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Locally Best Linear-quadratic Estimators *

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Abstract

In the simple case of measuring the one dimensional linear dependence passing through the origin by devices from a given class of precision the β_0-LBLUE (β_0-locally best linear unbiased estimator) and the β_0-LBLQUE (β_0-locally best linear-quadratic unbiased estimator) are compared using simulations.

Key words: Linear model,variances depending on mean value parameters, locally best linear unbiased estimator (LBLUE), locally best linear-quadratic unbiased estimator (LBLQUE).

1991 Mathematics Subject Classification: 62J05, 62F10

Meaurements by devices from a given class of precision lead often to a model linear in the mean value parameters. The covariance matrix depends quadratically on these parameters. That is why the model is of the form

$$(\boldsymbol{Y}, \boldsymbol{X}\boldsymbol{\beta}, \boldsymbol{\Sigma})\,. \tag{1}$$

Here $\boldsymbol{Y}_{n,1}$ is a normally distributed random vector. Its realization $\boldsymbol{y}_{n,1}$ are the measured values. The mean value is

$$\mathcal{E}(\boldsymbol{Y}) = \boldsymbol{X}\boldsymbol{\beta}\ ,$$

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where $\boldsymbol{X}_{n,k}$ is a known $n \times k$ design matrix and $\boldsymbol{\beta} \in \mathcal{R}^k$ (k-dimensional Euclidean space) is the vector of unknown mean value parameters of the model (1). The covariance matrix of the vector $\boldsymbol{Y}$ is

$$\boldsymbol{\Sigma} = \sigma^2 \boldsymbol{\Sigma}(\boldsymbol{\beta}) = \sigma^2 \begin{pmatrix} (a + b|e_1'\boldsymbol{X}\boldsymbol{\beta}|)^2 & 0 & \dots & 0 \\ 0 & (a + b|e_2'\boldsymbol{X}\boldsymbol{\beta}|)^2 & \dots & 0 \\ \vdots & & \ddots & \\ 0 & \dots & & (a + b|e_n'\boldsymbol{X}\boldsymbol{\beta}|)^2 \end{pmatrix},$$

where a, b and σ^2 are known positive constants, $\boldsymbol{e}_i'$ is the transpose of the i-th unit vector (i.e. the transpose of a vector of suitable dimension with the i-th component equal to 1 and with the other components equal to 0).

Generally does not exist in model (1) the UBLUE (uniformly best linear unbiased estimator, in more detail see in [1] and [2]) of $\boldsymbol{e}_i'\boldsymbol{\beta}$ ($i \in \{1, 2, \dots, k\}$).

Often we have an apriori information about the true value $\boldsymbol{\beta}^\circ$ of the parameter $\boldsymbol{\beta}$. If this information is of the form:

$$\boldsymbol{\beta}^\circ \in \mathcal{B} = \{\boldsymbol{\gamma} \in \mathcal{R}^k : \|\boldsymbol{e}_i'\boldsymbol{X}\boldsymbol{\gamma} - \boldsymbol{e}_i'\boldsymbol{X}\boldsymbol{\beta}_\circ\|^2 < \varrho^2,\ i = 1, 2, \dots, n\}$$

($\boldsymbol{\beta}_\circ$ and $\varrho > 0$ are known), we use for estimating $\boldsymbol{e}_i'\boldsymbol{\beta}$ ($i \in \{1, 2, \dots, k\}$) the $\boldsymbol{\beta}_\circ$-LBLUE ($\boldsymbol{\beta}_\circ$-locally best linear unbiased estimator, in more detail see in [1] and [2]).

Lemma 1 *The $\boldsymbol{\beta}_\circ$-LBLUE of $\boldsymbol{e}_i'\boldsymbol{\beta}$ ($i \in \{1, 2, \dots, k\}$) exists iff*

$$\boldsymbol{e}_i' \in \mu(\boldsymbol{X}') = \{\boldsymbol{X}'\boldsymbol{u} : \boldsymbol{u} \in \mathcal{R}^n\}$$

and its form is

$$\widehat{\boldsymbol{e}_i'\boldsymbol{\beta}} = \boldsymbol{e}_i'[(\boldsymbol{X}')^-_{m(\boldsymbol{\Sigma}(\boldsymbol{\beta}_\circ))}]'\boldsymbol{Y}.$$

Its dispersion is

$$\sigma^2 \boldsymbol{e}_i'[(\boldsymbol{X}')^-_{m(\boldsymbol{\Sigma}(\boldsymbol{\beta}_\circ))}]'\boldsymbol{\Sigma}(\boldsymbol{\beta}_\circ)(\boldsymbol{X}')^-_{m(\boldsymbol{\Sigma}(\boldsymbol{\beta}_\circ))}\boldsymbol{e}_i$$

where $(\boldsymbol{X}')^-_{m(\boldsymbol{\Sigma}(\boldsymbol{\beta}_\circ))}$ is an arbitrary but fixed minimum $\boldsymbol{\Sigma}(\boldsymbol{\beta}_\circ)$-norm g-inverse of the matrix $\boldsymbol{X}'$.

Proof see [2], Lemma 2.4 and its proof. □

We shall investigate the $\boldsymbol{\beta}_\circ$-LBLQUE ($\boldsymbol{\beta}_\circ$-locally best linear-quadratic unbiased estimator, see in [2] and [3]) of $\boldsymbol{e}_i'\boldsymbol{\beta}$ as improvement the $\boldsymbol{\beta}_\circ$-LBLUE.

Let us denote by $\mathcal{D}$ the class of matrices $\boldsymbol{B}_{n,n}$ satisfying the next three conditions:

$$\forall\{\boldsymbol{\beta} \in \mathcal{R}^k\} \qquad \operatorname{Tr} \boldsymbol{B} \begin{pmatrix} |e_1'\boldsymbol{X}\boldsymbol{\beta}| & 0 & \dots & 0 \\ 0 & |e_2'\boldsymbol{X}\boldsymbol{\beta}| & \dots & 0 \\ \vdots & & \ddots & \\ 0 & \dots & & |e_n'\boldsymbol{X}\boldsymbol{\beta}| \end{pmatrix} = 0,$$

$$\sum_{i=1}^n \boldsymbol{e}_i'\boldsymbol{B}\boldsymbol{e}_i = 0, \qquad \boldsymbol{X}'(\boldsymbol{B} + \sigma^2 b^2 \sum_{i=1}^n \boldsymbol{e}_i\boldsymbol{e}_i'\boldsymbol{B}\boldsymbol{e}_i\boldsymbol{e}_i')\boldsymbol{X} = \boldsymbol{0}.$$

Theorem 2 *In model* (1) *is the random variable* $a'Y + Y'AY$ *the* β_0-LBLQUE *of* $e_i'\beta$ $(i \in \{1, 2, \ldots, k\})$ *iff there exists a vector* $z \in \mathcal{R}^n$ *that*

$$a = -(A + A')X\beta_0 + (X')^-_{m(\Sigma(\beta_0))}X'z,$$

$$\forall\{D \in \mathcal{D}\}$$

$$\mathrm{Tr}(D + D')\{\sigma^2\Sigma(\beta_0)(A + A')\Sigma(\beta_0) + 2X\beta_0 z'X[(X')^-_{m(\Sigma(\beta_0))}]'\Sigma(\beta_0)\} = 0,$$

$$A \in \mathcal{D}$$

and

$$e_i = X'a.$$

Proof see in [4]. □

Theorem 3 *In the case*

$$k \le n \le k + 1$$

with $R(X) = n - 1$ *(*$R(X)$ *is the rank of the matrix* X*) there does not exist the* β_0*-LBLQUE as improvement the* β_0*-LBLUE (i.e. the* β_0*-LBLQUE is identical with the* β_0*-LBLUE).*

Proof see in [4]. □

Theorem 4 *In the case*

$$k \le n \le k + 2$$

with $R(X) = n - 2$ *there exists the* β_0*-LBLQUE as improvement the* β_0*-LBLUE of* $e_i'\beta$ *iff*

(i) two rows of the design matrix are obtained by multiplying one (say s-th) row by different nonzero numbers γ, δ*, where* $|\gamma| \neq 1$, $|\delta| \neq 1$ *and* $e_i \in \mu(X')$

or

(ii) one row of the design matrix is obtained by multiplying one (say s-th) row by γ *and another row is obtained by multiplying the l-th row by* δ*, where* $s \neq l$, $|\gamma| \neq 1$, $|\delta| \neq 1$, $\gamma \neq 0$, $\delta \neq 0$ *and* $e_i \in \mu(X')$.

The dispersion of the β_0*-LBLQUE of* $e_i'\beta$ *at* β_0 *is not greater than the dispersion of the* β_0*-LBLUE of* $e_i'\beta$.

Proof see in [4] where also examples are shown with β_0-LBLQUE having lower dispersion as the appropriate β_0-LBLUE. □

Finally let us have the simple case of measuring the one dimensional linear dependence passing through the origin. The measurements are made in three different points. So we have the model (1) with

$$X_{3,1} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \qquad \beta \in \mathcal{R}^1 \tag{2}$$

and

$$\boldsymbol{\Sigma} = \sigma^2 \begin{pmatrix} (a+b|x_1\beta|)^2 & 0 & 0 \\ 0 & (a+b|x_2\beta|)^2 & 0 \\ 0 & 0 & (a+b|x_3\beta|)^2 \end{pmatrix}.$$

According to Theorem 2 the $\boldsymbol{\beta}_\circ$-LBLQUE of β is $\boldsymbol{a}'\boldsymbol{Y} + \boldsymbol{Y}'\boldsymbol{A}\boldsymbol{Y}$, where

$$\boldsymbol{A} = \frac{\alpha}{2} \begin{pmatrix} 2A(|x_2|-|x_3|) & \frac{x_2(a+b|x_3\beta_\circ|)^2}{x_3(a+b|x_2\beta_\circ|)^2} & 1 \\ \frac{x_2(a+b|x_3\beta_\circ|)^2}{x_3(a+b|x_2\beta_\circ|)^2} & 2A(|x_3|-|x_1|) & \frac{x_2(a+b|x_1\beta_\circ|)^2}{x_1(a+b|x_2\beta_\circ|)^2} \\ 1 & \frac{x_2(a+b|x_1\beta_\circ|)^2}{x_1(a+b|x_2\beta_\circ|)^2} & 2A(|x_1|-|x_2|) \end{pmatrix}$$

and

$$\boldsymbol{a} = \begin{pmatrix} -\beta_\circ\alpha\{\frac{2K(|x_2|-|x_3|)x_1}{(1+\sigma^2b^2)\Delta} + \frac{x_2^2(a+b|x_3\beta_\circ|)^2}{x_3(a+b|x_2\beta_\circ|)^2} + x_3\} + \frac{Fx_1}{2\beta_\circ(a+b|x_1\beta_\circ|)^2} \\ -\beta_\circ\alpha\{\frac{x_1x_2(a+b|x_3\beta_\circ|)^2}{x_3(a+b|x_2\beta_\circ|)^2} + \frac{2K(|x_3|-|x_1|)x_2}{(1+\sigma^2b^2)\Delta}x_3 + \frac{x_2x_3(a+b|x_1\beta_\circ|)^2}{x_1(a+b|x_2\beta_\circ|)^2}\} + \frac{Fx_2}{2\beta_\circ(a+b|x_2\beta_\circ|)^2} \\ -\beta_\circ\alpha\{x_1 + \frac{x_2^2(a+b|x_1\beta_\circ|)^2}{x_1(a+b|x_2\beta_\circ|)^2} + \frac{2K(|x_1|-|x_2|)x_3}{(1+\sigma^2b^2)\Delta}\} + \frac{Fx_3}{2\beta_\circ(a+b|x_3\beta_\circ|)^2} \end{pmatrix}$$

with

$$K = \frac{x_1x_2^2(a+b|x_3\beta_\circ|)^2}{x_3(a+b|x_2\beta_\circ|)^2} + \frac{x_2^2x_3(a+b|x_1\beta_\circ|)^2}{x_1(a+b|x_2\beta_\circ|)^2} + x_1x_3$$

$$\Delta = x_1^2(|x_3|-|x_2|) + x_2^2(|x_1|-|x_3|) + x_3^2(|x_2|-|x_1|)$$

$$A = \frac{K}{(1+\sigma^2b^2)\Delta}, \qquad L = 1 + \alpha\frac{2\beta_\circ\sigma^2b^2K}{1+\sigma^2b^2},$$

$$S = \sum_{i=1}^{3} \frac{x_i^2}{(a+b|x_i\beta_\circ|)^2}, \qquad F = \frac{2\beta_\circ L}{S},$$

$$G = (a+b|x_1\beta_\circ|)^4(|x_2|-|x_3|)^2 + (a+b|x_2\beta_\circ|)^4(|x_3|-|x_1|)^2 + \\ + (a+b|x_3\beta_\circ|)^4(|x_1|-|x_2|)^2$$

and

$$\alpha = -\frac{2\beta_\circ b^2}{(1+\sigma^2b^2)S\left[\frac{2KG}{(1+\sigma^2b^2)\Delta^2} + \frac{(a+b|x_1\beta_\circ|)^2(a+b|x_3\beta_\circ|)^2}{x_1x_3} + \frac{4\beta_\circ^2\sigma^2b^4K}{(1+\sigma^2b^2)S}\right]}.$$

Results of simulations

In model (1) with $a = 0.25$, $\sigma^2 = 1$ and design matrix $\boldsymbol{X}$ given in (2) we obtained the realization of $\boldsymbol{Y}$ for various values of $\boldsymbol{\beta}^\circ$ (the true value of the parameter β). According to Lemma 1 and computations below Theorem 4 we obtained

$\hat{\beta}$ — realization of the $\boldsymbol{\beta}_0$-LBLQUE

$\tilde{\beta}$ — realization of the $\boldsymbol{\beta}_0$-LBLUE.

$\sigma_{(lin-q)}$ — standard deviation of the $\boldsymbol{\beta}^{\circ}$-LBLQUE

$\sigma_{(lin)}$ — standard deviation of the $\boldsymbol{\beta}^{\circ}$-LBLUE

for various values of $\boldsymbol{\beta}_0$ (the the localization of the estimator) and b. The results are rounded according to [5].

Results:

I. Linear dependence $y = 0.2x$ (i.e. $\beta^{\circ} = 0.2$)

1. $x_1 = 1,\ x_2 = 5,\ x_3 = 10$

$b = 0.1$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
0.2	0.174	0.174	0.037	0.037
0.1	0.173	0.173	0.038	0.038
0.3	0.175	0.176	0.038	0.038

$b = 0.01$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
0.2	0.212	0.212	0.024	0.024
0.1	0.211	0.211	0.024	0.024
0.3	0.212	0.212	0.024	0.024

$b = 0.001$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
0.2	0.164	0.164	0.022	0.022
0.1	0.164	0.164	0.022	0.022
0.3	0.164	0.164	0.022	0.022

$b = 0.00001$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
0.2	0.177	0.177	0.022	0.022
0.1	0.177	0.177	0.022	0.022
0.3	0.177	0.177	0.022	0.022

2. $x_1 = 1,\ x_2 = 9,\ x_3 = 10$

$b = 0.1$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
0.2	0.182	0.182	0.033	0.033
0.1	0.182	0.182	0.033	0.033
0.3	0.182	0.182	0.033	0.033

$b = 0.01$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
0.2	0.245	0.245	0.020	0.020
0.1	0.245	0.245	0.020	0.020
0.3	0.245	0.245	0.020	0.020

$b = 0.001$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
0.2	0.211	0.211	0.019	0.019
0.1	0.211	0.211	0.019	0.019
0.3	0.211	0.211	0.019	0.019

$b = 0.00001$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
0.2	0.194	0.194	0.019	0.019
0.1	0.194	0.194	0.019	0.019
0.3	0.194	0.194	0.019	0.019

3. $x_1 = 1,\ x_2 = 2,\ x_3 = 10$
$b = 0.1$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
0.2	0.166	0.166	0.042	0.042
0.1	0.166	0.166	0.043	0.043
0.3	0.165	0.165	0.043	0.043

$b = 0.01$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
0.2	0.174	0.174	0.026	0.026
0.1	0.174	0.174	0.026	0.026
0.3	0.174	0.174	0.026	0.026

$b = 0.001$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
0.2	0.211	0.211	0.025	0.025
0.1	0.211	0.211	0.025	0.025
0.3	0.211	0.211	0.025	0.025

$b = 0.00001$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
0.2	0.194	0.194	0.024	0.024
0.1	0.194	0.194	0.024	0.024
0.3	0.194	0.194	0.024	0.024

II. Linear dependence $y = x$ (i.e. $\boldsymbol{\beta}^{\circ} = 1.0$)

1. $x_1 = 1,\ x_2 = 5,\ x_3 = 10$
$b = 0.1$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
1.0	1.098	1.094	0.093	0.093
0.8	1.111	1.096	0.093	0.093
1.2	1.088	1.094	0.093	0.093

$b = 0.01$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
1.0	0.941	0.941	0.030	0.030
0.8	0.941	0.941	0.030	0.030
1.2	0.942	0.941	0.030	0.030

$b = 0.001$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
1.0	1.000	1.000	0.023	0.023
0.8	0.999	0.999	0.023	0.023
1.2	1.000	1.000	0.023	0.023

$b = 0.00001$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
1.0	1.011	1.011	0.022	0.022
0.8	1.011	1.011	0.022	0.022
1.2	1.011	1.011	0.022	0.022

2. $x_1 = 1,\ x_2 = 9,\ x_3 = 10$

$b = 0.1$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
1.0	1.044	1.044	0.087	0.087
0.8	1.037	1.039	0.087	0.087
1.2	1.050	1.049	0.087	0.087

$b = 0.01$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
1.0	1.000	1.000	0.026	0.026
0.8	1.000	1.000	0.026	0.026
1.2	1.000	1.000	0.026	0.026

$b = 0.001$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
1.0	0.949	0.949	0.019	0.019
0.8	0.949	0.949	0.019	0.019
1.2	0.949	0.949	0.019	0.019

$b = 0.00001$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
1.0	1.000	1.000	0.019	0.019
0.8	1.000	1.000	0.019	0.019
1.2	1.000	1.000	0.019	0.019

3. $x_1 = 1,\ x_2 = 2,\ x_3 = 10$
$b = 0.1$

β_o	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
1.0	1.03	1.04	0.10	0.10
0.8	1.03	1.03	0.10	0.10
1.2	1.04	1.04	0.10	0.10

$b = 0.01$

β_o	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
1.0	1.014	1.014	0.034	0.034
0.8	1.014	1.014	0.034	0.034
1.2	1.014	1.014	0.034	0.034

$b = 0.001$

β_o	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
1.0	1.022	1.022	0.025	0.025
0.8	1.022	1.022	0.025	0.025
1.2	1.022	1.022	0.025	0.025

$b = 0.00001$

β_o	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
1.0	0.985	0.985	0.024	0.024
0.8	0.985	0.985	0.024	0.024
1.2	0.985	0.985	0.024	0.024

III. Linear dependence $y = 5x$ (i.e. $\beta^\circ = 5.0$)

1. $x_1 = 1,\ x_2 = 5,\ x_3 = 10$
$b = 0.1$

β_o	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
5.0	4.37	4.38	0.34	0.34
4.0	4.39	4.39	0.34	0.34
7.0	4.34	4.38	0.34	0.34

$b = 0.01$

β_o	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
5.0	5.109	5.109	0.059	0.059
4.0	5.101	5.105	0.059	0.059
7.0	5.121	5.116	0.059	0.059

$b = 0.001$

β_o	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
5.0	5.049	5.049	0.026	0.026
4.0	5.049	5.049	0.026	0.026
7.0	5.049	5.049	0.026	0.026

$b = 0.00001$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
5.0	5.017	5.017	0.022	0.022
4.0	5.017	5.017	0.022	0.022
7.0	5.017	5.017	0.022	0.022

2. $x_1 = 1,\ x_2 = 9,\ x_3 = 10$

$b = 0.1$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
5.0	5.05	5.05	0.33	0.33
4.0	5.01	5.02	0.33	0.33
7.0	5.09	5.08	0.33	0.33

$b = 0.01$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
5.0	4.970	4.970	0.053	0.053
4.0	4.972	4.972	0.053	0.053
7.0	4.966	4.966	0.053	0.053

$b = 0.001$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
5.0	4.963	4.963	0.022	0.022
4.0	4.963	4.963	0.022	0.022
7.0	0.962	0.962	0.022	0.022

$b = 0.00001$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
5.0	5.004	5.004	0.019	0.019
4.0	5.004	5.004	0.019	0.019
7.0	5.004	5.004	0.019	0.019

3. $x_1 = 1,\ x_2 = 2,\ x_3 = 10$

$b = 0.1$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
5.0	5.37	5.39	0.35	0.35
4.0	5.30	5.38	0.35	0.35
7.0	5.46	5.41	0.35	0.36

$b = 0.01$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
5.0	5.039	5.039	0.067	0.067
4.0	5.059	5.054	0.067	0.067
7.0	4.998	5.012	0.068	0.068

$b = 0.001$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
5.0	4.957	4.957	0.029	0.029
4.0	4.957	4.957	0.029	0.029
7.0	4.956	4.956	0.029	0.029

$b = 0.00001$

β_0	$\hat{\beta}$	$\tilde{\beta}$	$\sigma_{(lin-q)}$	$\sigma_{(lin)}$
5.0	5.020	5.020	0.024	0.024
4.0	5.020	5.020	0.024	0.024
7.0	5.020	5.020	0.024	0.024

Final (dis)appointment *In all investigated simulated cases with different points of measurement x_1, x_2, x_3, different b, β_0 and β^0 there was no reasonable difference between realizations of the β_0-LBLQUE and β_0-LBLUE and also between the standard deviations of the β^0-LBLQUE and β^0-LBLUE.*

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