	0,337	4′ 36,6″	82° 4′ 56,5″	20° 30′ 05,0″	2,60″	20° 30′ 07,6″	20° 30′ 11,0″
501	0,311	4′ 15,3″	82° 4′ 59,0″	20° 30′ 10,9″	2,38″	20° 30′ 13,3″	
502	0,241	3′ 17,8″	81° 57′ 20,0″	20° 30′ 09,5″	2,13″	20° 30′ 11,6″	
503	0,246	3′ 21,9″	81° 57′ 22,0″	20° 30′ 11,0″	2,28″	20° 30′ 13,3″	
504	0,254	3′ 59,7″	82° 4′ 25,5″	20° 30′ 06,5″	2,03°	20° 30′ 08,5″	
505	0,269	3′ 40,8″	82° 4′ 22,5″	20° 30′ 10,4″	2,25″	20° 30′ 12,7″	
506	0,238	3′ 15,4″	81° 57′ 20,0″	20° 30′ 08,9″	2,35″	20° 30′ 11,3″	
507	0,231	3′ 09,6″	81° 57′ 20,0″	20° 30′ 07,4″	2,30″	20° 30′ 09,7″	

From Ga $K\alpha_1$ φ_1 = 20° 30′ 11,0;
and ZnS crystal d_1 = 1908,87 XU } .... Ga $K\alpha_1$ λ = 1337,19 XU

A series of measurements of the differential glancing angle $\varkappa_{1,2}$ of Ga $K\alpha_1$.

Table 7b.

Plate No	Δ mm	Δ'	α	$\varkappa_t$	$\Delta\varkappa$ for $\Delta t°$ C	$\varkappa$ 18° C	mean value of $\varkappa$ at 18° C
485	0,283	3′ 52,3″	47° 51′ 58,1″	23° 57′ 50,2″	4,02″	23° 57′ 54,2″	23° 57′ 44,4″
486	0,255	3′ 29,3″	47° 51′ 49,1″	23° 57′ 39,2″	4,50″	23° 57′ 43,7″	
490	0,201	2′ 45,0″	47° 58′ 21,4″	23° 57′ 48,2″	4,80″	23° 57′ 53,0″	
491	0,249	3′ 24,4″	47° 52′ 01,6″	23° 57′ 43,0″	4,60″	23° 57′ 47,6″	
492	0,250	3′ 25,2″	47° 58′ 25,6″	23° 57′ 30,2″	3,50″	23° 57′ 33,7″	
493	0,219	2′ 59,8″	47° 58′ 28,6″	23° 57′ 44,4″	3,05″	23° 57′ 47,5″	
494	0,250	3′ 25,2″	47° 58′ 35,6″	23° 57′ 35,2″	3,50″	23° 57′ 38,7″	
495	0,250	3′ 25,2″	47° 58′ 36,0″	23° 57′ 35,4″	4,26″	23° 57′ 39,7″	
496	0,243	3′ 19,5″	47° 51′ 55,9″	23° 57′ 37,7″	3,80″	23° 57′ 41,5″	

From Ga $K\alpha_1$ differential glancing angle
$\varkappa_{1,2}$ = 23° 57′ 44,4″;
and ZnS crystal d_1 = 1908,87 XU, } .... Ga $K\alpha_1$ λ = 1337,35 XU
d_2 = 1909,11 XU

λ = 1337,35 X. U. derived from the differential glancing angle $\varkappa_{m,n}$ is taken as the precise value of the wave-length of gallium $K\alpha_1$. Uhler and Cooksey[14]) have obtained λ = 1337,15 X. U. In their determination of the said value of λ they have taken a value for the grating constant of calcite ($CaCO_3$) other than the usual value which is used for the precise determination of the wave-lengths of other elements. Siegbahn[15]) has recalculated this value with the

[14]) H. S. Uhler-C. D. Cooksey, Phys. Rev. 10 (1917), 645.
[15]) M. Siegbahn, Spektroskopie, Berlin, 1931.

normal value of the grating constant $d_1 = 3029,04$ X. U. and has obtained 1337,8 X. U. Just either one of these two values of Uhler and Cooksey shows a discontinuity from the values given in its neighbourhood after Moseley's law. The values obtained from Uhler and Cooksey, therefore, must have a certain discrepancy. The precision of these values as well as that of the values which I have obtained from the glancing angle φ and the differential glancing angle $\varkappa_{m,n}$ can be compared with the help of Moseley's law.

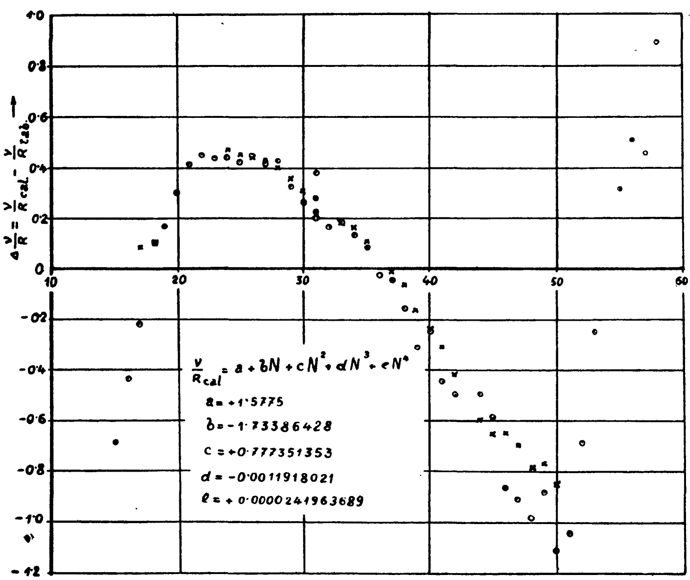

Fig. 7.

For a crucial test of the precision of the measurements of λ Ga $K\alpha_1$ I have made use of the Moseley's law after the modification of Dolejšek-Pestrecov.[16]) Dolejšek and Pestrecov have utilized the equation:

$$\frac{\nu}{R} = a + bN + cN^2 + dN^3 + eN^4,$$

[16]) V. Dolejšek-K. Pestrecov, C. R. **188** (1929), 164; Zs. f. Phys. **53** (1929), 566.

V. Dolejšek-K. Pestrecov, l. c. and Phys. Zs. **30** (1929), 898.

K. Pestrecov, Publications de la fac. des Sc. de l'univ. Charles, No. **90**, 1929.

V. Kunzl, Publications de la fac. des Sc. de l'univ. Charles, No. **180**, 1930.

where N is the atomic number and a, b, c, d and e are constants which are so determined that the equation holds true for the elements of the group of rare gases. It is with this equation, that the values of ν/R are calculated for the other elements. The difference between these calculated values and the values derived from the direct measurements are taken as the functions of the atomic numbers. With this modification we have thus a schematic course and the precision of the measurements can be tested very well through the survey of each and every discontinuity in the said course. In the Fig. 7, I give in the above mentioned system the graphical representation of the values $\Delta(\nu/R)$ between the measured values and the calculated values $\Delta(\nu/R) = \nu/R_{\text{calc.}} - \nu/R_{\text{mes.}}$, as a function of the atomic number. From this graph it can be seen for gallium and its neigbourhood that the value of λ obtained from φ_1 of Ga Kα_1 lies in the curve course of the older measurements which are signified by small circles. The value of λ obtained through $\varkappa_{1,2}$ lies on the curve obtained from the latest measurements, marked with the symbols of multiplication. I have denoted my measurements of gallium with black circular dots. Though the precision of our measurements has been limited by the precision of the scale, the value obtained with our method is of such a precision which is obtained only by the latest methods of precise measurements.

Now from these observations it can be seen that the value of the wave-length obtained through $\varkappa$ is more precise than that obtained from φ. A further test of the precision of this value of λ of Ga Kα_1 is also possible by new precise measurements of its neighbouring element germanium (Ge, atomic number 32) for which there is only one older value by Leide.[17]) From this graph it can further be seen that in this region of elements for Kα_1-lines with our value of the λ of Ga Kα_1 obtained from $\varkappa_{1,2}$ there exists no discontinuity greater than nearly 0,05 X. U.

In relation to the value of λ of Ga Kα_1 which I have obtained from the differential glancing angle $\varkappa_{1,2}$ I have measured the wavelength of Ga Kα_2. From the measurements I have obtained the difference $\Delta\varphi = 3'\,47{,}0''$ which corresponds to $\Delta\lambda = 3{,}94$ X. U. Thus the following values of λ are obtained for gallium

$$\mathrm{K}\alpha_1 = 1337{,}35 \text{ X. U.}$$
$$\mathrm{K}\alpha_2 = 1341{,}29 \text{ X. U.}$$

From the test of the precision of the measurements of λ of Kα_1 obtained from $\varkappa_{1,2}$, as can be noted from the above mentioned graph, it becomes evident that the use of the differential glancing angle $\varkappa$ for the measurement of the wave-lengths offers the same

[17]) Leide, Dissert. Lund, 1925.

advantages in diminishing or eliminating the errors due to the displacement of the Bragg's reflecting plane from the axis of the spectrograph as in the case of the measurements of the constants of crystal grating as it has been already shown. If we compare from the graph the $\Delta(\nu/R)$ of the older values of the elements at. number 30 and 32 which are the neighbouring elements of Ga (at. number 31) we note that they differ from the new and more precise values given by the Siegbahn's school by about 0,1 X. U. From these facts which I have mentioned it becomes obvious that such a difference can be caused also in the case of a perfect crystal through a small displacement error (some thousandths of millimetre) in the adjustment.

This research work, the results of which I have announced in these pages of this Journal, has been carried in the Spectroscopic Institute of Prof. Dr. V. Dolejšek (Charles University, Prague), to whom I offer my sincerest thanks not only for receiving me cordially as a research scholar in his laboratory, but also for kindly putting all the necessary requisites for my experimental work at my disposal.

*

Aplikace metody vzniklé kombinací metod φ a $\varkappa$ na určení mřížkové konstanty.

(Obsah předešlého článku.)

Výsledků uvedených v předešlé části, kde se zabýval měřením fiktivních mřížkových konstant, použil autor k přesné justaci Braggovy odrazové roviny do osy spektrografu. Tímto způsobem podařilo se mu docíliti toho, že odchylka Braggovy roviny reflekční od osy spektrografu byla asi 0,001 mm. Tohoto postavení krystalu lze dosáhnouti pomocí optické metody jen u krystalů s bezvadnými reflektujícími plochami. S ním provedl autor nová měření fiktivních konstant sfaleritu. Při diskusi výsledků měření ukázal, že přesnost měření byla omezena jedině přesností škály, a kdyby maximální chyba škály byla 1″, bylo by možno pomocí této metody zaručiti přesnost mřížkových konstant na 0,001 X jednotek.

V další části ukázal autor, že lze měřením rozdílového úhlu $\varkappa$ téže linie ve dvou různých řádech stanoviti absolutní hodnoty vlnových délek linií ze vzorce

$$m\lambda = \frac{2d_m \sin \varkappa_{m,n}}{\sqrt{\left(\frac{n}{m}\right)^2\left(\frac{d_m}{d_n}\right)^2 - 2\,\frac{n}{m}\,\frac{d_m}{d_n}\,.\cos \varkappa_{m,n} + 1}}.$$

Tento vzorec verifikoval autor experimentálně měřením čar $K\alpha$

u stříbra a galia. V diskusi výsledků ukázal, že užití metody $\varkappa$ pro měření vlnových délek má tytéž výhody jako pro měření mřížkových konstant. Způsob měření linií autorem udaný vychází z metody Kunzlovy a Köpplovy pro přesná měření mřížkových konstant.

Aplikace metody na měření mřížkových konstant pomocí dvou linií v témže řádě, jak ji udali Valouch, Bouchal a Dolejšek, není pro absolutní měření vlnových délek možná, nýbrž lze jí užíti jen pro měření relativní, podobně jak to učinili Lang a Schrör precisním měřením linií relativně k Cu Kα jako normálu.

Pomocí Dolejškovy a Pestrecovy modifikace Moseleyova zákona srovnává autor hodnoty, které měřil pro Ga Kα_1 (λ = 1337,35) a Kα_2 (λ = 1341,29) s hodnotami sousedních prvků a s hodnotami Ga Kα, které měřili Uhler a Cooksey. Ukazuje, že hodnoty stanovené metodou $\varkappa$ souhlasí s nejnovějšími přesnými hodnotami sousedních prvků.