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On the Generalization of the STER Distribution Applied to Generalized Hypergeometric Parents *

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Abstract

Two types of partial summation of parent and godparent distributions leading to generalized STER distributions used in inventory decision problems, income underreporting, etc. are presented. The summations are applied to two kinds of generalized hypergeometric distributions and demonstrated on many examples.

Key words: Discrete probability distributions, the generalization of the STER distribution, generalized hypergeometric distributions.

1991 Mathematics Subject Classification: 60E05, 62E10

1 Introduction

Let X be a discrete random variable (r.v.) (called *basic* r.v.) defined on nonnegative integers with probability mass function (pmf) $\{P_0^{**}, P_1^{**}, \ldots\}$ and probability generating function (pgf)

$$G^{**}(t) = \sum_{x \geq 0} P_x^{**} t^x. \tag{1}$$

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The sum in (1) may be finite or infinite. The generalization of the STER distribution (d.) corresponding to the basic d. has the pmf

$$P_x = \sum_{y \geqq x+k} \frac{1}{y-k+1} \frac{P_y^{**}}{\sum_{l \geqq k} P_l^{**}}, \qquad x = 0, 1, 2, \ldots, \tag{2}$$

where $k \in N_0 = \{0, 1, 2, \ldots\}$, which is a slight enlargement of Patil et al. [51, p. 90–91]. One can see that the generalization of the STER d. does not depend on $P_0^{**}, \ldots, P_{k-1}^{**}$ in case $k \in N = \{1, 2, \ldots\}$.

Since

$$\left\{0, 0, \ldots, \frac{P_k^{**}}{\sum_{l \geqq k} P_l^{**}}, \frac{P_{k+1}^{**}}{\sum_{l \geqq k} P_l^{**}}, \ldots\right\}$$

is for every $k = 0, 1, 2, \ldots$ a proper pmf belonging to the k-left truncated basic r.v. X with pgf

$$J(t) = \sum_{j \geqq k} \frac{P_j^{**} t^j}{\sum_{l \geqq k} P_l^{**}},$$

thus

$$P_i^* = \frac{P_{k+i-1}^{**}}{\sum_{l \geqq k} P_l^{**}}, \qquad i = 1, 2, \ldots \tag{3}$$

is again a proper pmf belonging to the $(k-1)$-shifted [i.e. $(k-1)$-times translated to the left] k-left truncated basic r.v. X. In case that $k = 0$ it means (-1)-shifted, i.e. translated one step to the right. This r.v. is called *parent* r.v. Its pgf is

$$G^*(t) = t^{-k+1} J(t).$$

Thus the generalization of the STER d. (2) can be written as

$$P_x = \sum_{y \geqq x+1} \frac{P_y^*}{y}, \qquad x = 0, 1, 2, \ldots \tag{4}$$

with pgf $G(t)$. It is easy to see that

$$G(t) = \frac{1}{1-t} \int_t^1 \frac{G^*(z)}{z} dz \tag{5}$$

and the recurrence formula for probabilities is $P_x = P_{x-1} - \frac{P_x^*}{x}$, $x = 1, 2, \ldots$

Evidently it is sufficient to consider an arbitrary parent pmf $\{P_i^*\}_{i \geqq 1}$ and its pgf $G^*(t)$. Then for arbitrary $k \in N_0$ we obtain the $(k-1)$-displaced r.v. [i.e. translated $(k-1)$-times to the right] with pmf $\{P_0^{**} = 0, P_1^{**} = 0, \ldots, P_k^{**} = P_1^*,\ P_{k+1}^{**} = P_2^*, \ldots\}$. (In the case $k = 0$ it means (-1)-displaced i.e. translated one step to the left.) It is the k-left truncated basic r.v. X. Thus we also obtain the generalized STER d. (2) which is of course the same as (4). Hence if the parent d. is given, from (4) and (5) we obtain the generalized STER d.

On the other hand let us know the generalized STER pmf $\{P_0, P_1, \ldots\}$. According to (4) it is necessary that

$$P_{i-1} \geqq P_i \quad i = 1, 2, \ldots, \tag{6}$$

thus from (4)

$$P_i^* = i(P_{i-1} - P_i) \quad i = 1, 2, \ldots \tag{7}$$

It also holds that

$$1 = \sum_{i \geqq 1} P_i^* = \sum_{i \geqq 1} i(P_{i-1} - P_i) = \sum_{i \geqq 1} iP_{i-1} - \sum_{i \geqq 1} iP_i, \tag{8}$$

thus if μ_1' (the mean of the generalized STER d.) exists then

$$\mu_1' < \sum_{i \geqq 1} iP_{i-1} < \infty$$

and (8) is true.

For the pgf $G^*(t)$ it holds that

$$\begin{aligned} G^*(t) &= \sum_{i \geqq 1} P_i^* t^i = \sum_{i \geqq 1} iP_{i-1}t^i - \sum_{i \geqq 1} iP_i t^i \\ &= t[tG(t)]' - tG'(t) = t\{[tG(t)]' - G'(t)\} \end{aligned} \tag{9}$$

providing μ_1' exists. It is easy to see that (9) can also be obtained from (5) and thus if (6) holds and μ_1' exists, $G^*(t)$ is given by (9). Equations (7) and (9) give us the pmf and pgf of the parent d. belonging to the generalized STER d. Again for arbitrary $k \in N_0$ we obtain the $(k-1)$-displaced r.v. with pmf $\{P_0^{**} = 0, P_1^{**} = 0, \ldots, P_k^{**} = P_1^*,\ P_{k+1}^{**} = P_2^*, \ldots\}$. This is the k-left truncated basic r.v. X. Hence we again have the generalized STER d.

In case $k = 1$ we obtain results for the simple STER d. (i.e. Sums successively Truncated from the Expectation of the Reciprocal of the original random variable) introduced by Bissinger in [5]. They arose in connection with inventory decision problems concerning the distribution of the number of demands as the parent distribution.

Example 1 Let the basic r.v. X has the pmf

$$P_i^{**} = \frac{e^{-a}a^i}{i!} \quad i = 0, 1, 2, \ldots$$

i.e. the basic distribution is a Poisson d. with

$$G^{**}(t) = e^{a(t-1)} = \frac{{}_0F_0(at)}{{}_0F_0(a)}.$$

For $k = 0, 1, \ldots$ the k-left truncated basic r.v. has the pmf

$$\frac{P_i^{**}}{\sum_{l \geqq k} P_l^{**}} = \frac{a^{i-k}}{i_{(i-k)}\, {}_1F_1(1; k+1; a)}, \qquad i = k, k+1, \ldots,$$

where $r_{(0)} = 1,\ r_{(s)} = r(r-1)\dots(r-s+1),\ s \in N$, and

$$_pF_q(\alpha_1, \alpha_2, \dots, \alpha_p; \beta_1, \beta_2, \dots, \beta_q; z) = \sum_{n=0}^{\infty} \frac{\alpha_1{}^{(n)} \alpha_2{}^{(n)} \dots \alpha_p{}^{(n)} z^n}{\beta_1{}^{(n)} \beta_2{}^{(n)} \dots \beta_q{}^{(n)}}$$

with $r^{(0)} = 1,\ r^{(s)} = r(r+1)\dots(r+s-1),\ s \in N$, is the generalized Gauss hypergeometric function (see e.g. in [29]). The pgf of this k-left truncated basic r.v. is

$$J(t) = \frac{t^k\ {}_1F_1(1; k+1; at)}{{}_1F_1(1; k+1; a)}.$$

The parent r.v. [i.e. the $(k-1)$-shifted k-left truncated Poisson d.] has the pgf

$$G^*(t) = t^{-k+1} J(t) = \frac{t\,{}_1F_1(1; k+1; at)}{{}_1F_1(1; k+1; a)}$$

and, according to (3), the pmf is

$$P_i^* = \frac{e^{-a} a^{k+i-1}}{(k+i-1)!} \left\{ \sum_{l=k}^{\infty} \frac{e^{-a} a^l}{l!} \right\}^{-1} = \frac{a^{i-1}}{(k+i-1)_{(i-1)}\,{}_1F_1(1; k+1; a)},\ i = 1, 2, \dots$$

According to (4) the generalization of the STER d. corresponding to the basic Poisson d. is given as

$$P_x = \frac{1}{{}_1F_1(1; k+1; a)} \sum_{y=x+1}^{\infty} \frac{a^{y-1}}{y(k+y-1)_{(y-1)}}, \quad x = 0, 1, 2, \dots$$

and its pgf given by (5) is

$$G(t) = \frac{1}{1-t} \int_t^1 \frac{z\,{}_1F_1(1; k+1; az)}{z\,{}_1F_1(1, k+1; a)} dz$$

$$= \frac{1}{(1-t)\,{}_1F_1(1; k+1; a)} \{{}_2F_2(1, 1; k+1, 2; a) - t\,{}_2F_2(1, 1; k+1, 2; at)\}.$$

In the literature one finds two special cases:
For $k = 0$ the parent pmf is

$$P_i^* = \frac{e^{-a} a^{i-1}}{(i-1)!}, \quad i = 1, 2, \dots,$$

i.e. one-displaced Poisson d. with pgf

$$G^*(t) = \frac{t\ {}_0F_0(at)}{{}_0F_0(a)} = t\ {}_0F_0(a(t-1)).$$

The generalized STER d. has

$$G(t) = \frac{1}{(1-t)\,{}_1F_1(1; 1; a)} \{{}_1F_1(1; 2; a) - t\,{}_1F_1(1; 2; at)\} = {}_1F_1(1; 2; a(t-1))$$

and

$$P_x = \frac{1}{{}_1F_1(1;1;a)} \sum_{y=x+1}^{\infty} \frac{a^{y-1}}{y!} = \frac{a^x}{(x+1)!}\,{}_1F_1(x+1;x+2;-a), \quad x = 0,1,\ldots$$

and can be found in [17]; [29,p. 448]; [36]. It is known e.g. under the names *Poisson's exponential binomial limit* or *Feller's Poisson-rectangular d.*

For $k = 1$ the parent pmf is

$$P_i^* = \frac{e^{-a}a^i}{i!(1-e^{-a})}, \quad i = 1,2,\ldots,$$

i.e. the zero-truncated Poisson d. with the pgf

$$G^*(t) = \frac{t\,{}_1F_1(1;2;at)}{{}_1F_1(1;2;a)} = \frac{e^{at}-1}{e^a-1}.$$

The generalized STER d. has then

$$\begin{aligned} G(t) &= \frac{1}{(1-t)\,{}_1F_1(1;2;a)}\{{}_2F_2(1,1;2,2;a) - t\,{}_2F_2(1,1;2,2;at)\} \\ &= \frac{e^{-a}}{(1-t)(1-e^{-a})} \sum_{j=1}^{\infty} \frac{a^j(1-t^j)}{j!j} \end{aligned}$$

and

$$\begin{aligned} P_x &= \frac{1}{{}_1F_1(1;2;a)} \sum_{y=x+1}^{\infty} \frac{a^{y-1}}{yy!} = \frac{e^{-a}}{1-e^{-a}} \sum_{y=x+1}^{\infty} \frac{a^y}{y!y} \\ &= \frac{e^{-a}a^{x+1}}{(1-e^{-a})(x+1)!(x+1)}\,{}_2F_2(1,x+1;x+2,x+2;a), \quad x = 0,1,\ldots \end{aligned}$$

which is known as *STER–Poisson* or *Bissinger–Poisson d.* ([5]; [29, p. 449]; [37]; [52]).

2 Investigations in the class of Generalized Hypergeometric Factorial Moment distributions (GHFD)

The generalized hypergeometric factorial moment distributions (GHFD), called also *Kemp–hypergeometric distributions*, have pgf's of the form

$$W(t) = {}_pF_q(a_1,\ldots,a_p;b_1,\ldots b_q;\lambda(t-1))$$

(see [33]; [29, p. 91]). The relationship between the generalized STER d. and its parent distribution in the GHFD family has been analyzed by Kemp and Kemp [38] applying Patil and Joshi's result in [51]. If for the parent d.

$$G^*(t) = t\,{}_pF_q(a_1,\ldots,a_p;b_1,\ldots b_q;\lambda(t-1))$$

is valid, then the corresponding $G(t)$ is

$$G(t) = {}_{p+1}F_{q+1}(a_1, \ldots, a_p, 1; b_1, \ldots b_q, 2; \lambda(t-1)). \tag{10}$$

On the other hand, any distribution with a valid pgf

$$G(t) = {}_pF_q(a_1, \ldots, a_p; b_1, \ldots b_q; \lambda(t-1))$$

and nonincreasing probabilities $\{P_0, P_1, \ldots\}$ is a generalized STER d. if

$$W(t) = {}_{p+1}F_{q+1}(a_1, \ldots, a_p, 2; b_1, \ldots b_q, 1; \lambda(t-1))$$

is a valid pgf. In that case

$$G^*(t) = t\, {}_{p+1}F_{q+1}(a_1, \ldots, a_p, 2; b_1, \ldots b_q, 1; \lambda(t-1)) \tag{11}$$

is the pgf of the corresponding parent distribution.

The following list in Table 1 shows that besides the well known Bissinger distributions in the literature a number of distributions has been considered whose mutual relations remained unknown.

3 Investigations in the class of Generalized Hypergeometric probability distributions

The generalized hypergeometric probability distributions (GHPD), called also *Kemp-Dacey-hypergeometric distributions*, have pgf's of the form

$$W(t) = \frac{{}_pF_q(a_1, \ldots, a_p; b_1, \ldots b_q; \lambda t)}{{}_pF_q(a_1, \ldots, a_p; b_1, \ldots b_q; \lambda t)}.$$

(cf. [33], [34], [12], and [29]). If

$$G(t) = \frac{{}_pF_q(a_1, \ldots, a_p; b_1, \ldots b_q; \lambda t)}{{}_pF_q(a_1, \ldots, a_p; b_1, \ldots b_q; \lambda)} = \frac{1}{D}\, {}_pF_q(a_1, \ldots, a_p; b_1, \ldots b_q; \lambda t) = \sum_{j \geqq 0} P_j t^j \tag{12}$$

is the pgf of the generalized STER d., i.e. if (6) is valid, then

$$[tG(t)]' = \left\{ \frac{t}{D} \sum_{j=0}^{\infty} \frac{a_1^{(j)} \ldots a_p^{(j)}}{b_1^{(j)} \ldots b_q^{(j)}} \frac{\lambda^j t^j}{j!} \right\}'$$

$$= \frac{1}{D} \left\{ \sum_{j=0}^{\infty} \frac{a_1^{(j)} \ldots a_p^{(j)}}{b_1^{(j)} \ldots b_q^{(j)}} \frac{\lambda^j t^{j+1}}{j!} \right\}' = \frac{1}{D}\, {}_{p+1}F_{q+1}(a_1, \ldots, a_p, 2; b_1, \ldots b_q, 1; \lambda t),$$

where $r^{(0)} = 1,\ r^{(s)} = r(r+1) \ldots (r+s-1),\ s \in N$, and

$$G'(t) = \frac{\lambda}{D} \frac{a_1 \ldots a_p}{b_1 \ldots b_q}\, {}_pF_q(a_1+1, \ldots, a_p+1; b_1+1, \ldots b_q+1; \lambda t).$$

TABLE 1.

name and/or some ref's of the parent d. $G^*(t)$ constrains on parameters	parent d. $G(t)$ P_x^*	generalized STER d. of the gen. STER d. P_x	name and/or some ref's
deterministic d. $n \in N_0$	$t\ {}_1F_0(-n; 1-t)$ $1, \quad x = n+1$	${}_2F_1(-n, 1; 2; 1-t)$ $\frac{1}{n+1}, \quad x = 0, 1, \ldots, n$	discrete uniform d.
1-displaced discrete uniform d. $n \in N_0$	$t\ {}_2F_1(-n, 1; 2; 1-t)$ $\frac{1}{n+1}, \quad x = 1, 2, \ldots, n+1$	${}_3F_2(-n, 1, 1; 2, 2; 1-t)$ $\frac{1}{n+1} \sum_{j=x+1}^{n+1} \frac{1}{j}, \quad x = 0, 1, \ldots, n$	[38]
1-displaced geometric d. $0 < q < \frac{1}{2}, \ p = 1-q$	$t\ {}_1F_0(1; \frac{q(t-1)}{p})$ $pq^{x-1}, \quad x = 1, 2, \ldots$	${}_2F_1(1, 1; 2; \frac{q(t-1)}{p})$ $\frac{1}{x+1} \left(\frac{q}{p}\right)^x {}_2F_1(x+1, x+1; x+2; -\frac{q}{p}),$ $x = 0, 1, \ldots$	[41]
1-displaced zero-one d. $0 \leqq p \leqq 1, \ q = 1-p$	$t\ {}_1F_0(-1; p(1-t))$ $p^{x-1}q^{2-x}, \quad x = 1, 2$	${}_2F_1(-1, 1; 2; p(1-t))$ $q^{1-x} + \frac{p}{2}, \quad x = 0, 1$	[73], [74], [29, p. 281]
1-displaced binomial d. $0 \leqq p \leqq 1, \ q = 1-p,$ $n \in N_0$	$t\ {}_1F_0(-n; p(1-t))$ $\binom{n}{x-1} p^{x-1} q^{n-x+1},$ $x = 1, 2, \ldots, n+1$	${}_2F_1(-n, 1; 2; p(1-t))$ $\frac{n_{(x)} p^x}{(x+1)!}\ {}_2F_1(-n+x, x+1; x+2; p),$ $x = 0, 1, \ldots, n$	[73], [74], [29, p. 281]
1-displaced negative binomial d. $0 < q < \frac{1}{2}, \ p = 1-q,$ $k \geqq 0$	$t\ {}_1F_0(k; \frac{q(t-1)}{p})$ $\binom{k+x-2}{x-1} p^k q^{x-1},$ $x = 1, 2, \ldots$	${}_2F_1(k, 1; 2; \frac{q(t-1)}{p})$ $\frac{1}{x+1} \binom{k+x-1}{x} \left(\frac{q}{p}\right)^x$ ${}_2F_1(k+x, x+1; x+2; -\frac{q}{p}),$ $x = 0, 1, \ldots$	[22], [23], [29, p. 282], [38]

TABLE 1 (CONT.).

name and/or some ref's of the parent d. $G^*(t)$ constrains on parameters	parent d. $G(t)$ P_x^*	generalized STER d. of the gen. STER d. P_x	name and/or some ref's
1-displaced Poisson d. $0 \leqq a$	$t\ {}_0F_0(a(t-1))$ $\frac{e^{-a}a^{x-1}}{(x-1)!}$, $x = 1, 2, \dots$	${}_1F_1(1; 2; a(t-1))$ $\frac{a^x}{(x+1)!}\ {}_1F_1(x+1; x+2; -a)$, $x = 0, 1, \dots$	Feller's Poisson-rectangular d. [29], [17], [76]
[20] $n \in N_0,\ 0 \leqq \alpha \leqq 1$	$t\ {}_2F_1(-n, 1-\alpha; 1; 1-t)$ $\binom{\alpha+n-x}{n-x+1}\binom{x-1-\alpha}{x-1}$, $x = 1, 2, \dots, n+1$	${}_2F_1(-n, 1-\alpha; 2; 1-t)$ $\frac{1}{n+1}\binom{x-\alpha}{x}\binom{\alpha+n-x}{n-x}$, $x = 0, 1, \dots, n$	negative hypergeometric d. [29]
[9], [29] $n \in N_0$	$t\ {}_2F_1(-n, \frac{1}{2}; 1; 1-t)$ $\binom{2n-2x+2}{n-x+1}\binom{2x-2}{x-1}2^{-2n}$, $x = 1, 2, \dots, n+1$	${}_2F_1(-n, \frac{1}{2}; 2; 1-t)$ $\frac{1}{n+1}\binom{x-\frac{1}{2}}{x}\binom{n+\frac{1}{2}-x}{n-x}$, $x = 0, 1, \dots, n$	negative hypergeometric d. [29]
[11], [12], [13] $n \in N$	$t\ {}_2F_1(-n+1, 2; n+2; 1-t)$ $\frac{x(2n-x-1)!(n+1)!}{(n-x)!(2n)!}$, $x = 1, 2, \dots, n$	${}_2F_1(-n+1, 1; n+2; 1-t)$ $\frac{\binom{2n-x-1}{n}}{\binom{2n}{n-1}}$, $x = 0, 1, \dots, n$	negative hypergeometric d. [29]
[38] $0 \leqq p \leqq \frac{1}{n+1},\ q = 1-p$, $n \in N_0$	$t\ {}_2F_1(-n, 2; 1; p(1-t))$ $\binom{n}{x-1}p^{x-1}q^{n-x+1}$ $\left[x - \frac{p(n-x+1)}{q}\right]$, $x = 1, 2, \dots, n+1$	${}_1F_0(-n; p(1-t))$ $\binom{n}{x}p^x q^{n-x}$, $x = 0, 1, \dots, n$	binomial d.

TABLE 1 (CONT.).

name and/or some ref's of the parent d. $G^*(t)$ constrains on parameters	parent d. $G(t)$ P_x^*	generalized STER d. of the gen. STER d. P_x	name and/or some ref's
[38] $0 \leqq q < \frac{1}{2}$, $0 \leqq k \leqq \frac{1}{q}$	$t\ {}_2F_1(k,2;1;\frac{q(t-1)}{p})$ $\binom{k+x-2}{x-1}p^k q^{x-1}[x-q(k+x-1)]$, $x=1,2,\ldots$	${}_1F_0(k;\frac{q(t-1)}{p})$ $\binom{k+x-1}{x}p^k q^x$, $x=0,1,\ldots$	negative binomial d.
[38], [57] $0 \leqq a \leqq 1$	$t\ {}_1F_1(2;1;a(t-1))$ $\frac{e^{-a}a^{x-1}}{(x-1)!}(x-a)$, $x=1,2,\ldots$	${}_0F_0(a(t-1))$ $\frac{e^{-a}a^x}{x!}$, $x=0,1,\ldots$	Poisson d.
[20] $0 \leqq B \leqq 1$, $n \in N_0$	$t\ {}_2F_0(-n,2;B(1-t))$ $\binom{n}{x-1}\sum_{j=0}^{n-x+1}\binom{n-x+1}{j}(-1)^j B^{x-1+j}\Gamma(x+j+1)$, $x=1,\ldots,n+1$	${}_2F_0(-n,1;B(1-t))$ $\binom{n}{x}\sum_{j=0}^{n-x}\binom{n-x}{j}(-1)^j B^{x+j}\Gamma(x+j+1)$, $x=0,1,\ldots,n$	binomial-exponential d. [20]
[73], [74] $N \geqq 2$, $n \in N_0$	$t\ {}_2F_1(-n,2;N+1;1-t)$ $\frac{x\binom{N+n-x-1}{n-x+1}}{\binom{N+n}{n}}$, $x=1,2,\ldots,n+1$	${}_2F_1(-n,1;N+1;1-t)$ $\frac{Nn_{(x)}}{(N+n)_{(x+1)}}$, $x=0,1,\ldots,n$	[25]
[73], [74] $\theta > 1$, $n \in N$	$t\ {}_2F_1(-n+1,2;\theta+1;1-t)$ $\binom{n-1}{x-1}\theta(\theta-1)B(x+1,n+\theta-x-1)$, $x=1,2,\ldots,n$	${}_2F_1(-n+1,1;\theta+1;1-t)$ $\theta\binom{n-1}{x}B(x+1,n+\theta-x-1)$, $x=0,1,\ldots,n-1$	[27], [32], [59]

TABLE 1 (CONT.).

name and/or some ref's of the parent d. $G^*(t)$ constrains on parameters	parent d. $G(t)$ P_x^*	generalized STER d. of the gen. STER d. P_x	name and/or some ref's
[31] $n \in N$	$t\,{}_1F_0(-n(n-1); \frac{1-t}{2})$ $\binom{n(n-1)}{x-1}2^{-n(n-1)}$, $x = 1, 2, \ldots, n(n-1)+1$	${}_2F_1(-n(n-1), 1; 2; \frac{1-t}{2})$ $\binom{n(n-1)}{x}\frac{1}{(x+1)2^x}$ ${}_2F_1(-n(n-1)+x; x+1; x+2; \frac{1}{2})$, $x = 0, 1, \ldots, n(n-1)$	[73], [74]
[56] $n \in N$	$t\,{}_1F_0(-n+1; \frac{1-t}{n+1})$ $\binom{n-1}{x-1}\frac{n^{n-x}}{(n+1)^{n-1}}$, $x = 1, 2, \ldots, n$	${}_2F_1(-n+1, 1; 2; \frac{1-t}{n+1})$ $\binom{n-1}{x}\frac{1}{(x+1)(n+1)^x}$ ${}_2F_1(-n+1+x; x+1; x+2; \frac{1}{n+1})$, $x = 0, 1, \ldots, n-1$	[73], [74]

Hence according to (9)

$$G^*(t) = \frac{t}{{}_pF_q(a_1,\ldots,a_p;b_1,\ldots b_q;\lambda)}$$
$$\Big\{{}_{p+1}F_{q+1}(a_1,\ldots,a_p,2;b_1,\ldots b_q,1;\lambda t) -$$
$$-\lambda\frac{a_1\ldots a_p}{b_1\ldots b_q}\,{}_pF_q(a_1+1,\ldots,a_p+1;b_1+1,\ldots b_q+1;\lambda t)\Big\} \tag{13}$$

is the pgf of the parent d. corresponding to the generalized STER d. with pgf (12).

From (7) we obtain also the probabilities $\{P_0^*, P_1^*, \ldots\}$ as

$$P_i^* = i(P_{i-1} - P_i), \quad i = 1, 2, \ldots \tag{14}$$

where

$$P_i = \frac{1}{{}_pF_q(a_1,\ldots,a_p;b_1,\ldots b_q;\lambda)}\frac{a_1^{(i)}\ldots a_p^{(i)}}{b_1^{(i)}\ldots b_q^{(i)}}\frac{\lambda^i}{i!}, \quad i = 0, 1, \ldots \tag{15}$$

On the other hand, let

$$\begin{aligned} G^*(t) &= \frac{t\,{}_pF_q(a_1,\ldots,a_p;b_1,\ldots b_q;\lambda t)}{{}_pF_q(a_1,\ldots,a_p;b_1,\ldots b_q;\lambda)} \\ &= \frac{t}{C}\,{}_pF_q(a_1,\ldots,a_p;b_1,\ldots b_q;\lambda t) \end{aligned} \tag{16}$$

be the pgf of the parent distribution. According to (5), the pgf of the corresponding generalized STER d. is given as

$$\begin{aligned} G(t) &= \frac{1}{(1-t)C}\int_t^1 {}_pF_q(a_1,\ldots,a_p;b_1,\ldots b_q;\lambda z)\,dz \\ &= \frac{1}{(1-t)\,{}_pF_q(a_1,\ldots,a_p;b_1,\ldots b_q;\lambda)} \\ &\qquad \Big\{{}_{p+1}F_{q+1}(a_1,\ldots,a_p,1;b_1,\ldots b_q,2;\lambda) \\ &\qquad -t\,{}_{p+1}F_{q+1}(a_1,\ldots,a_p,1;b_1,\ldots b_q,2;\lambda t)\Big\} \\ &= \frac{{}_{p+1}F_{q+1}(a_1,\ldots,a_p,1;b_1,\ldots b_q,2;\lambda)}{(1-t)\,{}_pF_q(a_1,\ldots,a_p;b_1,\ldots b_q;\lambda)} \\ &\qquad \left(1-\frac{t\,{}_{p+1}F_{q+1}(a_1,\ldots,a_p,1;b_1,\ldots b_q,2;\lambda t)}{{}_{p+1}F_{q+1}(a_1,\ldots,a_p,1;b_1,\ldots b_q,2;\lambda)}\right) \\ &= \frac{1-\frac{t\,{}_{p+1}F_{q+1}(a_1,\ldots,a_p,1;b_1,\ldots b_q,2;\lambda t)}{{}_{p+1}F_{q+1}(a_1,\ldots,a_p,1;b_1,\ldots b_q,2;\lambda)}}{(1-t)\left[\frac{d}{dt}\frac{t\,{}_{p+1}F_{q+1}(a_1,\ldots,a_p,1;b_1,\ldots b_q,2;\lambda t)}{{}_{p+1}F_{q+1}(a_1,\ldots,a_p,1;b_1,\ldots b_q,2;\lambda)}\right]_{t=1}}. \end{aligned} \tag{17}$$

Let us now consider a quite another type of summation process. If Y is a discrete r.v. defined on nonnegative integers with pmf $\{Q_0^*, Q_1^*, \ldots\}$, pgf $H^*(t)$ and finite mean ν^*, then it is easy to see that

$$Q_x = \frac{1}{\nu^*} \sum_{j \geqq x+1} Q_j^*, \qquad x = 0, 1, \ldots \tag{18}$$

is again a proper pmf (see e.g. [29, p. 448]) belonging to another type of partial-sums d. as the generalized STER d. defined in (4). In this case of summation let us name the r.v. Y a *godparent d.* The resulting partial-sums d. has now the pgf

$$H(t) = \frac{1 - H^*(t)}{(1-t)\nu^*} \tag{19}$$

and its r-th moment about zero is

$$\nu_r' = \frac{1}{\nu^*} \sum_{j \geqq 1} \left\{ Q_j^* \sum_{k=0}^{j-1} k^r \right\}, \qquad r = 1, 2, \ldots$$

provided it exists (see [29, p .449]). Comparing (17) and (19) we see that if

$$\frac{t\ {}_{p+1}F_{q+1}(a_1, \ldots, a_p, 1; b_1, \ldots b_q, 2; \lambda t)}{{}_{p+1}F_{q+1}(a_1, \ldots, a_p, 1; b_1, \ldots b_q, 2; \lambda)}$$

is a proper pgf of a r.v. Y with a finite mean

$$\left[\frac{d}{dt} \frac{t\ {}_{p+1}F_{q+1}(a_1, \ldots, a_p, 1; b_1, \ldots b_q, 2; \lambda t)}{{}_{p+1}F_{q+1}(a_1, \ldots, a_p, 1; b_1, \ldots b_q, 2; \lambda)} \right]_{t=1},$$

then Y is a godparent d. corresponding to the generalized STER d. with pgf $G(t)$. So the same generalized STER d. could be obtained via two summation processes. The correspondent parent d. has pgf $G^*(t)$ and the correspondent godparent d. has pgf $H^*(t)$.

It is also easy to obtain the relationship

$$P_i^* = \frac{iQ_i^*}{\nu^*}, \qquad i = 1, 2, \ldots \tag{20}$$

between the pmf of the parent d. and the godparent d. showing that the parent d. is a weighted godparent d. (moment godparent d.).

Example 2 Let the generalized STER d. be the Poisson d. with pgf

$$G(t) = \frac{{}_0F_0(at)}{{}_0F_0(a)}$$

with $0 < a \leqq 1$ in order to secure nonincreasing probabilities. According to (13) the pgf of the parent distribution is

$$\begin{aligned} G^*(t) &= \frac{t}{{}_0F_0(a)} \{ {}_1F_1(2; 1; at) - a\ {}_0F_0(at) \} \\ &= \frac{t}{e^a} \left[(1 + at)e^{at} - ae^{at} \right] = te^{a(t-1)}[1 + a(t-1)] = t\ {}_1F_1(2; 1; a(t-1)), \end{aligned}$$

(since ${}_1F_1(\beta+1;\beta;z) = \frac{1}{\beta}(\beta+z)e^z$). Thus we again obtain the *Poisson's exponential binomial limit* or *Feller's Poisson-rectangular d.* as above.

For the opposite direction, let the parent d. be the 1-displaced geometric d. with pgf $G^*(t) = tp\,{}_1F_0(1;qt)$, $0 < q < 1$. According to (17)

$$\begin{aligned}
G(t) &= \frac{1-q}{(1-t)}\{\,{}_2F_1(1,1;2;q) - t\,{}_2F_1(1,1;2;qt)\} \\
&= \frac{1-\frac{t\,{}_2F_1(1,1;2;qt)}{{}_2F_1(1,1;2;q)}}{(1-t)\frac{{}_1F_0(1;qt)}{{}_2F_1(1,1;2;q)}} = \frac{1-\frac{\log_e(1-qt)}{\log_e(1-q)}}{(1-t)\left[-\frac{q}{(1-q)\log_e(1-q)}\right]} \qquad (21) \\
&= \frac{1-q}{1-t}\left\{-\frac{\log_e(1-q)}{q} + t\frac{\log_e(1-qt)}{qt}\right\} = \frac{p}{q}\frac{1}{1-t}\log_e\frac{1-qt}{1-q} \\
&= \frac{1}{-\frac{q}{1-q}(t-1)}\log_e\left(1-\frac{q(t-1)}{1-q}\right) \qquad (22) \\
&= {}_2F_1(1,1;2;\frac{q}{1-q}(t-1)) \quad \text{if } 0<q<\frac{1}{2} \qquad (23)
\end{aligned}$$

(since $\ln(1-z) = -z\,{}_2F_1(1,1;2;z)$). Thus one can see from (21) that the godparent d. has pgf

$$H^*(t) = \frac{\log_e(1-qt)}{\log_e(1-q)}$$

(logarithmic d.). The resulting STER d. is proposed in [48, p. 115]; in the case $0 < q < \frac{1}{2}$ it yields the 1-displaced binomial-beta-geometric d. given in [41] as obtained in Table 1.

In Table 2 one finds some trinities of corresponding distributions (parent d., godparent d., generalized STER d.) where the parent d. and the godparent d. belong to GHPD (Kemp–Dacey-hypergeometric family).

TABLE 2.

parent d. name and/or some ref's $G^*(t)$ P_x^* constrains on parameters	godparent d. name and/or some ref's $H^*(t)$ Q_x^*	generalized STER d. name and/or some ref's $G(t)$ P_x
1-displaced Poisson d. $t\,\frac{{}_0F_0(at)}{{}_0F_0(a)}$ $\frac{e^{-a}a^{x-1}}{(x-1)!}, \quad x=1,2,\ldots$ $a>0$	positive Poisson d. $t\frac{{}_1F_1(1;2;at)}{{}_1F_1(1;2;a)}$ $\frac{a^x}{x!(e^a-1)}, \quad x=1,2,\ldots$	[55] ${}_1F_1(1;2;a(t-1))$ $\frac{a^x}{(x+1)!}\,{}_1F_1(x+1;x+2;-a),$ $x=0,1,\ldots$
[8], [62] $t\,\frac{{}_1F_0(2;t\sqrt{a})}{{}_1F_0(2;\sqrt{a})}$ $(1-\sqrt{a})^2x(\sqrt{a})^{x-1}, \quad x=1,2,\ldots$ $0\leqq a<1$	1-displaced geometric d. $t\,\frac{{}_1F_0(1;t\sqrt{a})}{{}_1F_0(1;\sqrt{a})}$ $(1-\sqrt{a})(\sqrt{a})^{x-1}, \quad x=1,2,\ldots$	geometric d. $\frac{1-\sqrt{a}}{1-t\sqrt{a}}$ $(1-\sqrt{a})(\sqrt{a})^x, \quad x=0,1,\ldots$
[18] $t\,\frac{{}_1F_0(2;\frac{t}{2})}{{}_1F_0(2;\frac{1}{2})}$ $x(\frac{1}{2})^{x+1}, \quad x=1,2,\ldots$	1-displaced geometric d. $t\,\frac{{}_1F_0(1;\frac{t}{2})}{{}_1F_0(1;\frac{1}{2})}$ $(\frac{1}{2})^x, \quad x=1,2,\ldots$	geometric d. $\frac{1}{2-t}$ $(\frac{1}{2})^{x+1}, \quad x=0,1,\ldots$
1-dispaced negative binomial d. $t\,\frac{{}_1F_0(k;qt)}{{}_1F_0(k;q)}$ $\binom{k+x-2}{x-1}(1-q)^kq^{x-1}, \ x=1,2,\ldots$ $k\geqq 0,\ k\neq 1,\ 0<q<1$	positive negative binomial d. $t\,\frac{{}_2F_1(1,k;2;qt)}{{}_2F_1(1,k;2;q)}$ $\binom{k+x-2}{x}\frac{(1-q)^{k-1}q^x}{1-(1-q)^{k-1}}, \ x=1,2,\ldots$	[22] if $0<q<\frac{1}{2}$ ${}_2F_1(1,k;2;\frac{q}{1-q}(t-1))$ if $0<q<\frac{1}{2}$ $\frac{1}{x+1}\binom{k+x-1}{x}\left(\frac{q}{1-q}\right)^x$ ${}_2F_1(k+x,x+1;x+2;-\frac{q}{1-q}),$ $x=0,1,\ldots,$ if $0\leqq q<\frac{1}{2}$

TABLE 2 (CONT.).

parent d. name and/or some ref's $G^*(t)$ P^*_x constrains on parameters	godparent d. name and/or some ref's $H^*(t)$ Q^*_x	generalized STER d. name and/or some ref's $G(t)$ P_x
1-displaced geometric d. $t\,\frac{{}_1F_0(1;tq)}{{}_1F_0(1;q)}$ $(1-q)q^{x-1}, \quad x=1,2,\dots$ $0<q<1$	logarithmic d. $t\,\frac{{}_2F_1(1,1;2;qt)}{{}_2F_1(1,1;2;q)}$ $\frac{q^x}{-x\log_e(1-q)},\ x=1,2,\dots$	[48] [41] if $0<q<\frac{1}{2}$ $\frac{-1}{\frac{q}{1-q}(t-1)}\log_e\left(1-\frac{q}{1-q}(t-1)\right)$ ${}_2F_1(1,1;2;\frac{q}{1-q}(t-1))$ if $0<q<\frac{1}{2}$ $\frac{(1-q)q^x}{x+1}\ {}_2F_1(1,x+1;x+2;q)$, $\frac{1}{x+1}\left(\frac{q}{1-q}\right)^x\ {}_2F_1(x+1,x+1;x+2;-\frac{q}{1-q})$, $x=0,1,\dots$ if $0<q<\frac{1}{2}$
[24] $t\,\frac{{}_0F_1(1;at)}{{}_0F_1(1;a)}$ $\frac{a^{x-1}}{[(x-1)!]^2}\frac{1}{{}_0F_1(1;a)},\ x=1,2,\dots$ $a\geqq 0$	[24] $t\,\frac{{}_0F_1(2;at)}{{}_0F_1(2;a)}$ $\frac{a^{x-1}}{x[(x-1)!]^2}\frac{1}{{}_0F_1(2;a)},\ x=1,2,\dots$	$\frac{{}_0F_1(2;a)}{(1-t)\ {}_0F_1(1;a)}\left(1-\frac{t\ {}_0F_1(2;at)}{{}_0F_1(2;a)}\right)$ $\frac{a^x\ {}_1F_2(1;x+1,x+2;a)}{(x+1)(x!)^2\ {}_0F_1(1;a)}$, $x=0,1,\dots$
positive Poisson d. $t\,\frac{{}_1F_1(1;2;at)}{{}_1F_1(1;2;a)}$ $\frac{a^x}{x!(e^a-1)},\quad x=1,2,\dots$ $a>0$	$t\,\frac{{}_2F_2(1,1;2,2;at)}{{}_2F_2(1,1;2,2;a)}$ $\frac{1}{a\ {}_2F_2(1,1;2,2;a)}\frac{a^x}{x!x},\ x=1,2,\dots$	[5] $\frac{1}{(1-t)(e^a-1)}\sum_{j=1}^{\infty}\frac{a^j(1-t^j)}{j!j}$ $\frac{a^{x+1}}{(e^a-1)(x+1)(x+1)!}$ ${}_2F_2(1,x+1;x+2,x+2;a)$, $x=0,1,\dots$

TABLE 2 (CONT.).

parent d. name and/or some ref's $G^*(t)$ P^*_x constrains on parameters	godparent d. name and/or some ref's $H^*(t)$ Q^*_x	generalized STER d. name and/or some ref's $G(t)$ P_x
[6] $t\,\frac{{}_1F_1(2;\frac{a}{n}+2;\frac{a}{n}t)}{{}_1F_1(2;\frac{a}{n}+2;\frac{a}{n})}$ $\frac{nx(\frac{a}{n})^x\Gamma(\frac{n+a}{n})}{a\Gamma(\frac{a+xn+n}{n})},\quad x=1,2,\dots$ $a,n>0$, Γ-Gamma function	1-dispaced Hyper-Poisson d., [3] $t\,\frac{{}_1F_1(1;2+\frac{a}{n};\frac{a}{n}t)}{{}_1F_1(1;2+\frac{a}{n};\frac{a}{n})}$ $\frac{(\frac{a}{n})^{x-1}}{{}_1F_1(1;2+\frac{a}{n};\frac{a}{n})(2+\frac{a}{n})^{(x-1)}},\quad x=1,2,\dots$	$\frac{{}_1F_1(1;\frac{a}{n}+2;\frac{a}{n})}{(1-t)\,{}_1F_1(2;\frac{a}{n}+2;\frac{a}{n})}\left(1-\frac{t\,{}_1F_1(1;\frac{a}{n}+2;\frac{a}{n}t)}{{}_1F_1(1;\frac{a}{n}+2;\frac{a}{n})}\right)$ $\frac{n}{n+a}\frac{(\frac{a}{n})^x}{(2+\frac{a}{n})^{(x)}}\,{}_1F_1(1;2+\frac{a}{n}+x;\frac{a}{n}),$ $x=0,1,\dots$
[36], [38] $t\,\frac{{}_1F_1(2-a;1-a;at)}{{}_1F_1(2-a;1-a;a)}$ $\frac{e^{-a}a^{x-1}}{(x-1)!}(x-a),\quad x=1,2,\dots$ $0<a<1$	$t\,\frac{{}_2F_2(1,2-a;1-a,2;at)}{{}_2F_2(1,2-a;1-a,2;a)}$ $\frac{1}{a(1-a)\,{}_2F_2(1,2-a;1-a,2;a)}\frac{a^x}{x!}(x-a),$ $x=1,2,\dots$	Poisson d. $\frac{{}_0F_0(at)}{{}_0F_0(a)}$ $\frac{e^{-a}a^x}{x!},\quad x=0,1,\dots$
[6] $t\,\frac{{}_2F_1(2,\frac{a}{b};\frac{n+a}{n+b}+1;\frac{b}{b+n}t)}{{}_2F_1(2,\frac{a}{b};\frac{n+a}{n+b}+1;\frac{b}{b+n})}$ $\frac{nx}{n+a}\frac{(\frac{a}{b})^{(x-1)}(\frac{b}{b+n})^{x-1}}{(\frac{a+b+2n}{b+n})^{(x-1)}},\quad x=1,2,\dots$ $a,b,n>0$	1-displaced Hyper-Pascal d., [73], [74] $t\,\frac{{}_2F_1(1,\frac{a}{b};\frac{n+a}{n+b}+1;\frac{b}{b+n}t)}{{}_2F_1(1,\frac{a}{b};\frac{n+a}{n+b}+1;\frac{b}{b+n})}$ $\frac{(\frac{a}{b})^{(x-1)}(\frac{b}{b+n})^{x-1}}{{}_2F_1(1,\frac{a}{b};\frac{n+a}{n+b}+1;\frac{b}{b+n})(\frac{a+b+2n}{b+n})^{(x-1)}},$ $x=1,2,\dots$	$\frac{{}_2F_1(1,\frac{a}{b};\frac{n+a}{n+b}+1;\frac{b}{b+n})}{(1-t)\,{}_2F_1(2,\frac{a}{b};\frac{n+a}{n+b}+1;\frac{b}{b+n})}$ $\left(1-\frac{t\,{}_2F_1(1,\frac{a}{b};\frac{n+a}{n+b}+1;\frac{b}{b+n}t)}{{}_2F_1(1,\frac{a}{b};\frac{n+a}{n+b}+1;\frac{b}{b+n})}\right)$ $\frac{n}{n+a}\frac{(\frac{a}{b})^{(x)}(\frac{b}{b+n})^x}{(\frac{a+b+2n}{b+n})^{(x)}}$ ${}_2F_1(1,\frac{a}{b}+x;\frac{n+a}{n+b}+1+x;\frac{b}{b+n}),$ $x=0,1,\dots$

TABLE 2 (CONT.).

parent d. name and/or some ref's $G^*(t)$ P^*_x constrains on parameters	godparent d. name and/or some ref's $H^*(t)$ Q^*_x	generalized STER d. name and/or some ref's $G(t)$ P_x
hyperlogarithmic d., [29], [72], [73] $t\,\frac{{}_2F_1(1,1;m+1;t)}{{}_2F_1(1,1;m+1;1)}$ $\frac{(x-1)!(m-1)}{m^{(x)}},\quad x=1,2,\ldots$ $m>1$, Ψ-Digamma function	trigamma d., [65], [66] $t\,\frac{{}_3F_2(1,1,1;2,m+1;t)}{{}_3F_2(1,1,1;2,m+1;1)}$ $\frac{1}{\Psi'(m)}\frac{(x-1)!}{xm^{(x)}},\ x=1,2,\ldots$	$\frac{m-1}{m(1-t)}[m\Psi'(m)-$ $t\,{}_3F_2(1,1,1;2,m+1;t)]$ $\frac{x!(m-1)}{m^{(x+1)}(x+1)}\,{}_3F_2(1,x+1,x+1;x+2,m+x+1;1)$, $x=0,1,\ldots$
[19], [46] $t\,\frac{{}_2F_1(2,2;1;pt)}{{}_2F_1(2,2;1;p)}$ $\frac{x^2p^{x-1}(1-p)^3}{1+p},\ x=1,2,\ldots$ $0<p<1$	[8], [62] $t\,\frac{{}_1F_0(2;pt)}{{}_1F_0(2;p)}$ $(1-p)^2xp^{x-1},\quad x=1,2,\ldots$	$\frac{(1-p)(1-p^2t)}{(1+p)(1-pt)^2}$ $\frac{p^x(1-p)[1+x(1-p)]}{1+p},\quad x=0,1,\ldots$
positive negative binomial d., [29, p. 225] $t\,\frac{{}_2F_1(1,k+1;2;(1-p)t)}{{}_2F_1(1,k+1;2;1-p)}$ $\binom{k+x-1}{x}\frac{p^k(1-p)^x}{1-p^k},\ x=1,2,\ldots$ $k>0,\ 0<p<1$	$t\,\frac{{}_3F_2(1,1,k+1;2,2;(1-p)t)}{{}_3F_2(1,1,k+1;2,2;1-p)}$ $\binom{k+x-1}{x}\frac{(1-p)^{x-1}}{kx\ {}_3F_2(1,1,k+1;2,2;1-p)}$, $x=1,2,\ldots$	[29, p. 449], [37] $\frac{p^k}{(1-p^k)(1-t)}\sum_{j=1}^{\infty}\binom{k+j-1}{j}\frac{(1-p)^j(1-t^j)}{j}$ $\binom{k+x}{x+1}\frac{p^k(1-p)^{x+1}}{(1-p^k)(x+1)}$ ${}_3F_2(1,x+1,k+x+1;x+2,x+2;1-p)$, $x=0,1,\ldots$
[29], [30], [65] $t\,\frac{{}_2F_1(1,a;a+2;t)}{{}_2F_1(1,a;a+2;1)}$ $\frac{a}{(a+x-1)(a+x)},\ x=1,2,\ldots$ $a>0$, Ψ-Digamma function	digamma d., [65], [66] $t\,\frac{{}_3F_2(1,1,a;2,a+2;t)}{{}_3F_2(1,1,a;2,a+2;1)}$ $\frac{a(a-1)}{[\Psi(a+1)-\Psi(2)]x(a+x-1)(a+x)}$, $x=1,2,\ldots$	$\frac{{}_3F_2(1,1,a;2,a+2;1)}{(1-t)\ {}_2F_1(1,a;a+2;1)}\left(1-\frac{t\ {}_3F_2(1,1,a;2,a+2;t)}{{}_3F_2(1,1,a;2,a+2;1)}\right)$ $\frac{a}{(x+1)(a+x)(a+x+1)}\,{}_3F_2(1,x+1,a+x;a+x+2,x+2;1)$, $x=0,1,\ldots$

TABLE 2 (CONT.).

parent d. name and/or some ref's $G^*(t)$ P^*_x constrains on parameters	godparent d. name and/or some ref's $H^*(t)$ Q^*_x	generalized STER d. name and/or some ref's $G(t)$ P_x
[71], [72] $t\,\frac{{}_2F_1(2,1;a+1;bt)}{{}_2F_1(2,1;a+1;b)}$ $\frac{b^{x-1}x!}{(a+1)^{(x-1)}\,{}_2F_1(2,1;a+1;b)},\ x=1,2,\dots$ $a>1,\ 0<b<1$	hyperlogarithmic d., [71], [72] $t\,\frac{{}_2F_1(1,1;a+1;bt)}{{}_2F_1(1,1;a+1;b)}$ $\frac{b^{x-1}(x-1)!}{(a+1)^{(x-1)}{}_2F_1(1,1;a+1;b)},\ x=1,2,\dots$	$\frac{{}_2F_1(1,1;a+1;b)}{(1-t)\,{}_2F_1(2,1;a+1;b)}\left(1-\frac{t\,{}_2F_1(1,1;a+1;bt)}{{}_2F_1(1,1;a+1;b)}\right)$ $\frac{b^x x!}{(a+1)^{(x)}\,{}_2F_1(2,1;a+1;b)}\,{}_2F_1(1,x+1;a+x+1;b),\ x=0,1,\dots$
[21], [44] $t\,\frac{{}_2F_1(1,1;m+3;t)}{{}_2F_1(1,1;m+3;1)}$ $\frac{(m+1)(x-1)!}{(m+2)^{(x)}},\quad x=1,2,\dots$ $m>-1$, Ψ-Digamma function	trigamma d., [65], [66] $t\,\frac{{}_3F_2(1,1,1;2,m+3;t)}{{}_3F_2(1,1,1;2,m+3;1)}$ $\frac{(x-1)!}{x(m+2)^{(x)}\Psi'(m+2)},\quad x=1,2,\dots$	$\frac{(m+1)\Psi'(m+2)}{(1-t)}\left(1-\frac{t\,{}_3F_2(1,1,1;2,m+3;t)}{(m+2)\Psi'(m+2)}\right)$ $\frac{(m+1)x!}{(x+1)(m+2)^{(x+1)}}\,{}_3F_2(1,x+1,x+1;x+2,m+x+3;1),$ $x=0,1,\dots$
positive Yule d., [40] $t\,\frac{{}_2F_1(2,1;b+3;t)}{{}_2F_1(2,1;b+3;1)}$ $\frac{x!b}{(b+2)^{(x)}},\quad x=1,2,\dots,$ $b>0$	[21], [44] $t\,\frac{{}_2F_1(1,1;b+3;t)}{{}_2F_1(1,1;b+3;1)}$ $\frac{(x-1)!(b+1)}{(b+2)^{(x)}},\quad x=1,2,\dots$	Yule d., [29, p. 275] $\frac{b}{b+1}\,{}_2F_1(1,1;b+2;t)$ $\frac{x!b}{(b+1)^{(x+1)}},\quad x=0,1,\dots$
[53] $t\,\frac{{}_2F_1(1,a;a+3;t)}{{}_2F_1(1,a;a+3;1)}$ $\frac{2a(a+1)}{(a+x-1)^{(3)}},\quad x=1,2,\dots$ $a>0$, Ψ-Digamma function	digamma d., [65], [66] $t\,\frac{{}_3F_2(1,1,a;2,a+3;t)}{{}_3F_2(1,1,a;2,a+3;1)}$ $\frac{(a-1)a(a+1)}{[\Psi(a+2)-\Psi(3)]x(a+x-1)^{(3)}},\ x=1,2,\dots$	$\frac{2[\Psi(a+2)-\Psi(3)]}{(1-t)(a-1)}\left(1-\frac{t\,{}_3F_2(1,1,a;2,a+3;t)}{{}_3F_2(1,1,a;2,a+3;1)}\right)$ $\frac{2a(a+1)}{(x+1)(a+x)^{(3)}}\,{}_3F_2(1,x+1,a+x;x+2,a+x+3;1),$ $x=0,1,\dots$

TABLE 2 (CONT.).

parent d. name and/or some ref's $G^*(t)$ P^*_x constrains on parameters	godparent d. name and/or some ref's $H^*(t)$ Q^*_x	generalized STER d. name and/or some ref's $G(t)$ P_x
[59] $t\,\frac{{}_2F_1(1,1;4;t)}{{}_2F_1(1,1;4;1)}$ $\frac{1}{\binom{x+1}{2}} - \frac{1}{\binom{x+2}{2}},\ x=1,2,\ldots$ Ψ-Digamma function	trigamma d., [65], [66] $t\,\frac{{}_3F_2(1,1,1;2,4;t)}{{}_3F_2(1,1,1;2,4;1)}$ $\frac{(x-1)!}{x3^{(x)}\Psi'(3)},\ x=1,2,\ldots$	$\frac{2\Psi'(3)}{(1-t)}\left(1-\frac{t\,{}_3F_2(1,1,1;2,4;t)}{3\Psi'(3)}\right)$ $\frac{4}{(x+1)^2(x+2)(x+3)}\,{}_3F_2(1,x+1,x+1;x+2,x+4;1),$ $x=0,1,\ldots$
[29], [67], [68] $t\,\frac{{}_2F_1(1,1;3;t)}{{}_2F_1(1,1;3;1)}$ $\frac{1}{x(x+1)},\quad x=1,2,\ldots$ Ψ-Digamma function	trigamma d., [65], [66] $t\,\frac{{}_3F_2(1,1,1;2,3;t)}{{}_3F_2(1,1,1;2,3;1)}$ $\frac{1}{x^2(x+1)\Psi'(2)},\quad x=1,2,\ldots$	$\frac{\Psi'(2)}{(1-t)}\left(1-\frac{t\,{}_3F_2(1,1,1;2,3;t)}{2\Psi'(2)}\right)$ $\frac{1}{(x+1)^2(x+2)}\,{}_3F_2(1,x+1,x+1;x+2,x+3;1),$ $x=0,1,\ldots$
[29], [37] $t\,\frac{{}_2F_1(1,n;n+b+1;t)}{{}_2F_1(1,n;a+b+1;1)}$ $\frac{bn^{(x-1)}}{(b+n)(b+n+1)^{(x-1)}},\ x=1,2,\ldots$ $b>0\ ,n>0,\ \Psi$-Digamma function	digamma d., [65], [66] $t\,\frac{{}_3F_2(1,1,n;2,n+b+1;t)}{{}_3F_2(1,1,1;2,n+b+1;1)}$ $\frac{(n-1)^{(x)}}{[\Psi(b+n)-\Psi(b+1)]x(n+b)^{(x)}},$ $x=1,2,\ldots$	$\frac{b[\Psi(b+n)-\Psi(b+1)]}{(1-t)(n-1)}\left(1-\frac{t\,{}_3F_2(1,1,n;2,n+b+1;t)}{{}_3F_2(1,1,n;2,n+b+1;1)}\right)$ $\frac{bn^{(x)}}{(x+1)(b+n)^{(x+1)}}\,{}_3F_2(1,x+1,x+n;x+2,b+n+1+x;1),$ $x=0,1,\ldots$
1-displaced Yule d., [29], [54] $t\,\frac{{}_2F_1(1,1;b+2;t)}{{}_2F_1(1,1;b+2;1)}$ $\frac{b(x-1)!}{(b+1)^{(x)}},\ x=1,2,\ldots$ $b>0,\ \Psi$-Digamma function	trigamma d., [65], [66] $t\,\frac{{}_3F_2(1,1,1;2,b+2;t)}{{}_3F_2(1,1,1;2,b+2;1)}$ $\frac{(x-1)!}{x(b+1)^{(x)}\Psi'(b+1)},\ x=1,2,\ldots$	$\frac{b\Psi'(b+1)}{(1-t)}\left(1-\frac{t\,{}_3F_2(1,1,1;2,b+2;t)}{(b+1)\Psi'(b+1)}\right)$ $\frac{bx!}{(x+1)(b+1)^{(x+1)}}\,{}_3F_2(1,x+1,x+1;x+2,b+2+x;1),$ $x=0,1,\ldots$

TABLE 2 (CONT.).

parent d. name and/or some ref's $G^*(t)$ P_x^* constrains on parameters	godparent d. name and/or some ref's $H^*(t)$ Q_x^*	generalized STER d. name and/or some ref's $G(t)$ P_x
1-displaced binomial d. $t\,\frac{{}_1F_0(-n;-\frac{p}{q}t)}{{}_1F_0(-n;-\frac{p}{q})}$ $\binom{n}{x-1}p^{x-1}q^{n-x+1}$, $x=1,2,\dots,n+1$ $n\in N_0$, $0\leqq p\leqq 1$, $q=1-p$	positive binomial d. $t\,\frac{{}_2F_1(-n,1;2;-\frac{p}{q}t)}{{}_2F_1(-n,1;2;-\frac{p}{q})}$ $\binom{n+1}{x}\frac{p^x q^{n+1-x}}{1-q^{n+1}}$, $x=1,2,\dots,n+1$	[73], [74] ${}_2F_1(-n,1;2;p(1-t))$ $\frac{n_{(x)}p^x}{(x+1)!}\,{}_2F_1(-n+x,x+1;x+2;p)$, $x=0,1,\dots,n$
[31] $t\,\frac{{}_1F_0(-n(n-1);-t)}{{}_1F_0(-n(n-1);-1)}$ $\binom{n(n-1)}{x-1}2^{-n(n-1)}$, $x=1,2,\dots,n(n-1)+1$ $n\in N$	1-displaced hyperbinomial d., [2], [48] $t\,\frac{{}_2F_1(-n(n-1),1;2;-t)}{{}_2F_1(-n(n-1),1;2;-1)}$ $\binom{n(n-1)}{x-1}\frac{n(n-1)+1}{x(2^{n(n-1)+1}-1)}$, $x=1,2,\dots,n(n-1)+1$	[73], [74] ${}_2F_1(-n(n-1),1;2;\frac{1-t}{2})$ $\binom{n(n-1)}{x}\frac{1}{(x+1)2^x}$ ${}_2F_1(-n(n-1)+x,x+1;x+2;\frac{1}{2})$, $x=0,1,\dots,n(n-1)$
[27], [32] $t\,\frac{{}_2F_1(-n+1,1;-n-\theta+2;t)}{{}_2F_1(-n+1,1;n-\theta+2;1)}$ $\frac{\theta}{n}\frac{\binom{n}{x}}{\binom{n+\theta-1}{x}}$, $x=1,2,\dots,n$ $\theta>0$, $n\in N$, $n+\theta\neq 2$	[14], [29] $t\,\frac{{}_3F_2(-n+1,1,1;-n-\theta+2,2;t)}{{}_3F_2(-n+1,1,1;-n-\theta+2,2;1)}$ $\frac{n_{(x)}}{x(n+\theta-1)_{(x)}}\left(\sum_{j=1}^{n}\frac{n_{(j)}}{j(n+\theta-1)_{(j)}}\right)^{-1}$, $x=1,2,\dots,n$	${}_3F_2(-n+1,1,1;\theta+1,2;1-t)$ $\frac{\theta}{n}\sum_{i=x}^{n-1}\frac{\binom{n}{i+1}}{(i+1)\binom{n+\theta-1}{i+1}}$, $x=0,1,\dots,n-1$
1-displaced zero-one d. $t\,\frac{{}_1F_0(-1;-\frac{p}{q}t)}{{}_1F_0(-1;-\frac{p}{q})}$ $p^{x-1}q^{2-x}$, $x=1,2$ $0<p<1$, $q=1-p$	positive binomial d. $t\,\frac{{}_2F_1(-1,1;2;-\frac{p}{q}t)}{{}_2F_1(-1,1;2;-\frac{p}{q})}$ $\frac{2p^x q^{2-x}}{x(1-q^2)}$, $x=1,2$	[73], [74] ${}_2F_1(-1,1;2;p(1-t))$ $q^{1-x}+\frac{p}{2}$, $x=0,1$

TABLE 2 (CONT.).

parent d. name and/or some ref's $G^*(t)$ P^*_x constrains on parameters	godparent d. name and/or some ref's $H^*(t)$ Q^*_x	generalized STER d. name and/or some ref's $G(t)$ P_x
Naor's urn model, [12], [29] $t\,\frac{{}_2F_0(-n+1,2;-\frac{t}{n})}{{}_2F_0(-n+1,2;-\frac{1}{n})}$ $\binom{n}{x}\frac{x!}{n^x}-\binom{n}{x+1}\frac{(x+1)!}{n^{x+1}}$, $x=1,2,\ldots,n$ $n\in N$	[2], [39] $t\,\frac{{}_2F_0(-n+1,1;-\frac{t}{n})}{{}_2F_0(-n+1,1;-\frac{1}{n})}$ $\frac{n^{n-x}}{(n-x)!}\left(\sum_{j=0}^{n-1}\frac{n^j}{j!}\right)^{-1}$, $x=1,2,\ldots,n$	$\frac{{}_2F_0(-n+1,1;-\frac{1}{n})}{(1-t)\;{}_2F_0(-n+1,2;-\frac{1}{n})}\left(1-\frac{t\;{}_2F_0(-n+1,1;-\frac{t}{n})}{{}_2F_0(-n+1,1;-\frac{1}{n})}\right)$ $\sum_{i=x}^{n-1}\frac{(n-1)_{(i)}}{n^{i+1}}$, $x=0,1,\ldots,n-1$
[2], [39] $t\,\frac{{}_2F_0(-R,1;-\frac{t}{a})}{{}_2F_0(-R,1;-\frac{1}{a})}$ $\frac{a^{R-x+1}}{(R-x+1)!}\left(\sum_{j=0}^{R}\frac{a^j}{j!}\right)^{-1}$, $x=1,2,\ldots,R+1$ $R\in N,\ a>0$	[1] $t\,\frac{{}_2F_1(-R,1,1;2;-\frac{t}{a})}{{}_3F_1(-R,1,1;2;-\frac{1}{a})}$ $\frac{a^{-x}}{x(R-x+1)!}\left(\sum_{j=1}^{R+1}\frac{1}{ja^j(R+1-j)!}\right)^{-1}$, $x=1,2,\ldots,R+1$	$\frac{{}_3F_1(-R,1,1;2;-\frac{1}{a})}{(1-t)\;{}_2F_0(-R,1;-\frac{1}{a})}\left(1-\frac{t\;{}_3F_1(-R,1,1;2;-\frac{t}{a})}{{}_3F_1(-R,1,1;2;-\frac{1}{a})}\right)$ $\left(\sum_{j=0}^{R}\frac{a^j}{j!}\right)^{-1}\sum_{j=x}^{R}\frac{a^{R-j}}{(j+1)(R-j)!}$, $x=0,1,\ldots,R$
1-displaced right trunc. Poisson d. $t\,\frac{{}_1F_1(-R,-R;at)}{{}_1F_1(-R,-R;a)}$ $\frac{a^{x-1}}{(x-1)!}\left(\sum_{j=0}^{R}\frac{a^j}{j!}\right)^{-1}$, $x=1,2,\ldots,R+1$ $R\in N,\ a>0$	doubly trunc. Poisson d., [29], [45] $t\,\frac{{}_2F_2(-R,1;-R,2;at)}{{}_2F_2(-R,1;-R,2;a)}$ $\frac{a^x}{x!}\left(\sum_{j=1}^{R+1}\frac{a^j}{j!}\right)^{-1}$, $x=1,2,\ldots,R+1$	$\frac{{}_2F_2(-R,1;-R,2;a)}{(1-t)\;{}_1F_1(-R;-R;a)}\left(1-\frac{t\;{}_2F_2(-R,1;-R,2;at)}{{}_2F_2(-R,1;-R,2;a)}\right)$ $\left(\sum_{j=0}^{R}\frac{a^j}{j!}\right)^{-1}\sum_{i=x}^{R}\frac{a^i}{(i+1)!}$, $x=0,1,\ldots,R$

TABLE 2 (CONT.).

parent d. name and/or some ref's $G^*(t)$ P^*_x constrains on parameters	godparent d. name and/or some ref's $H^*(t)$ Q^*_x	generalized STER d. name and/or some ref's $G(t)$ P_x
[56] $t\,\frac{{}_1F_0(-n+1;-\frac{t}{n})}{{}_1F_0(-n+1;-\frac{1}{n})}$ $\binom{n-1}{x-1}\frac{n^{n-x}}{(n+1)^{n-1}}$, $x=1,2,\dots,n$ $n\in N$	1-displaced hyperbinomial d., [2], [48] $t\,\frac{{}_2F_1(-n+1,1;2;-\frac{t}{n})}{{}_2F_1(-n+1,1;2;-\frac{1}{n})}$ $\binom{n-1}{x-1}\frac{1}{xn^{-n+x-1}[(n+1)^n-n^n]}$, $x=1,2,\dots,n$	[73], [74] ${}_2F_1(-n+1,1;2;\frac{1-t}{n+1})$ $\binom{n-1}{x}\frac{1}{(x+1)(n+1)^x}$ ${}_2F_1(-n+1+x,x+1;x+2;\frac{1}{n+1})$, $x=0,1,\dots,n-1$
[9], [29] $t\,\frac{{}_2F_1(-n,\frac{1}{2};-n+\frac{1}{2};t)}{{}_2F_1(-n,\frac{1}{2};-n+\frac{1}{2};1)}$ $\binom{2n-2x+2}{n-x+1}\binom{2x-2}{x-1}2^{-2n}$, $x=1,2,\dots,n+1$ $n\in N$	[13], [63], [64] $t\,\frac{{}_3F_2(-n,\frac{1}{2},1;-n+\frac{1}{2},2;t)}{{}_3F_2(-n,\frac{1}{2},1;-n+\frac{1}{2},2;1)}$ $\frac{\binom{2x}{x}\binom{2n+2-2x}{n+1-x}}{2x-1}\binom{2n+2}{n+1}^{-1}$, $x=1,2,\dots,n+1$	negative hypergeometric d., [29] ${}_2F_1(-n,\frac{1}{2};2;1-t)$ $2^{-2n}\sum_{i=x}^{n}\frac{1}{i+1}\binom{2n-2i}{n-i}\binom{2i}{i}$, $x=0,1,\dots,n$
[11], [12], [13] $t\,\frac{{}_2F_1(-n+1,2;-2n+2;t)}{{}_2F_1(-n+1,2;-2n+2;1)}$ $\frac{x(2n-x-1)!(n+1)!}{(n-x)!(2n)!}$, $x=1,2,\dots,n$ $n\in N$	[59] $t\,\frac{{}_2F_1(-n+1,1;-2n+2;t)}{{}_2F_1(-n+1,1;-2n+2;1)}$ $\frac{\binom{n}{x}}{\binom{2n-1}{x}}$, $x=1,2,\dots,n$	[12], [13] ${}_2F_1(-n+1,1;n+2;1-t)$ $\frac{\binom{2n-x-1}{n}}{\binom{2n}{n-1}}$, $x=0,1,\dots,n-1$

TABLE 2 (CONT.).

parent d. name and/or some ref's $G^*(t)$ P_x^* constrains on parameters	godparent d. name and/or some ref's $H^*(t)$ Q_x^*	generalized STER d. name and/or some ref's $G(t)$ P_x
[12], [13] $t\,\frac{{}_2F_1(-n,1;-n-m+2;t)}{{}_2F_1(-n,1;-n-m+2;1)}$ $\frac{\binom{m+n-x-1}{n-x+1}}{\binom{m+n-1}{n}}$, $x=1,2,\ldots,n+1$ $m,n\in N$	[14] $t\,\frac{{}_3F_2(-n,1,1;-n-m+2,2;t)}{{}_3F_2(-n,1,1;-n-m+2,2;1)}$ $\frac{(n+1)_{(x)}}{x(n+m-1)_{(x)}}\left(\sum_{j=1}^{n+1}\frac{(n+1)_{(j)}}{j(n+m-1)_{(j)}}\right)^{-1}$, $x=1,2,\ldots,n+1$	${}_3F_2(-n,1,1;m,2;1-t)$ $\binom{m+n-1}{n}^{-1}\sum_{i=x}^{n}\frac{1}{i+1}\binom{m+n-i-2}{n-i}$, $x=0,1,\ldots,n$
1-displaced discrete uniform d., [29] $t\,\frac{{}_2F_1(-n,1;-n;t)}{{}_2F_1(-n,1;-n;1)}$ $\frac{1}{n+1}$, $x=1,2,\ldots,n+1$ $n\in N_0$, Ψ-Digamma function	[16], [79] $t\,\frac{{}_3F_2(-n,1,1;-n,2;t)}{{}_3F_2(-n,1,1;-n,2;1)}$ $\frac{1}{x[\Psi(n+2)-\Psi(1)]}$, $x=1,2,\ldots,n+1$	[38] ${}_3F_2(-n,1,1;2,2;1-t)$ $\frac{1}{n+1}\sum_{i=x}^{n}\frac{1}{i+1}$, $x=0,1,\ldots,n$
[29], [30] $t\,\frac{{}_2F_1(-n,M;-n-N+M;t)}{{}_2F_1(-n,M;-n-N+M;1)}$ $\frac{\binom{M+x-2}{x-1}\binom{N-M+n-x+1}{n-x+1}}{\binom{N+n}{n}}$, $x=1,2,\ldots,n+1$ $n\in N$, $N>M>1$	positive negative hypergeometric d., [60] $t\,\frac{{}_3F_2(-n,M,1;-n-N+M,2;t)}{{}_3F_2(-n,M,1;-n-N+M,2;1)}$ $\frac{\binom{M+x-2}{x}\binom{N-M+n-x+1}{n-x+1}}{\binom{N+n}{n+1}-\binom{N-M+n+1}{n+1}}$, $x=1,2,\ldots,n+1$	${}_3F_2(-n,M,1;N+1,2;1-t)$ $\binom{N+n}{n}^{-1}\sum_{i=x}^{n}\frac{1}{i+1}\binom{M+i-1}{i}\binom{N-M+n-i}{n-i}$, $x=0,1,\ldots,n$

TABLE 2 (CONT.).

parent d. name and/or some ref's $G^*(t)$ P_x^* constrains on parameters	godparent d. name and/or some ref's $H^*(t)$ Q_x^*	generalized STER d. name and/or some ref's $G(t)$ P_x
doubly truncated geometric d., [61] $t\,\frac{{}_2F_1(1,-R+1;-R+1;qt)}{{}_2F_1(1,-R+1;-R+1;q)}$ $\left(\sum_{j=1}^R q^j\right)^{-1} q^x,\ x=1,2,\ldots,R$ $R\in N,\ q>0$	doubly truncated logarithmic d., [7], [49] $t\,\frac{{}_3F_2(1,1,-R+1;-R+1,2;qt)}{{}_3F_2(1,1,-R+1;-R+1,2;q)}$ $\left(\sum_{j=1}^R \frac{q^j}{j}\right)^{-1}\frac{q^x}{x},\ x=1,2,\ldots,R$	shifted right truncated logarithmic d. $\frac{{}_3F_2(1,1,-R+1;-R+1,2;q)}{(1-t)\,{}_2F_1(1,-R+1;-R+1;q)}\left(1-\frac{t\,{}_3F_2(1,1,-R+1;-R+1,2;qt)}{{}_3F_2(1,1,-R+1;-R+1,2;q)}\right)$ $\left(\sum_{j=1}^R q^j\right)^{-1}\sum_{i=x}^{R-1}\frac{q^{i+1}}{i+1},\ x=0,1,\ldots,R-1$
[25], [58] $t\,\frac{{}_2F_1(-n,1;-N-n+1;t)}{{}_2F_1(-n,1;-N-n+1;1)}$ $\frac{\binom{N+n-x}{n-x+1}}{\binom{N+n}{n}},$ $x=1,2,\ldots,n+1$ $N,n\in N$	[26] $t\,\frac{{}_3F_2(-n,1,1;-N-n+1,2;t)}{{}_3F_2(-n,1,1;-N-n+1,2;1)}$ $\frac{(n+1)_{(x)}}{x(N+n)_{(x)}}\left(\sum_{j=1}^{n+1}\frac{(n+1)_{(j)}}{j(N+n)_{(j)}}\right)^{-1},$ $x=1,2,\ldots,n+1$	${}_3F_2(-n,1,1;N+1,2;1-t)$ $\binom{N+n}{n}^{-1}\sum_{i=x}^{n}\frac{1}{i+1}\binom{N+n-i-1}{n-i},$ $x=0,1,\ldots,n$
1-disp. inverse hypergeometric d., [29] $t\,\frac{{}_2F_1(-N+M,k;-N+k;t)}{{}_2F_1(-N+M,k;-N+k;1)}$ $\frac{\binom{k+x-2}{x-1}\binom{N-k-x+1}{M-k}}{\binom{N}{M}},$ $x=1,2,\ldots,N-M+1$ $N,M,k\in N,\ N>M\geqq k$	positive negative hypergeometric d., [60] $t\,\frac{{}_3F_2(-N+M,1,k;-N+k,2;t)}{{}_3F_2(-N+M,1,k;-N+k,2;1)}$ $\frac{\binom{k+x-2}{x}\binom{N-k-x+1}{N-M+1-x}}{\binom{N}{N-M+1}-\binom{N-k+1}{N-M+1}},$ $x=1,2,\ldots,N-M+1$	${}_3F_2(-N+M,1,k;M+1,2;1-t)$ $\binom{N}{M}^{-1}\sum_{i=x}^{N-M}\frac{1}{i+1}\binom{k+i-1}{i}\binom{N-k-i}{M-k},$ $x=0,1,\ldots,N-M$

TABLE 2 (CONT.).

parent d. name and/or some ref's $G^*(t)$ P^*_x constrains on parameters	godparent d. name and/or some ref's $H^*(t)$ Q^*_x	generalized STER d. name and/or some ref's $G(t)$ P_x
[4], [42], [43], [70] $t\,\frac{{}_2F_1(-n+k,1;-n;t)}{{}_2F_1(-n+k,1;-n;1)}$ $\frac{\binom{n-x+1}{k}}{\binom{n+1}{k+1}}$, $x=1,2,\ldots,n-k+1$ $k,n\in N,\ n>k$	[26] $t\,\frac{{}_3F_2(-n+k,1,1;-n,2;t)}{{}_3F_2(-n+k,1,1;-n,2;1)}$ $\frac{(n-k+1)_{(x)}}{x(n+1)_{(x)}}\left(\sum_{j=1}^{n-k+1}\frac{(n-k+1)_{(j)}}{j(n+1)_{(j)}}\right)^{-1}$, $x=1,2,\ldots,n-k+1$	${}_3F_2(-n+k,1,1;k+2,2;1-t)$ $\binom{n+1}{k+1}^{-1}\sum_{i=x}^{n-k}\frac{1}{i+1}\binom{n-i}{k}$, $x=0,1,\ldots,n-k$
1-disp. negative hypergeometric d., [29] $t\,\frac{{}_2F_1(-n,M;-n-K+M+1;t)}{{}_2F_1(-n,M;-n-K+M+1;1)}$ $\frac{\binom{M+x-2}{x-1}\binom{K-M+n-x}{n-x+1}}{\binom{K+n-1}{n}}$, $x=1,2,\ldots,n+1$ $n\in N,\ K>M>1$	positive negative hypergeometric d., [60] $t\,\frac{{}_3F_2(-n,M,1;-n-K+M+1,2;t)}{{}_3F_2(-n,M,1;-n-K+M+1,2;1)}$ $\frac{\binom{M+x-2}{x}\binom{K-M+n-x}{n-x+1}}{\binom{K+n-1}{n+1}-\binom{K-M+n}{n+1}}$, $x=1,2,\ldots,n+1$	${}_3F_2(-n,M,1;K,2;1-t)$ $\binom{K+n-1}{n}^{-1}\sum_{i=x}^{n}\frac{1}{i+1}\binom{M+i-1}{i}\binom{K-M+n-1-i}{n-i}$, $x=0,1,\ldots,n$
1-displaced Pólya d., [29] $t\,\frac{{}_2F_1(-n,\frac{a}{c};-n-\frac{b}{c}+1;t)}{{}_2F_1(-n,\frac{a}{c};-n-\frac{b}{c}+1;1)}$ $\binom{n}{x-1}\frac{\left(\frac{a}{c}+x-2\right)_{(x-1)}\left(\frac{b}{c}+n-x\right)_{(n-x+1)}}{\left(\frac{a+b}{c}+n-1\right)_{(n)}}$, $x=1,2,\ldots,n+1$ $c\neq 0,\ \frac{a}{c}>1,\ \frac{b}{c}>0,\ n\in N$	positive negative hypergeometric d., [60] $t\,\frac{{}_3F_2(-n,\frac{a}{c},1;-n-\frac{b}{c}+1,2;t)}{{}_3F_2(-n,\frac{a}{c},1;-n-\frac{b}{c}+1,2;1)}$ $\frac{\binom{\frac{a}{c}+x-2}{x}\binom{\frac{b}{c}+n-x}{n-x+1}}{\binom{\frac{a+b}{c}+n-1}{n+1}-\binom{\frac{b}{c}+n}{n+1}}$, $x=1,2,\ldots,n+1$	${}_3F_2(-n,\frac{a}{c},1;\frac{a+b}{c},2;1-t)$ $\sum_{i=x}^{n}\binom{n}{i}\frac{\left(\frac{a}{c}+i-1\right)_{(i)}\left(\frac{b}{c}+n-i-1\right)_{(n-i)}}{(i+1)\left(\frac{a+b}{c}+n-1\right)_{(n)}}$, $x=0,1,\ldots,n$
[1-disp. right trunc. geom (q,R)] $t\,\frac{{}_2F_1(1,-R;-R;qt)}{{}_2F_1(1,-R;-R;q)}$ $\left(\sum_{j=0}^{R}q^j\right)^{-1}q^{x-1},\ x=1,2,\ldots,R+1$ $R\in N,\ q>0$	[doubly trunc. logar. (q,1,R+1)] $t\,\frac{{}_3F_2(1,1,-R;2,-R;qt)}{{}_3F_2(1,1,-R;2,-R;q)}$ $\left(\sum_{j=1}^{R+1}\frac{q^j}{j}\right)^{-1}\frac{q^x}{x},\ x=1,2,\ldots,R+1$	$\frac{{}_3F_2(1,1,-R;2,-R;q)}{(1-t)\ {}_2F_1(1,-R;-R;q)}\left(1-\frac{t\ {}_3F_2(1,1,-R;2,-R;qt)}{{}_3F_2(1,1,-R;2,-R;q)}\right)$ $\left(\sum_{j=0}^{R}q^j\right)^{-1}\sum_{i=x}^{R}\frac{q^i}{i+1},\ x=0,1,\ldots,R$

TABLE 2 (CONT.).

parent d. name and/or some ref's $G^*(t)$ P^*_x constrains on parameters	godparent d. name and/or some ref's $H^*(t)$ Q^*_x	generalized STER d. name and/or some ref's $G(t)$ P_x
1-disp. right trunc. neg. bin. d., [33], [49] $t\,\frac{{}_2F_1(k,-R;-R;qt)}{{}_2F_1(k,-R;-R;q)}$ $\left[\sum_{j=0}^{R}\binom{k+j-1}{j}q^j\right]^{-1}\binom{k+x-2}{x-1}q^{x-1}$, $x=1,2,\dots,R+1$ $R\in N,\ q>0,\ k>1$	doubly trunc. negative binomial d., [7] $t\,\frac{{}_3F_2(k,-R,1;-R,2;qt)}{{}_3F_2(k,-R,1;-R,2;q)}$ $\left[\sum_{j=1}^{R+1}\binom{k+j-2}{j}q^j\right]^{-1}\binom{k+x-2}{x}q^{x}$, $x=1,2,\dots,R+1$	$\frac{{}_3F_2(k,-R,1;-R,2;q)}{(1-t)\ {}_2F_1(k,-R;-R;q)}\left(1-\frac{t\ {}_3F_2(k,-R,1;-R,2;qt)}{{}_3F_2(k,-R,1;-R,2;q)}\right)$ $\left[\sum_{j=0}^{R}\binom{k+j-1}{j}q^j\right]^{-1}\sum_{i=x}^{R}\binom{k+i-1}{i}\frac{q^i}{i+1}$, $x=0,1,\dots,R$
[59] $t\,\frac{{}_2F_1(-N,1;-N-K+1;t)}{{}_2F_1(-N,1;-N-K+1;1)}$ $\frac{\binom{N+K-x}{K-1}}{\binom{N+K}{K}}$, $x=1,2,\dots,N+1$ $K,N\in N$	[14] $t\,\frac{{}_3F_2(-N,1,1;-N-K+1,2;t)}{{}_3F_2(-N,1,1;-N-K+1,2;1)}$ $\frac{(N+1)_{(x)}}{x(N+K)_{(x)}}\left(\sum_{j=1}^{N+1}\frac{(N+1)_{(j)}}{j(N+K)_{(j)}}\right)^{-1}$, $x=1,2,\dots,N+1$	${}_3F_2(-N,1,1;K+1,2;1-t)$ $\binom{N+K}{K}^{-1}\sum_{i=x}^{N}\frac{1}{i+1}\binom{N+K-1-i}{K-1}$, $x=0,1,\dots,N$
[59] $t\,\frac{{}_2F_1(-m+1,1;-2m+2;t)}{{}_2F_1(-m+1,1;-2m+2;1)}$ $\frac{\binom{m}{x}}{\binom{2m-1}{x}}$, $x=1,2,\dots,m$ $m\in N$	[26] $t\,\frac{{}_3F_2(-m+1,1,1;-2m+2,2;t)}{{}_3F_2(-m+1,1,1;-2m+2,2;1)}$ $\frac{(m)_{(x)}}{x(2m-1)_{(x)}}\left(\sum_{j=1}^{m}\frac{m_{(j)}}{j(2m-1)_{(j)}}\right)^{-1}$, $x=1,2,\dots,m$	${}_3F_2(-m+1,1,1;m+1,2;1-t)$ $\sum_{i=x}^{m-1}\frac{m_{(i+1)}}{(i+1)(2m-1)_{(i+1)}}$, $x=0,1,\dots,m-1$

TABLE 2 (CONT.).

parent d. name and/or some ref's $G^*(t)$ P_x^* constrains on parameters	godparent d. name and/or some ref's $H^*(t)$ Q_x^*	generalized STER d. name and/or some ref's $G(t)$ P_x
[77] $t\,\frac{{}_2F_1(-2^{m-1}+1,1;-2^m+2;t)}{{}_2F_1(-2^{m-1}+1,1;-2^m+2;1)}$ $\frac{2^{m-1}_{(x)}}{(2^m-1)_{(x)}}$, $x=1,2,\ldots,2^{m-1}$ $m \in \{2,3,\ldots\}$	[26] $t\,\frac{{}_3F_2(-2^{m-1}+1,1,1;-2^m+2,2;t)}{{}_3F_2(-2^{m-1}+1,1,1;-2^m+2,2;1)}$ $\frac{2^{m-1}_{(x)}}{x(2^m-1)_{(x)}}\left(\sum_{j=1}^{2^{m-1}}\frac{2^{m-1}_{(j)}}{j(2^m-1)_{(j)}}\right)^{-1}$, $x=1,2,\ldots,2^{m-1}$	${}_3F_2(-2^{m-1}+1,1,1;2^{m-1}+1,2;1-t)$ $\sum_{i=x}^{2^{m-1}-1}\frac{2^{m-1}_{(i+1)}}{(i+1)(2^m-1)_{(i+1)}}$, $x=0,1,\ldots,2^{m-1}-1$
[77] $t\,\frac{{}_2F_1(-2^{m-1}+1,2^{m-1};-2^{m-1}+1;t)}{{}_2F_1(-2^{m-1}+1,2^{m-1};-2^{m-1}+1;1)}$ $\frac{\binom{2^{m-1}+x-2}{x-1}}{\binom{2^m-1}{2^{m-1}-1}}$, $x=1,2,\ldots,2^{m-1}$ $m \in \{2,3,\ldots\}$	positive negative hypergeometric d., [60] $t\,\frac{{}_3F_2(-2^{m-1}+1,2^{m-1},1;-2^{m-1}+1,2;t)}{{}_3F_2(-2^{m-1}+1,2^{m-1},1;-2^{m-1}+1,2;1)}$ $\frac{\binom{2^{m-1}-2+x}{x}}{\binom{2^m-1}{2^m-1}-1}$, $x=1,2,\ldots,2^{m-1}$	${}_3F_2(-2^{m-1}+1,2^{m-1},1;2^{m-1}+1,2;1-t)$ $\binom{2^m-1}{2^{m-1}-1}^{-1}\sum_{i=x}^{2^{m-1}-1}\frac{1}{i+1}\binom{2^{m-1}+i-1}{i}$, $x=0,1,\ldots,2^{m-1}-1$
1-displaced hypergeometric d.,[29] $t\,\frac{{}_2F_1(-n,-M;N-M-n+1;t)}{{}_2F_1(-n,-M;N-M-n+1;1)}$ $\frac{\binom{M}{x-1}\binom{N-M}{n-x+1}}{\binom{N}{n}}$, $x=1,2,\ldots,\min(n,M)+1$ $n,N,M \in N,\ N>n+M$	positive hypergeometric d.,[12],[29] $t\,\frac{{}_3F_2(-n,-M,1;N-M-n+1,2;t)}{{}_3F_2(-n,-M,1;N-M-n+1,2;1)}$ $\frac{\binom{M+1}{x}\binom{N-M}{n+1-x}}{\binom{N+1}{n+1}-\binom{N-M}{n+1}}$, $x=1,2,\ldots,\min(n,M)+1$	${}_3F_2(-n,-M,1;-N,2;1-t)$ $\binom{N}{n}^{-1}\sum_{i=x}^{\min(n,M)}\frac{1}{i+1}\binom{M}{i}\binom{N-M}{n-i}$, $x=0,1,\ldots,\min(n,M)$

TABLE 2 (CONT.).

parent d. name and/or some ref's $G^*(t)$ P_x^* constrains on parameters	godparent d. name and/or some ref's $H^*(t)$ Q_x^*	generalized STER d. name and/or some ref's $G(t)$ P_x
[28], [69] $t\,\frac{_2F_1(-n_0,-n_1+1;2;t)}{_2F_1(-n_0,-n_1+1;2;1)}$ $\frac{\binom{n_1-1}{x-1}\binom{n_0+1}{n_0-x+1}}{\binom{n_0+n_1}{n_1}}$, $x=1,2,\ldots,\min(n_0+1,n_1)$ $n_0,n_1 \in N$	positive hypergeometric d., [12], [29] $t\,\frac{_3F_2(-n_0,-n_1+1,1;2,2;t)}{_3F_2(-n_0,-n_1+1,1;2,2;1)}$ $\frac{\binom{n_1}{x}\binom{n_0+1}{n_0+1-x}}{\binom{n_0+n_1+1}{n_0+1}-1}$, $x=1,2,\ldots,\min(n_0+1,n_1)$	$_3F_2(-n_0,-n_1+1,1;-n_0-n_1,2;1-t)$ $\binom{n_0+n_1}{n_1}^{-1}\sum_{i=x}^{\min(n_0+1,n_1)-1}\frac{1}{i+1}\binom{n_1-1}{i}\binom{n_0+1}{n_0-i}$, $x=0,1,\ldots,\min(n_0+1,n_1)-1$
[47] $t\,\frac{_2F_1(-R+k,-n+k;-R+k;-at)}{_2F_1(-R+k,-n+k;-R+k;-a)}$ $\left(\sum_{j=k}^{R}\binom{n}{j}\frac{j!a^j}{(j-k)!}\right)^{-1}\binom{n}{k+x-1}\frac{(k+x-1)!a^{k+x-1}}{(x-1)!}$, $x=0,1,\ldots,\min(n-k,R-k)+1$ $a>0,\ k,n,R\in N,\ n\geqq k,\ R\geqq k$	doubly truncated binomial d., [7], [29] $t\,\frac{_3F_2(-R+k,-n+k,1;-R+k,2;-at)}{_3F_2(-R+k,-n+k,1;-R+k,2;-a)}$ $\left(\sum_{j=1}^{R-k+1}\binom{n-k+1}{j}a^{j-1}\right)^{-1}\binom{n-k+1}{x}a^{x-1}$, $x=1,2,\ldots,\min(n-k,R-k)+1$	$\frac{_3F_2(-R+k,-n+k,1;-R+k,2;-a)}{(1-t)\ _2F_1(-R+k,-n+k;-R+k;-a)}\left(1-\frac{t\ _3F_2(-R+k,-n+k,1;-\boldsymbol{R+k},\boldsymbol{2};-at)}{_3F_2(-R+k,-n+k,1;-\boldsymbol{R+k},\boldsymbol{2};-a)}\right)$ $\left(\sum_{j=k}^{R}\binom{n}{j}\frac{j!a^j}{(j-k)!}\right)^{-1}\sum_{i=x}^{\min(n-k,R-k)}\binom{n}{k+i}\frac{(k+i)!a^{k+i}}{(i+1)!}$ $x=0,1,\ldots,\min(n-k,R-k)$
1-disp. right trunc. binomial d., [47], [49] $t\,\frac{_2F_1(-R,-n;-R;-at)}{_2F_1(-R,-n;-R;-a)}$ $\left(\sum_{j=0}^{R}\binom{n}{j}a^j\right)^{-1}\binom{n}{x-1}a^{x-1}$, $x=0,1,\ldots,\min(n,R)+1$ $a>0,\ n,R\in N$	doubly trunc. binomial d., [7], [29] $t\,\frac{_3F_2(-R,-n,1;-R,2;-at)}{_3F_2(-R,-n,1;-R,2;-a)}$ $\left(\sum_{j=1}^{R+1}\binom{n+1}{j}a^{j-1}\right)^{-1}\binom{n+1}{x}a^{x-1}$, $x=1,2,\ldots,\min(n,R)+1$	$\frac{_3F_2(-R,-n,1;-R,2;-a)}{(1-t)\ _2F_1(-R,-n;-R;-a)}\left(1-\frac{t\ _3F_2(-R,-n,1;-\boldsymbol{R},\boldsymbol{2};-\boldsymbol{at})}{_3F_2(-R,-n,1;-\boldsymbol{R},\boldsymbol{2};-\boldsymbol{a})}\right)$ $\left(\sum_{j=0}^{R}\binom{n}{j}a^j\right)^{-1}\sum_{i=x}^{\min(n,R)}\binom{n}{i}\frac{a^i}{i+1}$, $x=0,1,\ldots,\min(n,R)$

TABLE 2 (CONT.).

parent d.	godparent d.	generalized STER d.
name and/or some ref's	name and/or some ref's	name and/or some ref's
$G^*(t)$	$H^*(t)$	$G(t)$
P_x^*	Q_x^*	P_x
constrains on parameters		
[16], [79]	[15]	
$t\,\frac{{}_3F_2(-R+1,1,1;-R+1,2;t)}{{}_3F_2(-R+1,1,1;-R+1,2;1)}$	$t\,\frac{{}_4F_3(-R+1,1,1,1;-R+1,2,2;t)}{{}_4F_3(-R+1,1,1,1;-R+1,2,2;1)}$	$\frac{{}_4F_3(-R+1,1,1,1;-R+1,2,2;1)}{(1-t)\;{}_3F_2(-R+1,1,1;-R+1,2;1)}\left(1-\frac{t\;{}_4F_3(-R+1,1,1,1;-R+1,2,2;t)}{{}_4F_3(-R+1,1,1,1;-R+1,2,2;1)}\right)$
$\frac{1}{x[\Psi(R+1)-\Psi(1)]}$,	$\frac{1}{x^2[\Psi'(1)-\Psi'(R+1)]}$,	$\frac{1}{\Psi'(1)-\Psi'(R+1)}\sum_{i=x}^{R-1}\frac{1}{(i+1)^2}$,
$x=1,2,\ldots,R$	$x=1,2,\ldots,R$	$x=0,1,\ldots,R-1$
$R\in N$, Ψ-Digamma function		

References

[1] Abakuks, A.: *An invariance property of the Poisson distributions for a limited immigrant popupation.* J. of the Royal Statistical Society B **38** (1976), 99–101.

[2] Ancker, C. J. Jr., Gafarian, A. V.: *Some queueing problems with balking and reneging I.* Operations Research **11** (1963), 88–100.

[3] Bardwell, G. E., Crow, E. L.: *A two-parameter family of hyper-Poisson distributions.* J. of the American Statistical Association **59** (1964), 133–141.

[4] Becker, N. G.: *The spread of epidemic to fixed groups within the population.* Biometrics **24** (1968), 1007–1014.

[5] Bissinger, B. H.: *A type resistant distribution generated from consideration of an inventory decision model.* In: G. P. Patil (ed.) Classical and contagious Discrete Distributions. Calcutta: Statistical Publishing Society and Oxford: Pergamon Press, 15–17.

[6] Brockwell, P. J., Gani, J., Resnick, S. I.: *Birth, immigration and catastrophe processes.* Advances in Applied Probability **14** (1982), 709–731.

[7] Cacoullos, T.: *Best estimation for multiply truncated power series distributions.* Scandinavian J. of Statistics **4** (1977), 159–164.

[8] Cavender, J. A.: *Quasi-stationary distributions of birth-and-death processes.* Advances in Applied Probability **10** (1978), 570–586.

[9] Chung, K. L., Feller, W.: *On fluctuations in coin-tossing.* Proceedings of the National Academy of Sciences **35** (1949), 605–608.

[10] Conolly, B. W.: *More transient results for generalized state-dependent Erlangian queues.* Advances in Applied Probability **15** (1983), 688–690.

[11] Dacey, M. F.: *Probability distribution of number of networks in topologicaly random network patterns.* Water Resources Research **7**, (1971), 1652–1657.

[12] Dacey, M. F.: *A family of discrete probability distributions defined by the generalized hypergeometric series.* Sankhyā **B 34** (1972), 243–250.

[13] Dacey, M. F.: *Probability laws of topological properties of drainage basins.* In: G. P. Patil, S. Kotz and J. K. Ord (eds.). Statistical Distributions in Scientific Work, Dodrecht, Reidel **2**, 327–341.

[14] Darwin, J. H.: *An ecological distribution akin to Fisher's logarithmic distribution.* Biometrics **16** (1960), 51–60.

[15] Dubey, S. D.: *Asymptotically single observation best estimator of exponential expected life.* Sankhyā **A 27** (1965), 133–142.

[16] Estoup, J. B.: Les Gammes Sténographiques. *Gauthier–Villars, Paris: Institute Sténographique,* 1916.

[17] Feller, W.: *On a general class of contagious distributions.* Annals of Mathematical Statistics **14** (1943), 389–400.

[18] Flory, P. J.: *Intramolecular reaction between neighbouring substituents of vinyl polymers.* J. of the American Chemical Society **61** (1939), 1518–1521.

[19] Gani, J.: *Problems in the probability theory of storage systems.* J. of the Royal Statistical Society **B 19** (1957), 182–206.

[20] Gerstenkorn, T.: *The compounding of the binomial and generalized beta distributions.* In: W. Grossmann, G. Ch. Pflug, W. Wertz (eds.). Probability and Statistical Inference. Dodrecht: Reidel, 1982, 87–99.

[21] Gipps, P. G.: *A queueing model for traffic flow.* J. of the Royal Statistical Society **39** (1977), 276–282.

[22] Gurland, J.: *A generalized class of contagious distributions.* Biometrics **14** (1958), 229–249.

[23] Haight, F. A.: *On the effect of removing persons with N or more accidents from an accident prone popupation.* Biometrika **52** (1965), 298–300.

[24] Hall, P.: *On the roles of the Bessel and Poisson distributions in chemical kinetics.* J. of Applied Probability **20** (1983), 585–599.

[25] Harris, L. B.: *On a limiting case for the distribution of exceedances, with an application to life-testing.* Annals of Mathematical Statistics **23** (1952), 295–298.

[26] van Heerden, D. F. I., Gonin, H. T.: *The orthogonal polynomials of the factorial power series probability distributions.* South African Statistical J. **1** (1967), 49–53.

[27] Hoppe, F. M.: *The sampling theory of neutral alleles and an urn model in population genetics.* J. of Mathematical Biology **25** (1987), 123–159.

[28] Ising, E.: *Beitrag zur Theorie des Ferromagnetismus.* Zeischrift für Physik **31** (1925), 253–258.

[29] Johnson, N. L., Kotz, S., Kemp, A. W.: Univariate Discrete Distributions. *Wiley, New York,* 1992.

[30] Karlin, S., Taylor, H. M.: A Second Course in Stochastic Processes. *Academic Press, New York,* 1981.

[31] Katz, L., Powell, J. H.: *Probability distributions of random variables associated with a structure of the sample space of sociometric investigations.* Annals of Mathematical Statistics **28** (1957), 442–448.

[32] Kelly, F. P.: *Exact results for the Moran neutral allele model.* Advance in Applied Probability **9** (1977), 197–201.

[33] Kemp, A. W.: *A wide class of discrete distributions and the associated differential equations.* Sankhyā **A 30** (1968), 401–410.

[34] Kemp, A. W.: *A limited risk cPp.* Skandinavisk Aktuarietidskrift **51** (1968), 198–203.

[35] Kemp, A. W., Kemp, C. D.: *Branching and clustering models associated with the 'Lost games distribution'.* J. of Applied Probability **6** (1969), 700–703.

[36] Kemp, A. W., Kemp, C. D.: *A family of discrete distributions defined via their factorial moments.* Communications in Statistics **3** (1974), 1187–1196.

[37] Kemp, A. W., Kemp, C. D.: *Models for Gaussian hypergeometric distribution.* In: G. P. Patil, S. Kotz, J. K. Ord (eds.). Statistical Distributions in Scientific Work. Dodrecht: Reidel **1** (1975), 31–40.

[38] Kemp, C. D., Kemp, A. W.: *Some distributions arising from an inventory decision problem.* Bulletin of the International Statistical Institute **43** (1969), 367–369.

[39] Kleinrock, L.: Queueing Systems. Volume I: Theory. *Wiley, New York,* 1975.

[40] Krishnaji, N.: *A characteristic property of the Yule distribution.* Sankhyā **A 32** (1970), 343–346.

[41] Kupper, J.: *Wahrscheinlichkeitstheoretische Modelle in der Schadenversicherung. Teil I: Die Schadenzahl.* Blätter der Deutschen Gesellschaft für Versicherungsmathematik **5** (1960–62), 451–503.

[42] Martin-Löf, P.: *The notion of redundancy and its use as a quantitative measure of discrepancy between a statistical hypothesis and observational data.* Scandinavian J. of Statistics **1** (1974), 3–18.

[43] Martin-Löf, P.: *Reply to Sverdrup's polemical article Tests without power.* Scandinavian J. of Statistics **2** (1975), 161–165.

[44] Miller, A. J.: *A queueing model for road traffic flow.* J. of the Royal Statistical Society **B 23** (1961), 64–75.

[45] Moore, P. G.: *A note on truncated Poisson distribution.* Biometrics **10** (1954), 402–406.

[46] Moran P. A. P.: *A probability theory of dams and storage systems: modifications of the release rule.* Australian J. of Applied Science **6** (1955), 117–130.

[47] Moran P. A. P.: *The Engset distribution in telephone congestion.* Australian J. of Applied Science **12** (1961), 257–264.

[48] Ord, J. K.: Families of Frequency Distributions. *Griffin, London,* 1972.

[49] Patil, G. P.: *Estimation by two-moment method for generalized power series distribution and certain applications.* Sankhyā **B 24** (1962), 201–214.

[50] Patil, G. P. (ed.): Classical and contagious Discrete Distributions. *Calcutta: Statistical Publishing Society and Oxford: Pergamon Press,* 1965.

[51] Patil, G. P. Joshi, S. W.: A Dictionary and Bibliography of Discrete Distributions. *Oliver & Boyd, Edinburgh,* 1968.

[52] Patil, G. P. Boswell, M. T., Joshi, S. W., Ratnaparkhi, M. V.: Dictionary and Clasified Bibliography of Statistical Distributions in Scientific Work. *International Co-operative House, Fairland, Maryland,* 1984.

[53] Prasad, A.: *A new discrete distribution.* Sankhyā **17** (1957), 353–354.

[54] Price, B.: *A note on estimation in Bernoulli trials with dependence.* Communications in Statistics—Theory and Methods **A 45** (1976), 661–671.

[55] Quinkert, W.: Die kollektive Risikotheorie unter Berücksichtigung schwankender Grundwahrscheinlichkeiten mit endlichem Schwankungsbereich. *Diss., Karlsruhe,* 1957.

[56] Sachkov, V. N.: *Distribution of the number of fixed points of elements of a symmetric semigroup under the condition $\sigma^{h+1} = \sigma^h$ and the number of trees of height not exceeding h.* Theory of Probability and its Applications **16** (1971), 661–673.

[57] Samaniego, F. J.: *A characterization of convoluted Poisson distributions with applications to estimation.* J. of the American Statistical Association **71** (1976), 475–479.

[58] Schelling, H. v. *A second formula for the partial sum of hypergeometric series having unity as a fourth argument.* The Annals of Mathematical Statistics **21** (1950), 458–460.

[59] Schwarz, G., Tversky, A.: *On the reciprocity of proximity relations.* J. of Mathematical Psychology **22** (1980), 157–175.

[60] Seber, G. A. F.: The Estimation of Animal Abundance and Related Parameters. *Griffin, London,* 1973.

[61] Selvin, S.: *Maximum likelihood estimation in the truncated, single parameter discrete exponential family.* The Annals of Statistics **25** (1971), 41–42.

[62] Shahani, A. K., Crease, D. M.: *Towards models of screening for early detection of disease.* Advances in Applied Probability **9** (1977), 665–680.

[63] Shreve, R. L.: *Statistical law of stream numbers.* J. of Geology **74** (1966), 17–37.

[64] Shreve, R. L.: *Infinite topologically random channel networks.* J. of Geology **75** (1967), 178–186.

[65] Sibuya, M.: *Generalized hypergeometric, digamma and trigamma distributions.* Annals of the Institute of Statistical Mathematics **31** (1979), 373–390.

[66] Sibuya, M.: *A comparison of discrete distributions.* In: Matusita, K. (ed.). Recent Development in Statistical Inference and Data Analysis, Amsterdam, North Holland, 1980, 255–258.

[67] Simon, H. A.: *On a class of skew distribution functions.* Biometrika **42** (1955), 425–440.

[68] Simon, H. A.: Models of man. Mathematical essays on rational human behavior in a social setting. *Wiley, New York,* 1957.

[69] Stevens, W. L.: *Significance of groupings.* Annals of Eugenics **8** (1937), 57–69.

[70] Sverdrup, E.: *Tests without power.* Scandinavian J. of Statistics **2** (1975), 158–160.

[71] Tripathi, R. C., Gupta, R. C.: *A comparison between the ordinary and the length-biased modified power series distributions with applications.* Communications in Statistics—Theory and Methods **A 16** (1987), 1195–1206.

[72] Tripathi, R. C., Gupta, R. C., White, T. J.: *Some generalizations of the geometric distribution.* Sankhyā **B 49** (1987), 218–223.

[73] Tripathi, R. C., Gurland, J.: *A general family of discrete distributions with hypergeometric probabilities.* J. of the Royal Statistical Society **B 39** (1977), 349–356.

[74] Tripathi, R. C., Gurland, J.: *Some aspects of the Kemp families of distributions.* Communications in Statistics—Theory and Methods **A 8** (1979), 855–869.

[75] Wenocur, R. S.: *Waiting times and return periods to exceed the maximum of a previous sample.* In: Ch. Taillie, G. P. Patil, B. A. Baldessari (eds.). Statistical Distributions in Scientific Work, Dodrecht: Reidel **6** (1981), 411–418.

[76] Willmot, G. E.: *Mixed compound distributions.* ASTIN Bulletin **16** (1986), S59–S79.

[77] Wiorkowski, J. J.: *A curious aspect of knockout tournaments of size* 2^m. The American Statistician **26** (1972), 28–30.

[78] Xekalaki, E.: *Linear regression and the Yule distribution.* J. of Econometrics **24** (1984), 397–403.

[79] Zörnig, P., Altmann, G.: *Unified representation of Zipf distributions.* Computational Statistics & Data Analysis **19** (1995), 461–473.