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CHARACTERIZATION OF EPIMORPHISMS OF NETS BY NORMAL DECOMPOSITIONS

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This paper deals with projective nets of the same degree with singular points on the line. The notion of epimorphism of nets is studied in consequence with a normal decomposition. It will be shown that full pre-images of all non-singular points have a structure of nets and they all are of the same degree.

I. By a *projective net* (shortly a net) we mean a regular incidence structure (L, P), where P is a set and L is a set of some at least two-element subsets of P and where a set $s \subset P$ is privileged and the following axioms hold:

$(1)_N$ $s \in L$;

$(2)_N$ $\forall l \in L \setminus \{s\}, \quad \exists\, ! \, S \in s, \quad S \in l$;

$(3)_N$ $\forall P \in P \setminus s, \quad \forall S \in s, \quad \exists\, ! \, l \in L \setminus \{s\}, \quad S, P \in l$;

$(4)_N$ $l_1, l_2 \in L; \quad l_1 \neq l_2 \Rightarrow \# (l_1 \cap l_2) = 1$;

$(5)_N$ $P \setminus s \neq \emptyset$.

A net is termed non-trivial if

$(6)_N$ $\forall l \in L, \quad \# l \geqq 3$.

The elements of the set P are called *points*, the elements of the set L are called *lines*, the line s is called *singular*, its points the *singular points*. The points $A, B \in P$ are called *joinable* if there exists a line $l \in L$; $A, B \in l$, if they are distinct than we shall write $A \sqcup B := l$ and l is called the *join line* of A, B. The common point C of two distinct lines $a, b \in L$ we shall call the *point of intersection* of a, b and write $C =: a \sqcap b$. By a *degree of a net* it is meant the cardinality of the set s. It is obvious that $\# l_1 = \# l_2$ holds for every two lines $l_1, l_2 \in L \setminus \{s\}$.[1]) The common cardinality of all

1) Let $l_1, l_2 \in L \setminus \{s\}$; $l_1 \neq l_2$ and $S_1 =: s \sqcap l_1$, $S_2 =: s \sqcap l_2$, $S_3 \in s \setminus \{S_1, S_2\}$. We define such a mapping $f: l_1 \to l_2$ that for every point $P \in l_1$ the points P, P^f, S_3 are carried on the same line. Obviously the mapping f is bijective.

non-singular lines of the net we call the *order of the net*. If N^0 is a net where „^{0}“ is an index, than we designate $N^0 := (P^0, L^0, s^0)$.

Next let all nets be non-trivial and of the same degree m.

Let N, N' are nets. By a *homomorphism* of N into N' we shall mean a mapping $\varkappa : P \to P'$ for which

(i) $s^\varkappa = s'$, $\quad \varkappa/s$ is a bijection;

(ii) $\forall l \in L, \quad \exists \bar{l} \in L', \quad l^\varkappa \subseteq \bar{l}$; [2])

(iii) $\exists P \in P, \quad P^\varkappa \notin s$.

If $s'^{\varkappa^{-1}} = s$ holds then $\varkappa$ is called an *affine homomorphism*. If $P^\varkappa = P'$ we speak about *emimorphism of* N onto N'. A net N is the *subnet* of a net N' if $s = s'$, $P \subseteq P'$ and id_P is a homomorphism of N into N'.

Proposition 1. *Let $\varkappa$ be a homomorphism of a net N into N'. If for every line $l \in L \setminus \{s\}$ there is $\# l^\varkappa \geqq 3$, then $(P^\varkappa, \{l^\varkappa \mid l \in L\}, s^\varkappa)$ is a subnet of N'.*

Proof. Define:

$$P'' := P^\varkappa, \qquad s'' := s^\varkappa = s'$$
$$L'' := \{l^\varkappa \mid l \in L\}, \qquad N'' := (P'', L'', s'').$$

We need to verify the conditions $(1)N-(6)N$ for N''.

$(1)_N$, $(5)_N$, $(6)_N$: trivial.

$(2)_N$: Let $l \in L \setminus \{s\}$ be such a line that $l^\varkappa \neq s'$ and $S =: l \sqcap s$. Then $S^\varkappa \in l^\varkappa$. Suppose there exists a point $\bar{S} \in s \setminus \{S\}$ so that also $\bar{S}^\varkappa \in l^\varkappa$ is valid. Hence $l^\varkappa \subseteq s'$ by conditions (ii) and (i) of the definition of homomorphism of nets. By the hypothesis $l^\varkappa \neq s'$, it must be $l^\varkappa \subset s'$. Thus there exists a point $S^* \in s'$, $S^* \notin l^\varkappa$. Denote $\{S^0\} := s \cap S^{*\varkappa^{-1}}$. Let $P \in P \setminus s$ be an arbitrary non-singular point. Denote $\hat{l} := P \sqcup S^0$ and $\hat{P} := \hat{l} \sqcap l$. Then $\hat{P}^\varkappa \in s' \setminus \{S^*\}$ and thus $\hat{l}^\varkappa \subseteq s'$, it follows $P^\varkappa = s'$, in contradiction to (iii).

$(3)_N$: Let $P \in P \setminus s$ and $P^\varkappa \notin s'$. Further, let $S \in s$ ($\Rightarrow S^\varkappa \in s'$). If we put $l =: P \sqcup S$, then the $l^\varkappa$ contains $S^\varkappa$ as well as $P^\varkappa$.

$(4)_N$: Let $l_1^\varkappa \neq l_2^\varkappa$, for $l_1, l_2 \in L \setminus \{s\}$, then $l_1 \neq l_2$ and $(l_1 \sqcap l_2)^\varkappa \in l_1 \cap l_2 \Rightarrow$ $\Rightarrow \#(l_1^\varkappa \cap l_2^\varkappa) \geqq 1$. It remains to prove: $\#(l_1^\varkappa \cap l_2^\varkappa) = 1$. Let $\bar{l}_1, \bar{l}_2$ be the such lines that $l_1^\varkappa \subseteq \bar{l}_1$, $l_2^\varkappa \subseteq \bar{l}_2$, Suppose $\bar{l}_1 = \bar{l}_2$. There are two possible cases:

(a) $\bar{l}_1 \neq s'$. As $\varkappa/s$ is a bijection, it follows from this that $l_1 \sqcap l_2$ is a singular point S. Choose a singular point $S^* \neq S$. Let $Y = X^\varkappa \in l_1^\varkappa$, $X \in l_1$, $Y \neq S^\varkappa$, $Z = (S \sqcup X) \sqcap l_2$. Then $Z^\varkappa = X^\varkappa = Y$, therefore $l_1^\varkappa \subseteq l_2^\varkappa$. In the same way we obtain $l_2^\varkappa \subseteq l_1^\varkappa$, hence $l_1^\varkappa = l_2^\varkappa$, a contradiction.

(b) $\bar{l}_1 = s'$. If $l_1^\varkappa \neq s'$, then there exists a (singular) point $S^* \in s$ such, that $S^{*\varkappa} \in$ $\in s' \setminus l_1^\varkappa$. Let $P \setminus s$ be an arbitrary non-singular point. If we put $\hat{P} = l_1 \sqcap (S^* \sqcup P)$,

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2) If $\varkappa$: $M \to M'$ is a mapping then for every $N \subseteq M$ we define $N^\varkappa := \{X^\varkappa \mid X \in N\}$.

then $\mathrm{P} \in \mathrm{S}^* \sqcup \hat{\mathrm{P}} \Rightarrow \mathrm{P}^{\varkappa} \in s'$. Therefore $P^{\varkappa} \subset s'$ which gives a contradiction to (iii). Thus, we get $l_1^{\varkappa} = s'$. We can obtain in the same way that $l_2^{\varkappa} = s'$. It follows $l_1^{\varkappa} = l_2^{\varkappa}$ contradicting the assumption about $l_1^{\varkappa}, l_2^{\varkappa}$. It means, that $\bar{l}_1 \neq \bar{l}_2$ and the unequality $\#(l_1^{\varkappa} \cap l_2^{\varkappa}) \leqq 1$ is obvious.

Proposition 2. *Let $\varkappa$ be an epimorphism of N onto N'. Let* A, B *be two distinct points such that* $\mathrm{A}^{\varkappa}, \mathrm{B}^{\varkappa}$ *are distinct too. If* A, B *are joinable, then* $\mathrm{A}^{\varkappa}, \mathrm{B}^{\varkappa}$ *are joinable and $\varkappa$ mapps* $\mathrm{A} \sqcup \mathrm{B}$ *onto* $\mathrm{A}^{\varkappa} \sqcup \mathrm{B}^{\varkappa}$.

Proof. Let $p := \mathrm{A} \sqcup \mathrm{B}$. We can omit the trivial case $p = s$ and suppose $p \neq s$. Clearly, $p^{\varkappa}$ contains $\mathrm{A}^{\varkappa}, \mathrm{B}^{\varkappa}$ so that $\mathrm{A}^{\varkappa}, \mathrm{B}^{\varkappa}$ are joinable. It remains to prove that $p^{\varkappa} = \mathrm{A}^{\varkappa} \sqcup \mathrm{B}^{\varkappa}$. Two following cases will be distinguished:

(a) $\mathrm{A}^{\varkappa} \sqcup \mathrm{B}^{\varkappa} \neq s'$. Let $\mathrm{X}' \in \mathrm{A}^{\varkappa} \sqcup \mathrm{B}^{\varkappa}$ and let S^* be a singular point of P, $\mathrm{S}^* \notin p$. If $\mathrm{X}' \in s'$, then, evidently, $\mathrm{X}' = (p \sqcap s)^{\varkappa}$. Let X' be a non-singular point. Then there exists a non-singular point $\mathrm{X} \in P$ such that $\mathrm{X}^{\varkappa} = \mathrm{X}'$. Denoting by P the point $p \sqcap (\mathrm{S}^* \sqcup \mathrm{X})$ we get $\mathrm{P}^{\varkappa} = (\mathrm{A}^{\varkappa} \sqcup \mathrm{B}^{\varkappa}) \sqcap (\mathrm{S}^{*\varkappa} \sqcup \mathrm{X}^{\varkappa}) = \mathrm{X}'$.

(b) $\mathrm{A}^{\varkappa} \sqcup \mathrm{B}^{\varkappa} = s'$. Let S be the singular point of p, $\mathrm{X}' \in s'$, $\mathrm{X}' \neq \mathrm{S}^{\varkappa}$. Then there exists a unique singular point $\mathrm{X} \in P$ whose image under $\varkappa$ is X'. Let P be a point of P such that $\mathrm{P}^{\varkappa} \notin s'$. If we put $\mathrm{Y} = p \sqcap (\mathrm{X} \sqcup \mathrm{P})$, we obtain $\mathrm{Y}^{\varkappa} = s' \sqcap (\mathrm{X}^{\varkappa} \sqcup \mathrm{P}^{\varkappa}) = \mathrm{X}'$.

Corollary. *Let $\varkappa$ be an epimorphism of N onto N'. Then*

$$l \in L \Rightarrow l^{\varkappa} \in N'$$

II. Let N be a net. A *decomposition* $\bar{P}$ of the set P is termed *normal* if the following axioms hold:

$(1)_{\mathrm{D}}$ every class of the decomposition contains at most one singular point.
$(2)_{\mathrm{D}}$ There exists a class with no singular point.
$(3)_{\mathrm{D}}$ If two lines intersect two distinct classes then every class is intersected either by both of them or by none.
$(4)_{\mathrm{D}}$ Every line intersects at least three distinct classes.

Theorem 1. *Let $N = (P, L, s)$ be a net and $\bar{P}$ be a normal decomposition of the set P. Define*: $\bar{L} := \{\{\bar{\mathrm{P}} \in \bar{P} \mid \bar{\mathrm{P}} \cap l \neq \emptyset\} \mid l \in L\}$, $\bar{s} := \{\bar{\mathrm{S}} \in \bar{P} \mid \bar{\mathrm{S}} \cap s \neq \emptyset\}$. *Then* $\bar{N} := (\bar{P}, \bar{L}, \bar{s})$ *is a net and there exists an epimorphism $\varkappa$ of the net N onto the net $\bar{N}$.*

Proof. We need to verify axioms of a net for $\bar{N}$.

$(1)_N$: It is trivial by definition of $\bar{s}$.

$(2)_N$: Let $\bar{l} \in \bar{L} \setminus \{\bar{s}\}$, $\bar{l} := \{\bar{\mathrm{P}} \in \bar{P} \mid \bar{\mathrm{P}} \cap l \neq \emptyset\}$ for some $l \in L \setminus \{s\}$. Since $\#(l \cap s) = 1$ there exists exactly one common class of the decomposition for the lines $\bar{l}$ and $\bar{s}$.

$(3)_N$: Let $\bar{\mathrm{P}} \in \bar{P} \setminus \bar{s}$ be a fixed arbitrary class of the decomposition. For every $\mathrm{P} \in \bar{\mathrm{P}}$ and every $\mathrm{S} \in s$ there exists exactly one $l \in L \setminus \{s\}$ where $\mathrm{S}, \mathrm{P} \in l$ (by axiom $(3)_N$ of the definition of a net). Hence for $\bar{\mathrm{P}}$ and for every $\bar{\mathrm{S}} \in \bar{s}$ there must exist at least one line $\bar{l} \in \bar{L}$ where $\bar{\mathrm{S}}, \bar{\mathrm{P}} \in \bar{l}$. By axiom $(3)_{\mathrm{D}}$ there exists exactly one line of these properties.

$(4)_N$ Let $\bar{l}_1, \bar{l}_2$ be two distinct lines from $\bar{L} \setminus \{\bar{s}\}$. Then either $\bar{l}_1 \sqcap \bar{l}_2 = \bar{S} \in \bar{s}$ or there exist two distinct classes $\bar{S}_1, \bar{S}_2 \in \bar{s}$ such that $\bar{S}_1 \in \bar{l}_1$, $\bar{S}_2 \in \bar{l}_2$. Hence there exist lines $l_1, l_2 \in L \setminus \{s\}$ such that $\bar{l}_1 = \{\bar{P}_1 \in \bar{P} \mid \bar{P}_1 \cap \bar{l}_1 \neq \emptyset\}$, $\bar{l}_2 = \{\bar{P}_2 \in \bar{P} \mid \bar{P}_2 \cap \cap \bar{l}_2 \neq \emptyset\}$. Since $l_1, l_2 \in L \setminus \{s\}$ then there exists a point $P = l_1 \sqcap l_2$ and therefore there exists $P \in \bar{P} \in \bar{P}$ where $\bar{P} \cap l_1 \neq \emptyset$ and $\bar{P} \cap l_2 \neq \emptyset$, too. Hence $\bar{P} \in \bar{l}_1, \bar{l}_2$. Suppose that there exists a point $\bar{Q} \neq \bar{P}$, $\bar{Q} \in \bar{l}_1, \bar{l}_2$. Then there exist points $Q_1, Q_2 \in P$; $Q_1, Q_2 \in \bar{Q}$ and the lines $p_1 := P \sqcup Q_1$ and $p_2 := P \sqcup Q_2$. Therefore $\bar{l}_1 = \bar{l}_2$ by $(3)_D$, a contradiction.

Axioms $(5)_N$ and $(6)_N$ hold for $\bar{N}$ by $(2)_D$ and $(4)_D$. Sine $\bar{l} \subseteq \bar{P}$ by definition $\bar{l}$, $\#\bar{l} \geqq 3$ by $(4)_D$ and $\#(\bar{l}_1 \cap \bar{l}_2) = 1$ by this proof it is ibvious that $\bar{N}$ is a regular incidence structure. The mapping $\varkappa$:

$$P \to \bar{P},$$
$$P \mapsto \bar{P} \Leftrightarrow P \in \bar{P}$$

is obviously surjective. Axioms (i) and (ii) of the definition of homomorphism hold by definition $\bar{L}$ and by axiom $(1)_D$. Axiom (iii) by $(4)_D$. Hence $\varkappa$ is an epimorphism of N onto $\bar{N}$.

Theorem 2. *Let N, N' be nets and let there exists an epimorphism $\varkappa$ of N onto N'. Denote $\bar{P}$ such a decomposition of the set P that the classes of this decomposition are full pre-images of all points from P under an epimorphism $\varkappa$. Then $\bar{P}$ is a normal decomposition.*

Proof. We verify axioms $(1)_D-(4)_D$.

$(1)_D$: This axiom follows from (i) of definition of epimorphism.

$(2)_D$: Suppose that every class of the decomposition contains a singular point. Then $\varkappa$ maps every point from P onto some point of the line s', a contradiction to hypotheses that N is non-trivial and that $\varkappa$ is surjective.

$(3)_D$: Let $A', B' \in P'$ be two distinct points. Then $A'^{\varkappa^{-1}}$ and $B''^{\varkappa^{-1}}$ are distinct, too. Let $l_1, l_2 \in L$ be such distinct lines which both intersect $A'^{\varkappa^{-1}}$ as well as $B'^{\varkappa^{-1}}$. Further let $l_1 \cap C'^{\varkappa^{-1}} \neq \emptyset$ where C' is a point from P' distinct from A', B'. By (ii) $C' \in A' \sqcup B'$ and since $C' \in l_2^{\varkappa}$ there exists a point $N \in C'^{\varkappa^{-1}}$ by proposition 2; which was to be proved.

$(4)_D$: Let l be some line from L and $l^{\varkappa} \in L'$. $\#l^{\varkappa} \geqq 3$ because N' is a non-trivial net. Let A', B', C' be three mutually distinct points from l'. Since $\varkappa$ is an epimorphism, then $A'^{\varkappa^{-1}}, B'^{\varkappa^{-1}}, C'^{\varkappa^{-1}}$ are necessarily mutually distinct. The definition of epimorphism and proposition 3 follow: $A'^{\varkappa^{-1}} \cap l \neq \emptyset$, $B'^{\varkappa^{-1}} \cap l \neq \emptyset$, $C'^{\varkappa^{-1}} \cap l \neq \emptyset$.

Theorem 3. *Let N, N' be nets and let $\varkappa$ be an epimorphism of N onto N'. Let $P' \in \in P' \setminus s'$. Define*:

$$P_{P'} := \{X \mid X \in P, X \in P'^{\varkappa^{-1}}\} \cup s,$$
$$L_{P'} := \{l^* \mid l^* \neq \emptyset, l^* := (l \cap P'^{\varkappa^{-1}}) \cup S, l \in L, S = l \sqcap s\} \cup \{s\}.$$

Then $(P_{P'}, L_{P'}, s) =: N_{P'}$ is a net.

Proof. It is obvious that $P'^{\varkappa^{-1}} \neq \emptyset$. If $\# P'^{\varkappa^{-1}} = 1$ then $N_{P'}$ is a trivial net. Suppose that $\# P'^{\varkappa^{-1}} \geqq 2$. Then the axioms $(1)_N$, $(5)_N$, $(6)_N$ hold. Axioms $(2)_N$ and $(3)_N$ are cosequences of the definition $l^* \in L_{P'}$.

$(4)_N$: Let l_1^*, l_2^* be two distinct lines from $L_{P'} \setminus \{s\}$ and $S_1 = l_1^* \cap s$, $S_2 = l_2^* \cap s$. Denote by l_1, l_2 such two distinct lines from $L \setminus \{s\}$ for which $l_1^* = (l_1 \cap P'^{\varkappa^{-1}}) \cup \cup \{S_1\}$, $l_2 = (l_2 \cap P'^{\varkappa^{-1}}) \cup \{S_2\}$. There exists a point $R \in P$, $R = l_1 \sqcap l_2$ (by $(4)_N$ of the definition of a net N). Since the epimorphism $\varkappa$ makes a normal decomposition on P and $P'^{\varkappa^{-1}}$ is one of the classes of this decomposition (by theorem 2) then by $(3)_D$ either

– lines l_1, l_2 simultaneously intersect the same classes of the decomposition; hence $S_1 \in l_1 \cap l_2 = l_1^* \cap l_2^*$, a contradiction, or

– lines l_1, l_2 simultaneously intersect one class of the decomposition, class $P'^{\varkappa^{-1}}$ and we have $l_1 \cap l_2 = R \in P'^{\varkappa^{-1}}$ and consequently $R = l_2^* \sqcap l_2^* \in P_{P'}$, which was to be proved.

Proposition 3. *Let N and N' be nets and let $\varkappa$ be an epimorphism of N onto N'. Then $\# A'^{\varkappa^{-1}} = \# B'^{\varkappa^{-1}}$ for every $A', B' \in P' \setminus s'$.*

Proof. The epimorphism $\varkappa$ makes a decomposition $\bar{P}$ of the set P. Since $A'^{\varkappa^{-1}}$ has the structure of a net, then for every $A' \in P' \setminus s'$ it holds:

$$\# A'^{\varkappa^{-1}} = (\text{ord } N_{A'})^2,$$

and it is sufficient to prove the following statement:

The order of a net $N_{P'}$ is constant for every point $P' \in P' \setminus s'$.

If $A' \neq B'$, then $A'^{\varkappa^{-1}} \cap B'^{\varkappa^{-1}} = \emptyset$.

a) Let A', B', S_1' be points on the same line. Let $A' \sqcup B' = l'$ and $S_2 \in s$, $S_2 \notin S'^{\varkappa^{-1}}$. Then there exist two distinct lines $l_1, l_2 \in L$ throught a point S_2 such that $l_1 \cap A'^{\varkappa^{-1}} =: \; =: l_1^* \neq \emptyset$ and $l_2 \cap B'^{\varkappa^{-1}} =: l_2^\circ \neq \emptyset$; l_1^*, l_2° are lines from $L_{A'}$, $L_{B'}$, respectively. Since $l \cap A'^{\varkappa^{-1}} \neq \emptyset$ if and only if $l \cap B'^{\varkappa^{-1}} \neq \emptyset$ for every line $l \in L$ throught a point $S \in s$, $S^\varkappa = S_1'$ (by $(3)_D$) and since $N_{A'}$ and $N_{B'}$ are nets, there exists a bijective mapping $\pi : \{X \mid X \in l_1^*\} \to \{Y \mid Y \in l_2^\circ\}$, where $X^\pi = Y$, if the points X, Y, S lie on the line. Hence $\# l_1^* = \# l_2^\circ$ and ord $N_{A'} =$ ord $N_{B'}$.

b) Consider the points A', B' not joinable. Denote $l_1' := A' \sqcup S_1'$, $l_2' := B' \sqcup S_1'$, $l_3' := A' \sqcup S_2'$ where $S_1', S_2' \in s'$; $S_1' \neq S_2'$. According to axiom $(4)_N$, there exists a point $C' \in P'$, $C' := l_1' \sqcap l_3'$. Then the points A', C', S_2' lie on the same line, as well as the points B', C', S_1', and consequently by a)

$$\text{ord } N_{A'} = \text{ord } N_{C'} = \text{ord } N_{B'}.$$

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Shrnutí

CHARAKTERIZACE EPIMORFISMŮ TKÁNÍ NORMÁLNÍMI ROZKLADY

JAROSLAVA JACHANOVÁ

Článek se zabývá projektivními tkáněmi téhož stupně, jejichž singulární body jsou kolineární. Pojem epimorfismu tkání je studován v souvislosti s normálními rozklady. Rozklad $\bar{P}$ množiny P (množina bodů tkáně) nazveme normálním, platí-li: $(1)_D$ Každá třída rozkladu obsahuje nejvýše jeden singulární bod. $(2)_D$ Existuje třída, která neobsahuje žádný singulární bod. $(3)_D$ Protínají-li dvě přímky dvě různé třídy, pak každá třída je proťata současně oběma nebo žádnou z nich. $(4)_D$ Každá přímka protíná alespoň tři různé třídy rozkladu. Nechť $N = (P, L, s)$ je tkáň a $\bar{P}$ normální rozklad množiny P, pak $\bar{N} := (\bar{P}, \{\{\bar{\mathrm{P}} \in \bar{P} \mid \bar{\mathrm{P}} \cap l \neq \emptyset\} \mid l \in L\}, \{\bar{\mathrm{S}} \in \bar{P} \mid \bar{\mathrm{S}} \cap s \neq \emptyset\})$ je tkáň a existuje epimorfismus tkáně N na tkáň $\bar{N}$.

Nechť N, N' jsou tkáně a $\varkappa$ je epimorfismus prvé na druhou. Pak úplné vzory všech bodů z P' při $\varkappa$ jsou třídami normálního rozkladu množiny P.

Nechť N, N' jsou tkáně, $\varkappa$ epimorfismus N na N' a nechť P' je nesingulární bod tkáně N'. Pak $N_{\mathrm{P}'} := (P_{\mathrm{P}'}, L_{\mathrm{P}'}, s)$ je tkáň, kde $P_{\mathrm{P}'} := \{\mathrm{X} \mid \mathrm{X} \in P, \mathrm{X} \in \mathrm{P}'^{\varkappa^{-1}}\} \cup s$

$$L_{\mathrm{P}'} := \{l^* \mid l^* \neq \emptyset, l^* := (l \cap \mathrm{P}'^{\varkappa^{-1}}) \cup \{\mathrm{S}\}, l \in L, \mathrm{S} \in l\} \cup \{s\}.$$

Řád tkáně $N_{\mathrm{P}'}$ je týž pro každý nesingulární bod P' tkáně N'.

Резюме

ХАРАКТЕРИЗАЧИЯ ЭПИМОРФИЗМОВ СЕТЕЙ НОРМАЛЬНЫМИ РАЗЛОЖЕНИЯМИ

Ярослава Яханова

Статья описывает проективные сети того же рода, сингулярные точки которых находятся на одной линии. Понятие этиморфизма сетей изучается в связи с нормальными разложениями. Разложение $\bar{P}$ множества P (множества всех точек сети) называется нормальным когда: $(1)_D$ В каждом смежном классе не больше одной сингулярной точки. $(2)_D$ Существует такой класс который не содержит ни одной сингулярной точки. $(3)_D$ Если две линии пересекают два розных класса, то каждый класс пересечен либо овеими, либо никакой. $(4)_D$ Всякая линия пересекает по крайней мере три смежные класса.

Пусть $N = (P, L, s)$ сеть и $\bar{P}$ нормальное разложение множества P, то

$$\bar{N} := (\bar{P}, \{\{\bar{\mathrm{P}} \in \bar{P} \mid \bar{\mathrm{P}} \cap l \neq \emptyset\} \mid l \in L\}, \{\bar{\mathrm{S}} \in \bar{P} \mid \bar{\mathrm{S}} \cap s \neq \emptyset\})$$

также сеть и существует этиморфизм сети N на сеть $\bar{N}$.

Пусть N, N' сети и $\varkappa$ эпиморфизм N на N'. То полные прообраза всез точек $\bar{P}'$ в эпиморфизме $\varkappa$ являются смежными классами нормального разложения множества P. Пусть N, N' сети и $\varkappa$ эпиморфизм N на N' и пусть P' несингулярная точка сети N'. То $N_{\mathrm{P}'} := (P_{P'}, L_{\mathrm{P}'}, s)$ сеть, в которой $P_{\mathrm{P}'} := \{\mathrm{X} \mid \mathrm{X} \in P,\ \mathrm{X} \in \mathrm{P}'^{\varkappa^{-1}}\} \cap s$

$$L_{\mathrm{P}'} := \{l^* \mid l^* \neq \emptyset,\ l^* := (l \cap \mathrm{P}'^{\varkappa^{-1}}) \cup \{\mathrm{S}\}, l \in L, \mathrm{S} \in l\} \cup \{s\}.$$

Порядок сети $N_{\mathrm{P}'}$ тот же для всякую точку P' сети N'.