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ON INFLECTION POINTS OF SOME TYPE OF QUARTICS WITH A SINGLE TRIPLE POINT

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Let us consider a plane algebraic curve (**K**) of order 4 possessing a single three-fold point A. Let us suppose that this quartic (**K**) is situated in a complex projective plane S_2. There exist three types of such point-singularities.

(a) A is an ordinary triple point, i.e. A is the centre of three distinct linear places with different tangents.

(b) A is a centre of one quadratic place and one linear place with different tangents.

(c) A is a three-fold cusp, i.e. A is a centre of just one place of degree three.

By a suitable choice of a coordinate frame in the plane S_2 and by a simple analysis of places with the centre A and according to the Hessian of the curve (**K**) we establish the divisor of the inflection points to be of order 6 in the case (a), of order 4 in the case (b) and of order 2 in the case (c). As an illustration we will carry out the analysis of the case (b) to which our paper is devoted.

Let the coordinate frame (A_0, A_1, A_2, E) in S_2 be chosen so that $A_0 = A$, A_0A_1 is the tangent of the quadratic place and A_0A_2 is the tangent of the linear place with common centre A. Then the curve (**K**) is determined by the form

$$\mathbf{K}(x_0, x_1, x_2) = x_1 x_2^2 x_0 + \mathbf{u}(x_1, x_2), \tag{1}$$

where

$$\mathbf{u}(x_1, x_2) = (\alpha x_1^4 + \beta x_1^3 x_2 + \gamma x_1^2 x_2^2 + \delta x_1 x_2^3 + \varepsilon x_2^4), \tag{2}$$
$$\alpha \neq 0, \qquad \varepsilon \neq 0.$$

The places P_1 (the quadratic place), P_2 (the linear place) with the centre A have the following analytic expressions (parametrisations):

$$\begin{aligned} P_1 : \bar{x}_0 &= 1, & P_2 : \bar{x}_0' &= 1, \\ \bar{x}_1 &= t^2, & \bar{x}_1' &= t^2(m' + \dots), \\ \bar{x}_2 &= t^3(m + \dots), & \bar{x}_2' &= t, \\ m &\neq 0, m^2 + \alpha = 0, & m' &= 0, m' + \varepsilon \neq 0. \end{aligned} \tag{3}$$

By a mechanical counting we obtain for the Hessian **H** of the form **K**

$$\mathbf{H}(x_0, x_1, x_2) = 6x_2^2[x_0x_1x_2^2 + (-8\alpha x_1^4 - 2\beta x_1^3x_2 + \gamma x_1^2x_2^2 + \delta x_1x_2^3 - 2\varepsilon x_2^4)].$$

It follows from (4) and (3) that $\mathbf{H}(\bar{x}_0, \bar{x}_1, \bar{x}_2) = 54\alpha^2t^{14}$ + members of degree $\geqq 15$, hence the order $\mathsf{O}_{\mathsf{P}_1}(\mathbf{H})$ of the place P_1 on the form $\mathbf{H}$ equals 14; $\mathbf{H}(\bar{x}_0', \bar{x}_1', \bar{x}_2') = -18\varepsilon t^6$ + members of degree $\geqq 7$, hence $\mathsf{O}_{\mathsf{P}_2}(\mathbf{H}) = 6$.

Now from (1), (2) and (4) we get

$$\mathbf{H}(x_0, x_1, x_2) + 12x_2^2\mathbf{K}(x_0, x_1, x_2) = 18x_1x_2^2\vartheta(x_0, x_1, x_2), \tag{5}$$

where

$$\vartheta(x_0, x_1, x_2) = x_0x_2^2 + (-2\alpha x_1^3 + \gamma x_1^2x_2^2 + \delta x_2^3), \tag{6}$$

which implices that the inflection-point divisor I of the curve (**K**) is on (**K**) cut out by the cubic form ϑ. Using (3) and (6) we may easily find that the form ϑ determines the divisor

$$6\mathsf{P}_1 + 2\mathsf{P}_2 + \mathsf{I}$$

on (**K**).

It is a natural question whether the divisor I may be determined by a linear form $\boldsymbol{l}$ (c.f. [1]—[6]) on the curve (**K**). But in this case the line ($\boldsymbol{l}$) must be a component of the cubic (ϑ) and consequently ($\boldsymbol{l}$) must contain the unique singular point $\boldsymbol{A}$ of (ϑ). Hence ($\boldsymbol{l}$) intersects (**K**) into a divisor of order at least 7, which is a contradiction.

We have proved the following

Theorem. *Let* (**K**) *be a plane quartic possessing a single triple point* **A** *which is a centre of one quadratic place* P_1 *and of one linear place* P_2 *with different tangents. Then the divisor* I *of inflection points of* (**K**) *is of order 4 and there exists a cubic form* ϑ *determining the divisor* $6\mathsf{P}_1 + 2\mathsf{P}_2 + \mathsf{I}$ *on* (**K**) *but there doesn't exist any line* ($\boldsymbol{l}$) *intersecting* (**K**) *into divisor* I.

Remark: The first polar of the point $\boldsymbol{A}_2$ is the form $\mathbf{K}_2 = \partial\mathbf{K}/\partial x_2$. By the method described in [5] pg. 115—117 we find that our quartic (**K**) is of classes $m = 6$ (for the case (a) "in generally"), $m = 4$ (for the case (c)).

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INFLEXNÍ BODY NĚKTERÝCH TYPŮ KVARTIK S JEDNÍM TROJNÁSOBNÝM BODEM

Souhrn

V článku se vyšetřuje divizor I inflexních bodů kvartiky s trojnásobným bodem, který je středem jedné kvadratické a jedné lineární větve o různých tečnách. Divizor inflexních bodů uvažované kvartiky má řád 4. Je dokázáno, že I není nikdy určen lineární formou, avšak divizor $6P_1 + 2P_2 + I$, kde P_1 je kvadratická a P_2 lineární větev se středem v trojnásobném bodě kvartiky, je určen formou kubickou.

ТОЧКИ ПЕРЕГИБА НЕКОТОРОГО ТИПА АЛГЕБРАИЧЕСКОЙ КРИВОЙ 4-ОГО ПОРЯДКА ОБЛАДАЮЩЕЙ ОДНОЙ ТРЕХКРАТНОЙ ТОЧКОЙ

Резюме

В статье рассматривается дивизор I точек перегиба алгебраической кривой (K) 4-го порядка с одной трехкратной точкой ***A*** являющейся центром одной квадратичной и одной линейной ветви с различными касательными. ***I*** имеет порядок 4. О дивизоре I доказывается, что он никогда необразует полное пересечение с кривой (K), но всегда существует форма 3-го порядка определяющая на нашей кривой дивизор $6P_1 + 2P_2 + I$ (здесь P_1 и P_2 означают по очереди квадратичную и линейную ветв с центром в точке ***A***).