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PLACES OF ALTERNATIVE FIELDS

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In the following text, the connection between places of alternative fields and its total subrings is studied. The results obtained are similar to those derived for places and valuation rings of fields.

Definition.

An alternative field is an algebraic structure $(\mathbf{A}, +, .)$ with two binary operations such that $(\mathbf{A}, +)$ is a group, $(\mathbf{A} \div \{0\}, .)$ is a loop, both distributive and both alternative[1]) laws are satisfied.

It can be proved, that the additive group is commutative and the left and right inverse property is satisfied:

$$\text{(IP) for } a \neq 0, \quad a^{-1} . (a . b) = (b . a) . a^{-1} = b,$$
$$a . (a^{-1} . b) = (b . a^{-1}) . a = b.$$

Moreover, $(x . y)^{-1} = y^{-1} . x^{-1}$.

Definition.

A place of alternative fields[2]) is a mapping Θ from an alternative field $(\mathbf{A}, +, .)$ to an alternative field $(\mathbf{A}', +', .')$ satisfying:

(i) For $x, y \in \mathbf{A}$, if $x^\Theta \neq \infty$ and $y^\Theta \neq \infty$, then $(x - y)^\Theta = x^\Theta -' y^\Theta$ and $(x . y)^\Theta = x^\Theta .' y^\Theta$.

(ii) For $x, y \in \mathbf{A}$, if $x^\Theta \neq 0'$ and $y^\Theta = \infty$, then $(x . y)^\Theta = (y . x)^\Theta = \infty$, where ∞ is a symbol not belonging to $\mathbf{A}'$ and the notation $a^\Theta \neq \infty$, $b^\Theta = \infty$ means that a belongs and b does not belong to the domain of Θ.

It can be verified, that $(\operatorname{Im} \Theta, +', .')$ is an alternative field. Therefore it can be supposed, that Θ is surjective:

(iii) $\mathbf{A}^\Theta = \mathbf{A}'$.

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1) A left alternative law: for all $a, b \in \mathbf{A}$, $a . (a . b) = (a . a) . b$; right alternative law: for all $a, b \in \mathbf{A}$, $(a . b) . b = a . (b . b)$.

2) In [1], this mapping is called *pseudohomomorphism*.

It can be easily proved that a place Θ of alternative fields satisfies:
$0^\Theta = 0'$, $1^\Theta = 1'$;
if $a^\Theta = \infty$ and $b^\Theta \neq \infty$, then $(a \pm b)^\Theta = (b \pm a)^\Theta = \infty$;
if $x \neq 0$, then $x^\Theta = 0'$ is equivalent to $(x^{-1})^\Theta = \infty$.

Definition.

A subring[3]) $\mathbf{O}$ of the alternative field $\mathbf{A}$ is said to be *total* in $\mathbf{A}$, if either $x \in \mathbf{O}$ or $x^{-1} \in \mathbf{O}$ for every $x \in \mathbf{A} \dot{-} \{0\}$.

In a total ring, there are no zero divisors and every subring of the alternative field $\mathbf{A}$, which contains a ring total in $\mathbf{A}$, is also total in $\mathbf{A}$.

If Θ is a place of alternative fields, let us denote

$$\mathbf{O}_\Theta = \{x \in \mathbf{A}/x^\Theta \neq \infty\}.$$

If $x^\Theta \neq 0'$, then $(x^{-1})^\Theta \neq \infty$ and so $x^{-1} \in \mathbf{O}_\Theta$. We shall use the following notation:
$U_\Theta = \{x \in \mathbf{A}/x \in \mathbf{O}_\Theta$ and $x^{-1} \in \mathbf{O}_\Theta\}$, so called units;
$I_\Theta = \{x \in \mathbf{A}/x \in \mathbf{O}_\Theta$ and $x^{-1} \notin \mathbf{O}_\Theta$, or $x = 0\}$, so called non-units;
$J_\Theta = \{x \in \mathbf{A}/x \notin \mathbf{O}_\Theta\} = \{x \in \mathbf{A}/x^\Theta = \infty\}$.
It can be seen, that $\mathbf{O}_\Theta$ is a disjoint union of U_Θ and I_Θ and for every $x \in U_\Theta$, $(x^{-1})^\Theta = (x^\Theta)^{-1}$.

Theorem 1.

Let Θ be a place of an alternative field $(\mathbf{A}, +, .)$ onto an alternative field $(\mathbf{A}', +', .')$[4]). *Then $\mathbf{O}_\Theta$ is a total subring in $\mathbf{A}$, I_Θ being its unique maximal ideal, and the factorring $\mathbf{O}_\Theta/I_\Theta$* is isomorphic to $\mathbf{A}'$.

Proof. It is clear, that $\mathbf{O}_\Theta$ is a ring. If $x \notin \mathbf{O}_\Theta$, then $x^\Theta = \infty$ and $(x^{-1})^\Theta = 0'$, which implies $x^{-1} \in \mathbf{O}_\Theta$. Hence $\mathbf{O}_\Theta$ is total in $\mathbf{A}$. Suppose that $x, y \in I_\Theta$. From this it follows $(x - y)^\Theta = x^\Theta -' y^\Theta = 0' - 0' = 0'$, and thus $x - y \in I_\Theta$. If $x \in I_\Theta$ and $z \in \mathbf{O}_\Theta$, then $(x . z)^\Theta = 0' .' z^\Theta = 0' = z^\Theta .' 0' = (x . z)^\Theta$. Hence I_Θ is a both-sided ideal. We shall show now, that I_Θ is maximal. Let $x \in U_\Theta$. Then each ideal generated by x contains a unite 1 and is equal to the whole ring $\mathbf{O}_\Theta$. This implies, that each ideal different from $\mathbf{O}_\Theta$ is contained in I_Θ. The restriction of Θ to $\mathbf{O}_\Theta$ is a homomorphism of $\mathbf{O}_\Theta$ onto $\mathbf{A}'$ with a kernel I_Θ, thus $\mathbf{O}_\Theta/I_\Theta \cong \mathbf{A}'$.

Note that $\Theta: \mathbf{A} \to \mathbf{A}' \cup \{\infty\}$ is a place of alternative fields, if and only if it satisfies:

(a) There exists a subring $\mathbf{O}$ in $\mathbf{A}$ such that the restriction $\Theta_1 = \Theta/\mathbf{O}$ is a homomorphism of $\mathbf{O}$ onto the ring $\mathbf{A}'$;

(b) If $x \in \mathbf{A} \dot{-} \mathbf{O}$, then $x^\Theta = \infty$, $x^{-1} \in \mathbf{O}$ and $(x^{-1})^\Theta = 0'$;

(c) There exists $x \in \mathbf{O}$ such that $x^\Theta \neq 0'$.

Theorem 2.

Let $\mathbf{O}$ be a total subring in the alternative field $\mathbf{A}$. A set $I = \{x \in \mathbf{A}/x \in \mathbf{O}$ and

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[3]) Not necessary associative.

[4]) In short, $\mathbf{A} \to \mathbf{A}' \cup \{\infty\}$.

$x^{-1} \notin \mathbf{O}$, *or* $x = 0\}$ *forms a unique maximal ideal in* $\mathbf{O}$. *Moreover, a factorring* $\mathbf{O}/I$ *is an alternative field.*

We shall call I a set of non-units again. Note that each total ring is local.

Before proving our theorem, we shall establish several lemmas.

Lemma 1.

Let $\mathbf{O}$ *be a total subring in an alternative field* $\mathbf{A}$ *and denote by* I *a set of non-units in* $\mathbf{O}$. *If* $x \in I$ *and* $y \in \mathbf{O}$, *then* $x \,.\, y$ *and* $y \,.\, x$ *are in* I *for all* x, y *from* $\mathbf{A}$. *Especially,* $I \,.\, I \subseteqq I$.

Let us denote $U = \{x \in \mathbf{A}/x \in \mathbf{O}$ and $x^{-1} \in \mathbf{O}\}$. Obviously, $U = \mathbf{O} \dot{-} I$. If $x = 0$ or $y = 0$, the proof is trivial. Thus suppose $x \,.\, y \neq 0$. In the first step, let $x \in I \dot{-} \{0\}$, $y \in U$. Thus $x, y \in \mathbf{O}$, $x^{-1} \notin \mathbf{O}$ and $y^{-1} \in \mathbf{O}$. Suppose that $z = x \,.\, y \notin I$. This implies $z \in \mathbf{O}$ and $z^{-1} \notin \mathbf{O}$. Further, $x^{-1} = (z \,.\, y^{-1})^{-1} = y \,.\, z^{-1}$. We obtain $x^{-1} \in \mathbf{O}$, a contradiction. Hence $x \,.\, y \in I$. Similarly for $y \,.\, x$. In the second step, let $x, y \in \in I \dot{-} \{0\}$. Suppose $z = x \,.\, y \notin I$. Then $z^{-1} = y^{-1} \,.\, x^{-1}$ and $y^{-1} = z^{-1} \,.\, x \in \mathbf{O}$, which is a contradiction. Thus $x \,.\, y \in I$.

Lemma 2.

Under the same assumptions as above, let $J = \mathbf{A} \dot{-} \mathbf{O}$. *Then*

(i) $a^{-1} \in I \Leftrightarrow a \in J$ *for every* $a \in \mathbf{A} \dot{-} \{0\}$,

(ii) $J \,.\, J \subseteqq J$,

(iii) *if* $x \in J$ *and* $y \in U$, *then* $x \,.\, y \in J$ *and* $y \,.\, x \in J$,

(iv) $U \,.\, U \subseteqq U$.

The proof is easy.

Now let us return to the proof of Theorem 2. We shall show first that $(I, +)$ is a subgroup of $(\mathbf{O}, +)$. For every $a, b \in I$ we have $a - b \in \mathbf{O}$. Suppose $(a - b)^{-1} \in \in \mathbf{O}$. Since $\mathbf{O}$ is total in $\mathbf{A}$, it must be either $a^{-1} \,.\, b \in \mathbf{O}$, or $(a^{-1} \,.\, b)^{-1} = b^{-1} \,.\, a \in \mathbf{O}$. If $a^{-1} \,.\, b \in \mathbf{O}$, we shall use the relation

$$a^{-1} = (a^{-1} \,.\, (a - b)) \,.\, (a - b)^{-1} = (1 - a^{-1} \,.\, b) \,.\, (a - b)^{-1}.$$

Obviously, $1 \in \mathbf{O}$ and $1 - a^{-1} \,.\, b \in \mathbf{O}$. Further $(a - b)^{-1} \in \mathbf{O}$, thus the right side belongs to $\mathbf{O}$, in contrary to the assumption $a^{-1} \notin \mathbf{O}$. The case $b^{-1} \,.\, a$ is symmetric. Therefore $(a - b)^{-1} \notin \mathbf{O}$ and $a - b \in I$. By Lemma 1., I is an ideal. The same arguments as in the proof of Theorem 1. shows that I is maximal. We observe that a decomposition of $\mathbf{O}$ modulo I is compatible with addition and multiplication.

It remains to prove that $(\mathbf{O}/I \dot{-} \{0\}, .)$ is a loop. The coset $[1]$ is obviously a unit element. Suppose $[a], [b] \neq [0]$. Thus $a, b \in U$ and further $a^{-1} \in U$, $b \,.\, a^{-1} \in U$. Since $[b] = [(b \,.\, a^{-1}) \,.\, a] = [b \,.\, a^{-1}] \,.\, [a]$, the coset $[b \,.\, a^{-1}]$ is a solution of the equation $[b] = [a] \,.\, [x]$ in $\mathbf{O}/I \dot{-} \{0\}$. It can be checked that the solution is unique and that both distributive and alternative laws holds.

Theorem 3.

Let $\mathbf{O}$ be a total subring in the alternative field $\mathbf{A}$. Then there exists a place Θ of $\mathbf{A}$ such that $\mathbf{O}_\Theta = \mathbf{O}$.[5])

Proof. Let $\varkappa : \mathbf{O} \to \mathbf{O}/I$ denotes a canonical homomorphism, I being a maximal ideal. Define $x^\Theta := x^\varkappa$ for $x \in \mathbf{O}$, $x^\Theta := \infty$ for $x \in \mathbf{A} \dot{-} \mathbf{O}$, $\mathbf{A}' := \mathbf{O}/I$.

It can be verified that Θ has the properties (i) – (iii) from the definition of a place.

The following theorem shows that a place has no proper extension.

Theorem 4.

Let $\Theta : \mathbf{A} \to \mathbf{A}' \cup \{0\}$ be a place and φ a homomorphism of the subring $\mathbf{R} \subseteqq \mathbf{A}$ to an alternative field. Let $\mathbf{O}_\Theta \subseteqq \mathbf{R}$ and suppose that the equality $\varphi = \Theta$ is true on $\mathbf{O}_\Theta$. Then $\mathbf{O}_\Theta = \mathbf{R}$.

Proof. Let $x \in \mathbf{R}$. Suppose $x \notin \mathbf{O}_\Theta$. Then $x^{-1} \in \mathbf{O}_\Theta$ and $(x^{-1})^\varphi = (x^{-1})^\Theta = 0'$, Since $1 \in \mathbf{O}_\Theta$ and $1^\Theta = 1' \in \mathbf{A}'$, we have

$$1' = 1 = (x \,.\, x^{-1})^\varphi = x^\varphi \,.\, (x^{-1})^\varphi = x^\varphi \,.\, 0' = 0',$$

which is impossible, since we suppose that $\mathbf{A}'$ contains at least two different elements. Thus $x \in \mathbf{O}_\Theta$ and therefore $\mathbf{R} \subseteqq \mathbf{O}_\Theta$.

A place is said to be *trivial,* if it is an isomorphism, i.e. $\mathbf{A} = \mathbf{O}_\Theta$ and $I_\Theta = \{0\}$.

Every place of the alternative field $\mathbf{A}$ induces a place of each alternative field $\mathbf{A}_1$ contained in $\mathbf{A}$. If $\mathbf{A}_1 \subseteqq \mathbf{O}_\Theta$, the place is trivial.

The set of all places of the given alternative field $\mathbf{A}$ can be decomposed into equivalence classes as so: two places $\Theta : \mathbf{A} \to \mathbf{A}_1 \cup \{\infty\}$, $\varphi : \mathbf{A} \to \mathbf{A}_2 \cup \{\infty\}$ are said to be *equivalent* ($\Theta \sim \varphi$), if there exists an isomorphism $\lambda : \mathbf{A}_1 \to \mathbf{A}_2$ of alternative fields such that $\varphi = \Theta \circ \lambda$. As in the associative commutative case, $\Theta \sim \varphi$ if and only if $\mathbf{O}_\Theta = \mathbf{O}_\varphi$.

Also a notion of specialisation can be introduced as in the classical case.

Definition.

Let Θ, Θ' be places of an alternative field $\mathbf{A}$. We say that Θ' is a *specialisation* of Θ

(and we write $\Theta \to \Theta'$), if $\mathbf{O}_{\Theta'} \subseteqq \mathbf{O}_\Theta$.

It can be verified, that $\Theta \to \Theta'$ if and only if one of the following equivalent conditions holds:

(i) $x^{\Theta'} \neq \infty \Rightarrow x^\Theta \neq \infty$;

(ii) $x^\Theta = 0' \Rightarrow x^{\Theta'} = 0''$.

In the other words, $\Theta \to \Theta'$ if and only if

$$I_\Theta \subseteqq I_{\Theta'} \quad \text{and} \quad \mathbf{O}_{\Theta'} \subseteqq \mathbf{O}_\Theta \ldots \text{(B)}.$$

Note that each place is a specialisation of a trivial place. Further, φ and Θ are equivalent, if and only if $\varphi \to \Theta$ and $\Theta \to \varphi$. This assertion is generalized in the following:

[5]) Such place is unique up to an equivalence relation, defined later on.

Theorem 5.

Let $\Theta_1 : \mathbf{A} \to \mathbf{A}_1 \cup \{\infty\}$, $\Theta_2 : \mathbf{A} \to \mathbf{A}_2 \cup \{\infty\}$ are places of the alternative field $\mathbf{A}$. Then $\Theta_1 \to \Theta_2$ if and only if there exists a place $\varphi : \mathbf{A}_1 \to \mathbf{A}_2 \cup \{\infty\}$ such that $\Theta_2 = \Theta_1 \circ \varphi$ is true on $\mathbf{O}_{\Theta_2}$.

Proof. Let $\Theta_1 \to \Theta_2$. Then $\mathbf{R} = (\mathbf{O}_{\Theta_2})^{\Theta_1}$ is a subring in $\mathbf{A}_1$. Let us define a mapping $\varphi : \mathbf{A}_1 \to \mathbf{A}_2 \cup \{\infty\}$ in this way:

$$x^{\varphi} := \infty \qquad \text{for } x \in \mathbf{A}_1 \dot{-} \mathbf{R},$$

$$x^{\varphi} := \xi^{\Theta_2}, \qquad \text{where} \qquad \xi \in \mathbf{O}_{\Theta_2} \qquad \text{and} \qquad \xi^{\Theta_1} = x, \qquad \text{for } x \in \mathbf{R}.$$

We must show that this definition is correct. Suppose that $\xi^{\Theta_1} = \eta^{\Theta_1} = x$ for some $\xi, \eta \in \mathbf{O}_{\Theta_2}$. Thus $(\eta - \xi)^{\Theta_1} = \eta^{\Theta_1} -' \xi^{\Theta_1} = 0'$. This implies $\eta - \xi \in I_{\Theta_1}$. Since $I_{\Theta_1} \subseteqq I_{\Theta_2}$, we have $0' = (\eta - \xi)^{\Theta_2} = \eta^{\Theta_2} - '\xi^{\Theta_2}$. Therefore $\eta^{\Theta_2} = \xi^{\Theta_2}$. It can be verified that φ is a demanded place. Moreover, φ is uniquely determined by Θ_1, Θ_2.

Conversely, let $\Theta_2 = \Theta_1 \circ \varphi$ is true on $\mathbf{O}_{\Theta_2}$, $\varphi : \mathbf{A}_1 \to \mathbf{A}_2 \cup \{\infty\}$ being a place. If $x \in \mathbf{O}_{\Theta_2}$, i.e. $x^{\Theta_2} \neq \infty$, then $(x^{\Theta_1})^{\varphi} \neq \infty$ and therefore $x^{\Theta_1} \in \mathbf{O}_{\varphi} \subseteqq \mathbf{A}_1$. This implies that $x^{\Theta_1} \neq \infty$ and $x \in \mathbf{O}_{\Theta_1}$. Thus $\mathbf{O}_{\Theta_2} \subset \mathbf{O}_{\Theta_1}$, hence $\Theta_1 \to \Theta_2$.

The mappings Θ_2 and $\Theta_1 \circ \varphi$ are identical on the whole $\mathbf{O}_{\Theta_1}$ in the following sence: if $x \in \mathbf{O}_{\Theta_1} \dot{-} \mathbf{O}_{\Theta_2}$, then $x^{\Theta_2} = (x^{\Theta_1})^{\varphi} = \infty$. Really, if $x \notin \mathbf{O}_{\Theta_2}$, then $(x^{-1})^{\Theta_2} = 0$, thus $x^{-1} \in \mathbf{O}_{\Theta_2}$ and $(x^{-1})^{\Theta_1 \circ \varphi} = ((x^{-1})^{\Theta_1})^{\varphi} = 0$, which yealds the previous result.

If Θ, Θ' are equivalent, then φ is trivial. The following theorem solves a problem of finding all places φ such that the given place Θ is a specialisation of φ ($\varphi \to \Theta$).

Theorem 6.

Let $\mathbf{A}$ be an alternative field containing a total subring $\mathbf{O}$. Each subring $\mathbf{R}$ in $\mathbf{A}$ such that $\mathbf{O} \subseteqq \mathbf{R}$, can be expressed in the form

$$\mathbf{R} = \{x \in \mathbf{A} / x \in a \,.\, b^{-1} \text{ for some } a, b \in \mathbf{O},\ b \notin \mathfrak{M}\},$$

where $\mathfrak{M}$ is a prime-ideal in $\mathbf{O}$.

Proof. $\mathbf{R}$ is total in $\mathbf{A}$, for it contains a total ring $\mathbf{O}$. By Theorem 3, there exist places Θ and φ of $\mathbf{A}$ such that $\mathbf{O}_{\Theta} = \mathbf{O}$, $\mathbf{O}_{\varphi} = \mathbf{R}$ and $\varphi \to \Theta$. Since $I_{\varphi} \subseteqq I_{\Theta}$, I_{φ} is a prime-ideal in $\mathbf{O}_{\Theta}$. Let $\mathbf{Z} = \{x \in \mathbf{A}/x = a \,.\, b^{-1};\ a, b \in \mathbf{O}_{\Theta},\ b \notin I_{\varphi}\}$. Since $a \in \mathbf{O}_{\varphi}$, $b \in \mathbf{O}_{\Theta} \subseteqq \mathbf{O}_{\varphi}$ and $b \notin I_{\varphi}$, we have $b \in U_{\varphi}$ and $b^{-1} \in U_{\varphi} \subseteqq \mathbf{O}_{\varphi}$. Thus the product $a \,.\, b^{-1} \in \mathbf{O}_{\varphi}$ and $\mathbf{Z} \subseteqq \mathbf{O}_{\varphi}$. Now we shall prove, that the converse inclusion is also true. Suppose $x \in \mathbf{O}_{\varphi}$. If $x \in \mathbf{O}_{\Theta}$, then $x \in \mathbf{Z}$, since $x = x \,.\, 1^{-1}$ and $1^{-1} = 1 \notin I_{\varphi}$. Let $x \notin \mathbf{O}_{\Theta}$; then $x^{-1} \in \mathbf{O}_{\Theta}$ and $x \notin I_{\varphi}$. Thus $x, x^{-1} \in U_{\varphi}$. The element $y = x^{-1}$ satisfies $y^{-1} \in U_{\varphi}$ and $y^{-1} \notin I_{\varphi}$. Now we can write $x = 1 \,.\, y^{-1} \in Z$. Hence $\mathbf{Z} = \mathbf{O}_{\varphi} = \mathbf{R}$.

Similarly, it can be proved that

$$\mathbf{R} = \{x \in \mathbf{A}/x = b^{-1} \,.\, a;\ a \in \mathbf{O},\ b \in \mathbf{O} \dot{-} \mathfrak{M}\}$$

for a certain ideal $\mathfrak{M}$ in $\mathbf{O}$.

Theorem 8.

Let $\mathfrak{M}_1$ and $\mathfrak{M}_2$ be left ideals in a total ring $\mathbf{O}$ of the alternative field $\mathbf{A}$. Then either $\mathfrak{M}_1 \subseteqq \mathfrak{M}_2$, or $\mathfrak{M}_2 \subseteqq \mathfrak{M}_1$.

Proof. Suppose $\mathfrak{M}_1 \nsubseteq \mathfrak{M}_2$. Then there exists $x \in \mathfrak{M}_1 \dot{-} \mathfrak{M}_2$. Let $y \in \mathfrak{M}_2, y \neq 0$. Then $x \, . \, y^{-1} \notin \mathbf{O}$. In fact, the assumption $x \, . \, y^{-1} \in \mathbf{O}$ would imply $x = (x \, . \, y^{-1}) \, . \, y \in \in \mathfrak{M}_2$, a contradiction. From this follows $y \, . \, x^{-1} = (x \, . \, y^{-1})^{-1} \in \mathbf{O}$. Further, $y = = (y \, . \, x^{-1}) \, . \, x \in \mathfrak{M}_1$ and thus $\mathfrak{M}_2 \subseteqq \mathfrak{M}_1$.

Remark.

An analogous theorem holds for right and for both-sided ideals, too.

Corollary.

A set of all subrings in an alternative field, containing the given total ring, is fully ordered by the set inclusion.

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SOUHRN

UMÍSTĚNÍ ALTERNATIVNÍCH TĚLES

ALENA VANŽUROVÁ

V článku se studuje vztah mezi umístěními a totálními okruhy alternativních těles. Dokázané výsledky jsou obdobné větám platným pro umístění a valuační okruhy komutativních těles.

РЕЗЮМЕ

ТОЧКИ АЛЬТЕРНАТИВНЫХ ТЕЛ

АЛЕНА ВАНЖУРОВА

В статье изучается связь между точками и тотальными кольцами альтернативных тел. Достигнутые результаты аналогичны теоремам известным для точек и колец нормирования полей.