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EXISTENCE THEOREMS FOR ALMOST PERIODIC SOLUTIONS OF FIRST-ORDER DIFFERENTIAL EQUATIONS

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1. In this paper there are given existence criterions for almost periodic solutions of the scalar differential equation

$$x' = f(t,x), \tag{1}$$

where $f: R \times I \to R$ is an almost periodic function in the variable t uniformly for $x \in I$, $I = \langle \alpha, \beta \rangle$, $-\infty < \alpha < \beta < \infty$ (see e.g. [5]), by the method of subsolutions and supersolutions of (1). There are given applications of this criterions to concrete differential equations.

2. We shall prove the following lemma, which will be used later.

Lemma 1. Let u, v be almost periodic C^1-functions satisfying

$$\alpha \le v(t) \le u(t) \le \beta \qquad \text{for } t \in R , \tag{2}$$

and either

$$u'(t) \leqq f(t,u(t)), \quad v'(t) \geqq f(t,v(t)) \quad \text{for } t\in R \ , \tag{3}$$

or

$$u'(t) \geqq f(t,u(t)), \quad v'(t) \leqq f(t,v(t)) \quad \text{for } t\in R \ . \tag{4}$$

If every Cauchy problem for equation (1) has (locally) the unique solution and $u(t_0) = v(t_0)$ for some t_0, $t_0 \in R$, then

$$u(t) = v(t) \quad \text{for } t\in R. \tag{5}$$

Proof. Let y be the solution of (1), $y(t_0) = u(t_0)$ $(=v(t_0))$ for some $t_0 \in R$. Using (3) ((4)) and a differential inequality theorem (see e.g. [1]) we obtain $u(t) \geqq y(t) \geqq v(t)$ $(v(t) \geqq y(t) \geqq u(t))$ for $t\in(-\infty,t_0\rangle$ and $v(t) \geqq y(t) \geqq u(t)$ $(u(t) \geqq y(t) \geqq v(t))$ for $t\in\langle t_0,\infty)$. Then $u(t) = v(t)$ for $t\in\langle t_0,\infty)$ $(u(t) = v(t)$ for $t\in(-\infty,t_0\rangle)$ by (2) and since u-v is an almost periodic function, the equality (5) holds.

Theorem 1. Let $f: R \times I \to R$ be an almost periodic function in the variable t uniformly for $x\in I$. Assume that there exist almost periodic C^1-functions u, v satisfying (2) and (3). If $\frac{\partial f}{\partial x}(t,x)$ exists on $H := \{(t,x);\ t\in R,\ v(t) \leqq x \leqq u(t)\} \subset R \times I$ and

$$0 < m \leqq \frac{\partial f}{\partial x}(t,x) \leqq M \text{ for } (t,x)\in H, \tag{6}$$

where m, M are positive constants, then equation (1) has an almost periodic solution in H.

Proof. If $u(t_0) = v(t_0)$ for some t_0, $t_0\in R$, then $u = v$ $(:=y)$ by Lemma 1 and y is an almost periodic solution in H of (1).

Assume $v(t) < u(t)$ for $t\in R$. Let $t_1\in R$ be a number and let y_1 be the solution (on R) of the Cauchy problem

$$y' - ay = f(t,u(t)) - au(t),$$
$$y(t_1) = u(t_1),$$

with a positive constant a, $\sup_{(t,x)\in H} \frac{\partial f}{\partial x}(t,x) < a$. Using (3) we get

$$(y_1(t) - u(t))' \geq a(y_1(t) - u(t)) \text{ for } t\in R$$

and thus $y_1(t) \leq u(t)$ for $t\in(-\infty,t_1\rangle$, $y_1(t) \geq u(t)$ for $t\in\langle t_1,\infty)$. To prove $v(t) < y_1(t)$ on R suppose $y_1(t_0) = v(t_0)$ for some $t_0\in R$, $t_0 < t_1$, and $y_1(t) > v(t)$ for $t\in(t_0,\infty)$. Then

$$y_1'(t_0) \geq v'(t_0) . \tag{7}$$

On the other hand

$$y_1'(t_0)-v'(t_0) \leq f(t_0,u(t_0))-f(t_0,v(t_0))+a(v(t_0)-u(t_0)) =$$

$$= (\frac{\partial f}{\partial x}(t_0,\xi)-a)(u(t_0)-v(t_0)) ,$$

where $\xi\in(v(t_0), u(t_0))$ is an appropriate number, consequently, $y_1'(t_0)-v'(t_0)< 0$ contradicting (7).

Let $\{t_n\}$, $t_n\in R$, be an increasing sequence, $\lim_{n\to\infty} t_n = \infty$ and y_n be the solution of the Cauchy problem

$$y' - ay = f(t,u(t)) - au(t),$$

$$y(t_n) = u(t_n).$$

Then, of course, $v(t)< y_n(t) \leq y_{n+1}(t)$ for $t\in R$, $n\in N$, and $y_n(t) \leq u(t)$ for $t\in(-\infty,t_n\rangle$, $y_n(t) \geq u(t)$ for $t\in\langle t_n,\infty)$, $n\in N$. Thus there exists $\lim_{n\to\infty} y_n(t) = y(t)$ for $t\in R$ and

$$v(t) \leq y(t) \leq u(t) \quad \text{for } t\in R. \tag{8}$$

Since u, v are bounded, the sequence $\{y_n(t)\}$ is locally uniformly bounded on R and from the equalities $y_n'(t) = f(t,u(t)) + a(y_n(t)-u(t))$, $t\in R$, $n\in N$, it follows that $\{y_n'(t)\}$ is locally uniformly bounded on R, too. By the Ascoli's theorem $\lim_{n\to\infty} y_n(t) = y(t)$ locally uniformly on R and taking the limit for $n \to\infty$ in the equalities

$$y_n(t) = y_n(0) + \int_0^t [a(y_n(s)-u(s)) + f(s,u(s))]ds, \quad t\in R, \quad n\in N,$$

we see y is a solution in H of the equation

$$y' - ay = f(t,u(t)) - au(t).$$

The solution y is bounded by (8) and since $f(t,u(t))-au(t)$ is an almost periodic function (see Theorem 2.7, [5]), y is also almost periodic (see [2], p.124-126).

Putting $z_1:=y$, then

$$z_1'(t)-f(t,z_1(t)) = az_1(t)+f(t,u(t))-au(t)-f(t,z_1(t)) =$$
$$= (\frac{\partial f}{\partial x}(t, \xi_1(t))-a)(u(t)-z_1(t)) ,$$

where $\xi_1(t)$ lies in the interval with the end points $z_1(t)$, $u(t)$, thus

$$z_1'(t) \leqq f(t,z_1(t)) \quad \text{for } t\in R.$$

If we take the almost periodic C^1-function z_1 in place of u, assumptions (2), (3) are satisfied and as above we may prove the existence of an almost periodic function z_2 such that $v(t) \leqq z_2(t) \leqq z_1(t) \leqq u(t)$, $z_2'(t) \leqq f(t,z_2(t))$ for $t\in R$ and z_2 is a solution of the equation

$$y' - ay = f(t,z_1(t)) - az_1(t) .$$

By this method we obtain a sequence $\{z_n(t)\}$ of almost periodic C^1-functions z_n such that $v(t) \leqq z_{n+1}(t) \leqq z_n(t) \leqq u(t)$ for $t\in R$ and z_{n+1} is a solution of the equation

$$y' - ay = f(t,z_n(t)) - az_n(t)$$

for all $n\in N$. The sequences $\{z_n(t)\}$, $\{z_n'(t)\}$ are uniformly bounded on R and therefore

$$\lim_{n\to\infty} z_n(t) = z(t) \tag{10}$$

locally uniformly on R. Taking the limit for $n \to \infty$ in the equalities

$$z_{n+1}(t) = z_{n+1}(0) + \int_0^t [f(s,z_n(s))+a(z_{n+1}(s)-z_n(s))]ds, \quad t\in R, \quad n\in N,$$

we see that z is a solution in H of (1).

If $z_{n_0}(t_0) = z_{n_0+1}(t_0)$ for some $t_0 \in R$ and some $n_0 \in N$, then $z_n = z$ for all $n \geq n_0$ by Lemma 1 and z is an almost periodic solution in H of (1).

In the opposite case $z_{n+1}(t) < z_n(t)$ for $t \in R$ and all $n \in N$. Putting $w_n := z_n - z_{n+1}$, then we have

$$w_n'(t) - aw_n(t) = f(t,z_{n-1}(t)) - f(t,z_n(t)) - a(z_{n-1}(t) - $$

$$- z_n(t)) = (\frac{\partial f}{\partial x}(t, \eta_n(t)) - a)w_{n-1}(t) \ ,$$

where $\eta_n(t) \in (z_{n+1}(t), z_n(t))$ is an appropriate number. From

$$w_n'(t) - aw_n(t) = (\frac{\partial f}{\partial x}(t, \eta_n(t)) - a)w_{n-1}(t)$$

we obtain

$$w_n(t) = e^{at}\left[w_n(0) - \int_0^t e^{-as}(a - \frac{\partial f}{\partial x}(s, \eta_n(s))w_{n-1}(s)ds\right] \ ,$$

and since w_n is bounded on R and $\lim_{t \to \infty} e^{at} = \infty$,

$$w_n(0) = \int_0^\infty e^{-as}(a - \frac{\partial f}{\partial x}(s, \eta_n(s))w_{n-1}(s)ds$$

and thus

$$w_n(t) = e^{at}\int_t^\infty e^{-as}(a - \frac{\partial f}{\partial x}(s, \eta_n(s))w_{n-1}(s)ds, \quad t \in R.$$

Since $a - \frac{\partial f}{\partial x}(t, \eta_n(t)) \leq a-m$, we have

$$w_n(t) \leq (a-m)e^{at}\sup_{t \in R} w_{n-1}(t)\int_t^\infty e^{-as}ds = (1 - \frac{m}{a})\sup_{t \in R} w_{n-1}(t),$$

consequently,

$$\sup_{t\in R} w_n(t) \leqq \left(1-\frac{m}{a}\right)\sup_{t\in R} w_{n-1}(t)$$

and

$$\sup_{t\in R} w_n(t) \leqq \left(1-\frac{m}{a}\right)^{n-1} \sup_{t\in R} w_1(t) \quad \text{for all } n\in N.$$

Then $\sum_{n=1}^{\infty} w_n(t)$ is convergent uniformly on R, thus (10) holds uniformly on R and z is an almost periodic solution in H of (1). This completes the proof.

<u>Example 1</u>. Consider the Riccati equation

$$x' = a(t)x^2 + b(t)x + c(t), \tag{11}$$

where a, b, c are almost periodic functions, $b(t) \geqq m > 0$, $a(t) \geqq 0$, $c(t) \leqq 0$ for $t\in R$, m is a positive constant. Putting $v(t):= 0$, $u(t):= A$, $H_1:= R \times \langle 0,A\rangle$, where $A = \sup_{t\in R}\left(-\frac{c(t)}{m}\right)$ $(\geqq 0)$, then we have $u'(t)-f(t,u(t)) = -A^2a(t) - Ab(t) - c(t) \leqq - mA - c(t) \leqq 0$, $v'(t)-f(t,v(t)) = - c(t) \geqq 0$ for $t\in R$ and $\frac{\partial f}{\partial x}(t,x) = 2a(t)x + b(t) \geqq m$ for $(t,x)\in H_1$. Thus the assumptions of Theorem 1 are satisfied and there exists an almost periodic solution in H_1 of (11).

<u>Example 2</u>. Consider the Riccati equation

$$x' = x^2 + q(t) , \tag{12}$$

where q is an almost peirodic function, $q(t) \leqq -m < 0$ for $t\in R$, m is a positive constant. Let $A:= \sup_{t\in R}\sqrt{-q(t)}$, $B:= \inf_{t\in R}\sqrt{-q(t)}$. The functions $u(t):=A$, $v(t):=B$ for $t\in R$ satisfy assumptions of Theorem 1 and $2B \leqq \frac{\partial f}{\partial x}(t,x) = 2x \leqq 2A$ for $(t,x)\in H_1:=R \times \langle B,A\rangle$. Thus there exists an almost periodic solution in H_1 of (12).

<u>Remark 1</u>. If the equation $y'' = q(t)y$ with an almost periodic coefficient q is generally (specially) disconjugate on R, then equation (12) has exactly two (one) almost periodic solutions (see [3], [4]).

Example 3. Consider the equation

$$x' = \mu x + \lambda x^{2n+1} + g(t,x) , \tag{13}$$

where $g: H:=R \times \langle \alpha, \beta \rangle \longrightarrow R$ is an almost periodic function in the variable t uniformly for $x \in \langle \alpha, \beta \rangle$ $(-\infty < \alpha < \beta < \infty)$ and n is a positive integer. Let $A = \sup_{(t,x)\in H} |g(t,x)|$, $m \leqq \frac{\partial g}{\partial x}(t,x) \leqq M$ for $(t,x) \in H$ with constants m, M and let $\mu > 0$, $\lambda > 0$ be such positive constants that $\mu > -m$, $\left\langle -(\frac{A}{\lambda})^{\frac{1}{2n+1}}, (\frac{A}{\lambda})^{\frac{1}{2n+1}} \right\rangle \subset \langle \alpha, \beta \rangle$. Putting $u(t):= (\frac{A}{\lambda})^{\frac{1}{2n+1}}$, $v(t):=-(\frac{A}{\lambda})^{\frac{1}{2n+1}}$ for $t \in R$, then $u'(t) - \left[\mu u(t) + \lambda (u(t))^{2n+1} + g(t,u(t))\right] = -\left[\mu (\frac{A}{\lambda})^{\frac{1}{2n+1}} + A + g(t,(\frac{A}{\lambda})^{\frac{1}{2n+1}})\right] \leqq 0$, $v'(t) - \left[\mu v(t) + \lambda (v(t))^{2n+1} + g(t,v(t))\right] = -\left[-\mu (\frac{A}{\lambda})^{\frac{1}{2n+1}} - A + g(t,-(\frac{A}{\lambda})^{\frac{1}{2n+1}})\right] \geqq 0$ for $t \in R$ and for $\frac{\partial}{\partial x}\left[\mu x + \lambda x^{2n+1} + g(t,x)\right] = \mu + \lambda (2n+1)x^{2n} + \frac{\partial g}{\partial x}(t,x)$ we have

$$\mu + \lambda (2n+1)(\frac{A}{\lambda})^{\frac{2n}{2n+1}} + M \geqq \mu + \lambda (2n+1)x^{2n} + \frac{\partial g}{\partial x}(t,x) \geqq$$

$$\geqq \mu + m, \quad (t,x) \in H.$$

By Theorem 1 there exists an almost periodic solution in H of (13).

Theorem 2. Let $f: R \times I \longrightarrow R$ be almost periodic function in the variable t uniformly for $x \in I$. Assume that there exist almost periodic C^1-functions u, v satisfying (2) and (4). If $\frac{\partial f}{\partial x}(t,x)$ exists on H, where H is defined in Theorem 1, and for positive constants m, M the inequalities

$$-M \leqq \frac{\partial f}{\partial x}(t,x) \leqq -m, \quad (t,x) \in H, \tag{14}$$

hold, then equation (1) has an almost periodic solution in H.

Proof. The proof is similar to that of Theorem 1 and therefore we are not going to all details.

Assume, without loss of generality, that $v(t) < u(t)$ in R. Let $t_1 \in R$ be a number and y_1 be the solution (on R) of the Cauchy problem

$$y' + ay = f(t,u(t)) + au(t) ,$$
$$y(t_1) = u(t_1) ,$$

where a is a positive constant, $-a < \inf_{(t,x)\in H} \frac{\partial f}{\partial x}(t,x)$. Then $(y_1(t)-u(t))' \leqq -a(y_1(t)-u(t))$, consequently, $y_1(t) \geqq u(t)$ for $t \in (-\infty,t_1\rangle$, $y_1(t) \leqq u(t)$ for $t \in \langle t_1,\infty)$. If $y_1(t_0) = v(t_0)$ for some t_0, $t_0 > t_1$ and $y_1(t) > v(t)$ for $t \in (-\infty,t_0)$, then

$$y_1'(t_0) \leqq v'(t_0) \quad . \tag{15}$$

On the other hand $y_1'(t_0)-v'(t_0) \geqq f(t_0,u(t_0))-f(t_0,v(t_0)) +$

$+ a(u(t_0)-v(t_0)) = (\frac{\partial f}{\partial x}(t_0,\xi)+a)(u(t_0)-v(t_0)) > 0$, contradicting (15), consequently, $v(t) < y_1(t)$ on R.

Let $\{t_n\}$, $t_n \in R$, be a decreasing sequence, $\lim_{n\to\infty} t_n = -\infty$ and y_n be the solution of the Cauchy problem

$$y' + ay = f(t,u(t)) + au(t),$$
$$y(t_n) = u(t_n) .$$

Then $v(t) < y_{n+1}(t) \leqq y_n(t)$ for $t \in R$. $y_n(t) \geqq u(t)$ for $t \in (-\infty,t_n\rangle$, $y_n(t) \leqq u(t)$ for $t \in \langle t_n,\infty)$ and all $n \in N$. The function z_1 defined by $z_1(t) := \lim_{n\to\infty} y_n(t)$ for $t \in R$, is an almost periodic solution of the equation

$$y' + ay = f(t,u(t)) + au(t)$$

satisfying $v(t) \leqq z_1(t) \leqq u(t)$, and $z_1'(t) - f(t,z_1(t)) \geqq 0$ for $t \in R$. If we take the function z_1 in place of u, then assumptions (2) and (4) hold and there exists an almost periodic solution z_2 of the equation

$$y' + ay = f(t,z_1(t)) + az_1(t)$$

such that $v(t) \leqq z_2(t) \leqq z_1(t) \leqq u(t)$, $z_2'(t) - f(t,z_2(t)) \geqq 0$ for $t \in R$. By this method we obtain a sequence $\{z_n(t)\}$ of almost periodic functions z_n, $v(t) \leqq z_{n+1}(t) \leqq z_n(t) \leqq u(t)$ for $t \in R$, z_{n+1} is a solution of the equation

$$y' + ay = f(t,z_n(t)) + az_n(t)$$

and

$$\lim_{n\to\infty} z_n(t) = z(t) \tag{16}$$

locally uniformly on R. The function z is a solution of (1). Putting $w_n := z_n - z_{n+1}$, then

$$w_n'(t) + aw_n(t) = f(t,z_{n-1}(t)) - f(t,z_n(t)) + aw_{n-1}(t) =$$
$$= \left(\frac{\partial f}{\partial x}(t,\eta_n(t))+a\right)w_{n-1}(t) ,$$

where $\eta_n(t)$ lies in the interval with the end points $z_{n-1}(t)$, $z_n(t)$, thus

$$w_n(t) = e^{-at}\left[w_n(0) + \int_0^t e^{as}\left(\frac{\partial f}{\partial x}(s,\eta_n(s))+a\right)w_{n-1}(s)ds\right] .$$

Since w_n is bounded on R and $\lim_{t\to-\infty} e^{-at} = \infty$, $w_n(0) = = -\int_0^{-\infty} e^{as}\left(\frac{\partial f}{\partial x}(s,\eta_n(s))+a\right)w_{n-1}(s)ds$ and $w_n(t) = e^{-at}\int_{-\infty}^t e^{as}\left(\frac{\partial f}{\partial x}(s, \eta_n(s))+a\right)w_{n-1}(s)ds$. Therefore

$$w_n(t) \leqq \left(1 - \frac{m}{a}\right)\sup_{t\in R} w_{n-1}(t)$$

and

$$\sup_{t\in R} w_n(t) \leqq \left(1 - \frac{m}{a}\right)^{n-1} \sup_{t\in R} w_1(t) \text{ for } n \in N.$$

Since $\sum_{n=1}^{\infty} w_n(t)$ is convergent uniformly on R, (16) holds uniformly on R and z is an almost periodic solution in H of (1).

Example 4. Consider the equation

$$x' = \mu + \lambda x^{\nu} + g(t,x) \ , \tag{17}$$

where $g(t,x)$ is an almost periodic function in the variable t uniformly for $x \in \langle 0,1 \rangle$. Let $g(t,1) \leqq A$, $g(t,0) \geqq -A$ for $t \in R$, $m \leqq \frac{\partial g}{\partial x}(t,x) \leqq M < 0$ for $(t,x) \in H_2 := R \times \langle 0,1 \rangle$ with constants $A > 0$, m, M. Let μ, λ, $\nu \in R$, $\nu \geqq 1$, $\mu \geqq A$ and $\lambda < -A - \mu$. Putting $v := 0$, $u := 1$, then $u' - \mu - \lambda u^{\nu} - g(t,u) \geqq -\mu - \lambda - A \geqq 0$, $v' - \mu - \lambda v^{\nu} - g(t,v) \leqq -\mu + A \leqq 0$ and $\nu \lambda + m \leqq \frac{\partial}{\partial x}(\mu + \lambda x^{\nu} + g(t,x)) = \lambda \nu x^{\nu - 1} + \frac{\partial g}{\partial x}(t,x) \leqq M < 0$ for $(t,x) \in H_2$. By Theorem 2 there exists an almost periodic solution in H_2 of (17).

Example 5. Consider Riccati equation (12) by the same assumptions on q as in Example 2. The function $u(t) := A_1$, $v(t) := B_1$ for $t \in R$ with $A_1 = -\inf\sqrt{-q(t)}$, $B_1 = -\sup\sqrt{-q(t)}$ satisfy the assumptions of Theorem 2 and $2B_1 \leqq \frac{\partial f}{\partial x}(t,x) = 2x \leqq 2A_1$ for $(t,x) \in H_2 := R \times \langle B_1, A_1 \rangle$. Thus there exists an almost periodic solution in H_2 of (12).

Remark 2. From Examples 2 and 5 it follows that Riccati equation (12) with an almost periodic function q and $\inf_{t \in R} q(t) < 0$ has two almost periodic solution (see Remark 1).

SOUHRN

VĚTY O EXISTENCI SKOROPERIODICKÝCH ŘEŠENÍ DIFERENCIÁLNÍCH ROVNIC 1.ŘÁDU

SVATOSLAV STANĚK

V práci je dokázán následující výsledek: Nechť u, v jsou skoroperiodické C^1-funkce, $\alpha \leq v(t) \leq u(t) \leq \beta$ pro $t \in R$, kde $\alpha, \beta \in R$. Nechť $f: R \times \langle \alpha, \beta \rangle \to R$ je skoroperiodická funkce v proměnné t stejnoměrně pro $x \in \langle \alpha, \beta \rangle$ a

$$\nu(u'(t) - f(t,u(t))) \leq 0, \quad \nu(v'(t) - f(t,v(t))) \geq 0, \quad t \in R,$$

kde $\nu \in \{-1, 1\}$. Jestliže $\frac{\partial f}{\partial x}(t,x)$ existuje na množině $H :=$ $:= \{(t,x);\ t \in R,\ v(t) \leq x \leq u(t)\}$ a

$$m \leq \nu \frac{\partial f}{\partial x}(t,x) \leq M \text{ pro } (t,x) \in H,$$

kde m, M jsou kladné konstanty, pak rovnice

$$x' = f(t,x)$$

má skoroperiodické řešení v H. Aplikace výsledku je ukázána na pěti konkrétních diferenciálních rovnicích.

РЕЗЮМЕ

ТРЕХТОЧЕЧНАЯ КРАЕВАЯ ЗАДАЧА ДЛЯ СОДЕРЖАЩЕГО ПАРАМЕТРА ФУНКЦИОНАЛЬНОГО ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ 2-ОГО ПОРЯДКА С ЗАПАЗДЫВАНИЕМ

С. СТАНЕК

Пусть h- положительная постоянная и $X = \{y;\ y \in C^0(\langle -h, 0\rangle)\}$ пространство Банаха с нормой $\|y\| = \max_{t \in \langle -h, 0\rangle} |y(t)|$. В работе исследуется функциональное дифференциальное уравнение с запаздыванием

$$y'' - q(t)y = f(t, y_t, \mu), \qquad (1)$$

где $q : J := \langle t_1, t_3\rangle \to (0, \infty), f : J \times X \times \langle a, b\rangle \to R$ - непрерывные функции. Пусть $t_2 \in (t_1, t_3)$. Приведены условия для функций g, f которые достаточны для того, чтобы для каждой начальной функции $\varphi \in X_0 \subset X$ существовало $\mu_0 \in \langle a, b\rangle$ такое, что уравнение (1) с $\mu = \mu_0$ имело решение y удовлетворяющие краевым условиям

$$y(t_1) = y(t_2) = y(t_3) = 0. \qquad (2)$$

Исследуется также проблема однозначности решения краевой задачи (1), (2).

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