Kybernetika

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Kybernetika, Vol. 20 (1984), No. 6, 473--482

Persistent URL: `http://dml.cz/dmlcz/124605`

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KYBERNETIKA — VOLUME 20 (1984), NUMBER 6

ROBUSTNESS OF DECENTRALIZED CONTROL SUBJECT TO NONLINEAR PERTURBATION IN THE SYSTEM DYNAMICS*

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This paper considers the problem of robustness in decentralized control system designs subject to nonlinear perturbation in the system dynamics. New relationship is established between perturbation bounds and dominant poles of the nominal closed loop system. The theory is applied to characterize the robustness properties of a power system example which employ five DC terminals to damp out interarea oscillations due to the AC power system dynamics.

1. INTRODUCTION

In recent years, there has been an increased interest in the development of satisfactory control design methods implemented in a decentralized way. One of the most basic issues that arise in this class of problems is the robustness of the decentralized design, i.e. its ability to maintain stability and performance in the face of uncertainties. In this light, stability is a primary concern since an unstable system is obvious useless. The major objective of this paper is to show that the decentralized control systems with a prescribed degree of stability, can accommodate different types of nonlinear perturbations in the system dynamics, so that the perturbed system remains stable. New relationships are established between perturbation bounds and dominant poles of the nominal closed loop system, thus helping a designer to select an appropriate degree of stability to attain a robust design.

Consider the decentralized control system described by

$$\dot{x}(t) = A\,x(t) + \sum_{i=1}^{k} B_i\,u_i(t) \tag{1}$$

$$u_i(t) = F_i C_i\,x(t)\,, \quad i = 1, 2, \ldots, k. \tag{2}$$

At least two approaches can be used to determine the decentralized feedback matrices F_i. The first one is based on minimization of the decentralized quadratic performance

* Research supported in part by the U.S.-Yugoslav Scientific and Technological Cooperation under Grant ENERGY-401.

index (cf. [1], [6])

$$J = \tfrac{1}{2}\int_0^\infty e^{2\alpha t}(x^T Q x + \sum_{i=1}^{k} u_i^T R_i u_i)\,dt \tag{3}$$

$Q = Q^T \geqq 0$, $R_i = R_i^T > 0$, $i = 1, 2, \ldots, k$ and α is a nonnegative constant. The second approach (cf. [2], [6]) is based on computation of a complete state feedback (by LQ methodology) and reduction to a specified control with a decentralized structure.

We assume that the system, eqns. (1) and (2), is stabilizable and that, by one of the above approaches, the feedback matrices F_i have been selected so that the closed loop matrix

$$A_c = A + \sum_{i=1}^{k} B_i F_i C_i \tag{4}$$

is a stability matrix so that

$$\mathrm{Re}\,[\lambda(A_c)] \leqq -\alpha \tag{5}$$

where α defines the minimal degree of stability. Therefore, we can define the following Lyapunov matrix equation

$$A_c^T P + P A_c + 2\alpha P + \sum_{i=1}^{k} C_i^T F_i^T R_i F_i C_i + Q = 0\,. \tag{6}$$

Notice that the value of the corresponding performance index is

$$J = e^{2\alpha t_0} x^T(t_0)\, P\, x(t_0) \tag{7}$$

where P is defined by (6). For ease in subsequent calculation it is assumed that P is positive definite.

2. ROBUSTNESS RESULTS

Let the perturbed version of the nominal system, eqns. (1) and (2), satisfies

$$(\hat{S})\left\{\begin{aligned} \dot{\hat{x}} &= A\hat{x} + \sum_{i=1}^{k} B_i \hat{u}_i + f(\hat{x}, \hat{u}, t) && (8)\\ \hat{u}_i &= F_i C_i \hat{x} && (9)\end{aligned}\right.$$

where A, B_i, F_i and C_i are the same as in eqns. (1) and (2); so that all the parameter variations, nonlinearities in the open loop system dynamics and the design lineriza- tion are lumped into the vector function $f(\hat{x}, \hat{u}, t)$.

Therefore, we are not restricting our attention to linear systems with nonlinear parameters variations in the system dynamics but $(\hat{S})$ can also be interpreted as a linearization of a general nonlinear system. The control engineer's task would indeed not be an enviable one if the mathematical description of all physical systems requires, for the purpose of control system design, a general nonlinear model of the process dynamics. The presence of the nonlinear disturbance vector $\boldsymbol{f}(\hat{\boldsymbol{x}}, \hat{\boldsymbol{u}}, t)$ makes the overall system $(\hat{\mathrm{S}})$ a nonlinear one, and despite the avaliability of modern design techniques the generation of algorithm for the stabilization of nonlinear systems, as is well known, is extremely complicated. Even if a satisfactory algorithm is found, the data-processing difficulties arising from the high dimensionality of the system make the determination of the necessary control functions quite formidable.

The major objective of this paper is to show that when the nonlinear perturbation vector $\boldsymbol{f}(\hat{\boldsymbol{x}}, \hat{\boldsymbol{u}}, t)$ admits certain norm bounds or conforms to certain symmetry requirements the perturbed system can be stabilized by using the nominal model of the system for the purpose of control system design.

A reasonable measure of robustness for a feedback system is the magnitude of the otherwise arbitrary perturbations which may be tolerated without instability. The stability margines of the proposed decentralized output feedback control schemes are characterized by the following theorem, which gives sufficient conditions for the closed loop stability of the actual nonlinear system $(\hat{\mathrm{S}})$.

Theorem 1. Let $\boldsymbol{f}(\hat{\boldsymbol{x}}, \hat{\boldsymbol{u}}, t)$ be memoryless, time-varying vector function. If the following inequality is satisfied

$$\frac{\|\boldsymbol{f}(\hat{\boldsymbol{x}}, t)\|_{\mathrm{E}}}{\|\hat{\boldsymbol{x}}\|_{\mathrm{E}}} \leqq \frac{\lambda_{\min}(\boldsymbol{K})}{2\lambda_{\max}(\boldsymbol{P})} \tag{10}$$

where the matrix $\boldsymbol{K}$ is defined by

$$\boldsymbol{K} = \sum_{i=1}^{k} \boldsymbol{C}_i^{\mathrm{T}} \boldsymbol{F}_i^{\mathrm{T}} \boldsymbol{R}_i \boldsymbol{F}_i \boldsymbol{C}_i + \boldsymbol{Q} + 2\alpha \boldsymbol{P} \tag{11}$$

and the matrix $\boldsymbol{P}$ is the solution of the equation (6); for all $t \in [0, \infty)$, then the perturbed system (8) and (9) is asymptotically stable.

Proof. Choose the positive definite Lyapunov function as $V(\hat{\boldsymbol{x}}) = \hat{\boldsymbol{x}}^{\mathrm{T}} \boldsymbol{P} \hat{\boldsymbol{x}}$. Taking the time derivative along the solution of (8) and (9) and using (6), $\dot{V}(\hat{\boldsymbol{x}})$ may be represented as,

$$\dot{V}(\hat{\boldsymbol{x}}) = -\hat{\boldsymbol{x}}^{\mathrm{T}}\Big(\sum_{i=1}^{k} \boldsymbol{C}_i^{\mathrm{T}} \boldsymbol{F}_i^{\mathrm{T}} \boldsymbol{R}_i \boldsymbol{F}_i \boldsymbol{C}_i + \boldsymbol{Q} + 2\alpha \boldsymbol{P}\Big) \hat{\boldsymbol{x}} + 2 \boldsymbol{f}^{\mathrm{T}}(\hat{\boldsymbol{x}}, t) \boldsymbol{P} \hat{\boldsymbol{x}}. \tag{12}$$

Notice that

$$\boldsymbol{f}^{\mathrm{T}}(\hat{\boldsymbol{x}}, t) \boldsymbol{P} \hat{\boldsymbol{x}} \leqq \|\boldsymbol{f}(\hat{\boldsymbol{x}}, t)\|_{\mathrm{E}} \|\boldsymbol{P} \hat{\boldsymbol{x}}\|_{\mathrm{E}} \leqq \|\boldsymbol{f}(\hat{\boldsymbol{x}}, t)\|_{\mathrm{E}} \|\boldsymbol{P}\|_{\mathrm{E}} \|\hat{\boldsymbol{x}}\|_{\mathrm{E}} \tag{13}$$

since,

$$\frac{\|\boldsymbol{P} \hat{\boldsymbol{x}}\|_{\mathrm{E}}}{\|\hat{\boldsymbol{x}}\|_{\mathrm{E}}} \leqq \max_{\hat{x}} \frac{\|\boldsymbol{P} \hat{\boldsymbol{x}}\|_{\mathrm{E}}}{\|\hat{\boldsymbol{x}}\|_{\mathrm{E}}} = \|\boldsymbol{P}\|_{\mathrm{E}}. \tag{14}$$

Therefore, from (13) and condition (10), it follows that

$$\boldsymbol{f}^{\mathrm{T}}(\hat{\boldsymbol{x}}, t) \boldsymbol{P} \hat{\boldsymbol{x}} \leqq \tfrac{1}{2} \lambda_{\min}(\boldsymbol{K}) \|\hat{\boldsymbol{x}}\|_{\mathrm{E}}^2 \tag{15}$$

and $\dot{V}(\hat{x})$, eqn. (12), becomes

$$\dot{V}(\hat{x}) = -\hat{x}^{\mathrm{T}}\left(\sum_{i=1}^{k} C_i^{\mathrm{T}} F_i^{\mathrm{T}} R_i F_i C_i + Q + 2\alpha P - \lambda_{\min}(K)\right) \hat{x} . \tag{16}$$

It is easy to see that $\dot{V}(\hat{x}) \leqq 0$ for all $\hat{x}(t)$ and, hence, the perturbed system is stable for all nonlinear perturbations satisfying (10). □

An alternative expression for the results of Theorem 1 is given in the following corollary.

Corollary 1. Let the nonlinear perturbation $f(\hat{x}, t)$ satisfy the inequality

$$\|f(\hat{x}, t)\|_{\mathrm{E}} \leqq \sum_{i}^{n} d_i \|\hat{x}\|_{\mathrm{E}} \tag{17}$$

for all $(t, \hat{x}) \in \mathbb{R}^{n+1}$, $\hat{x} \in \mathbb{R}^n$, where d_i are n nonnegative numbers, and let $d = \sum_{i}^{n} d_i$ satisfy

$$\lambda_{\min}(K) \geqq 2d\, \lambda_{\max}(P) \tag{18}$$

then the perturbed system (8) and (9) is asymptotically stable.

Proof. Straightforward from Theorem 1 and observation that

$$\sum_{i}^{n} d_i \|\hat{x}\|_{\mathrm{E}} \leqq d \|\hat{x}\|_{\mathrm{E}} . \tag{19}$$ □

These results provide a framework within which quantitive bounds on the nonlinear perturbations and on the imprecisions in an engineering model may be used to test whether or not the model is sufficiently precise to be of use in assessing the actual system's behavior with given feedback law. In this framework, nonlinear systems emerge as a special case in which one is primarily interested in testing the validity of a linear approximation.

If the above analysis finds out that the proposed decentralized control system has unsatisfactory robustness characteristics, it is necessary to develop the procedure which leads to control system designs with improved robustness characteristics. For that reason, in what follows the allowable perturbations will be interpreted in terms of dominant eigenvalues of the nominal closed loop system and in terms of a prescribed degree of stability parameter α.

Theorem 2. The following relationship between the perturbation bounds on the nonlinear vector $f(\hat{x}, t)$ and the closed loop poles can be established

$$\frac{\|f(\hat{x}, t)\|_{\mathrm{E}}}{\|\hat{x}\|_{\mathrm{E}}} \leqq -\operatorname{Re}\left[\lambda_{\max}(A_{\mathrm{c}})\right] \tag{20}$$

where A_{c} is given by (4).

Proof. Denote by q the eigenvector corresponding to the greatest eigenvalue function for the perturbed system, $\dot{V}(\hat{\boldsymbol{x}})$ can be calculated as before to give

$$(27)\qquad \dot{V}(\hat{\boldsymbol{x}}) = -\hat{\boldsymbol{x}}^{\mathrm{T}}\Big(\sum_{i=1}^{k} \boldsymbol{C}_i^{\mathrm{T}}\boldsymbol{F}_i^{\mathrm{T}}\boldsymbol{R}_i\boldsymbol{F}_i\boldsymbol{C}_i + \boldsymbol{Q} + 2\alpha\boldsymbol{P}\Big)\,\hat{\boldsymbol{x}} + \boldsymbol{f}^{\mathrm{T}}(\hat{\boldsymbol{x}}, t)\,\boldsymbol{P}\hat{\boldsymbol{x}} + \hat{\boldsymbol{x}}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{f}(\hat{\boldsymbol{x}}, t).$$

For $\boldsymbol{f}(\hat{\boldsymbol{x}}, t)$ defined by (25), expression (27) becomes

$$(28)\qquad \dot{V}(\hat{\boldsymbol{x}}) = -\hat{\boldsymbol{x}}^{\mathrm{T}}\Big(\sum_{i=1}^{k} \boldsymbol{C}_i^{\mathrm{T}}\boldsymbol{F}_i^{\mathrm{T}}\boldsymbol{R}_i\boldsymbol{F}_i\boldsymbol{C}_i + \boldsymbol{Q} + 2\alpha\boldsymbol{P} - \boldsymbol{P}(\boldsymbol{T}^{\mathrm{T}}(\hat{\boldsymbol{x}}, t) + \boldsymbol{T}(\hat{\boldsymbol{x}}, t))\,\boldsymbol{P} + \\ + \boldsymbol{P}(\boldsymbol{S}^{\mathrm{T}}(\hat{\boldsymbol{x}}, t) + \boldsymbol{S}(\hat{\boldsymbol{x}}, t))\,\boldsymbol{P}\Big)\,\hat{\boldsymbol{x}}.$$

But,

$$(29)\qquad \boldsymbol{P}(\boldsymbol{T}^{\mathrm{T}}(\hat{\boldsymbol{x}}, t) + \boldsymbol{T}(\hat{\boldsymbol{x}}, t))\,\boldsymbol{P} = 0$$

since $\boldsymbol{T}(\hat{\boldsymbol{x}}, t) = -\boldsymbol{T}^{\mathrm{T}}(\hat{\boldsymbol{x}}, t)$ for all $t \in [0, \infty)$.

Now, if conditions (i) holds, then (28) becomes

$$(30)\qquad \dot{V}(\hat{\boldsymbol{x}}) = -\hat{\boldsymbol{x}}^{\mathrm{T}}\Big(\sum_{i=1}^{k} \boldsymbol{C}_i^{\mathrm{T}}\boldsymbol{F}_i^{\mathrm{T}}\boldsymbol{R}_i\boldsymbol{F}_i\boldsymbol{C}_i + \boldsymbol{Q} + 2\alpha\boldsymbol{P} + 2\boldsymbol{P}\boldsymbol{S}(\hat{\boldsymbol{x}}, t)\,\boldsymbol{P}\Big)\,\hat{\boldsymbol{x}}.$$

Therefore,

$$(31)\qquad \dot{V}(\hat{\boldsymbol{x}}) \leqq -\hat{\boldsymbol{x}}^{\mathrm{T}}\Big(\sum_{i=1}^{k} \boldsymbol{C}_i^{\mathrm{T}}\boldsymbol{F}_i^{\mathrm{T}}\boldsymbol{R}_i\boldsymbol{F}_i\boldsymbol{C}_i + \boldsymbol{Q} + 2\alpha\boldsymbol{P}\Big)\,\hat{\boldsymbol{x}}.$$

So, it can be said that the system with nonlinear perturbation vector of the form (25) which satisfies the condition (i), is "at least" as stable as the nominal unperturbed system.

The proof of conditions (ii) follows directly from (28) and its omitted. □

It is important to emphasize that the decentralized pole placement approach for large scale control system design present no difficulty to the theory, and that the robustness results developed, can all be applied directly.

3. NUMERICAL EXAMPLE: FIVE TERMINAL MTDC SYSTEM DESIGN

To illustrate the concepts developed in this paper a five-terminal MTDC system, with typical parameters, is used throughout. A one-line diagram of this system is shown in Figure 1. The AC bases in this system represent coherent areas identified by a dynamic-equivalent algorithm and the AC lines represent the equivalent tie-lines connecting the different areas. In this illustrative system, the terminal 1 is considered to be the voltage controller and acting as the slack terminal for the MTDC system. The other terminals are assumed to be current controllers and their current orders may be perturbed by respective modulating signals. For more detailed discussion on this issue and physical interpretation of five machine model see reference [5].

The linear quadratic state feedback is used as a starting point in the decentralized control system design. The control penalty matrix $\boldsymbol{R}$ is selected as an identity matrix of A_c, and by $\hat{q}$ the complex conjugate of q. Premultipying and post-multiplying (6) by $\hat{q}$ and q it follows

$$- \operatorname{Re}\left[\lambda_{\max}(A_c)\right] = \frac{\hat{q}\left(\sum_{i=1}^{k} C_i^T F_i^T R_i F_i C_i + Q + 2\alpha P\right) q}{2\hat{q} P q}. \tag{21}$$

Notice that the matrices on the right side of eqn. (21) are symmetric ones. Therefore, by applying the Rayleigh's principle (cf. [3], [4]) it follows

$$\lambda_{\min}\left(\sum_{i=1}^{k} C_i^T F_i^T R_i F_i C_i + Q + 2\alpha P\right) \leqq \hat{q}\left(\sum_{i=1}^{k} C_i^T F_i^T R_i F_i C_i + Q + 2\alpha P\right) q \tag{22}$$

$$\lambda_{\min}(P) \leqq \hat{q} P q \leqq \lambda_{\max}(P) \tag{23}$$

and

$$- \operatorname{Re}\left[\lambda_{\max}(A_c)\right] \geqq \frac{\lambda_{\min}\left(\sum_{i=1}^{k} C_i^T F_i^T R_i F_i C_i + Q + 2\alpha P\right)}{2\,\lambda_{\max}(P)} \tag{24}$$

From (10) and (24) the inequality (20) follows directly. □

Having in mind that $-\operatorname{Re}\left[\lambda_{\max}(A_c)\right] \geqq \alpha$, where α is a prescribed degree of stability parameter, the inequality (20) also establishes a relationship between the allowable perturbation and the parameter α. In other words, the inequality (20) provides an explicit link between the allowable perturbation and the choice of the parameter α, helping a designer to use the parameter α as a design parameter.

The following theorem gives an explicit parametrization of the subclass of disturbances that never destabilize the originally stable closed loop system. In essence the theorem gives alternative conditions for stability, which do not take into account the norms bounds on the nonlinear vector $f(\hat{x}, t)$ but rather constraint the structural arrangement of $f(\hat{x}, t)$.

Theorem 3. Let the nonlinear perturbation vector $f(\hat{x}, t)$ be of the form

$$f(\hat{x}, t) = \left[T(\hat{x}, t) - S(\hat{x}, t)\right] P \hat{x}(t) \tag{25}$$

If one of the following conditions holds

(i) $T(\hat{x}, t)$ is an arbitrary skew-symmetric matrix for all $t \in [0, \infty)$, and $S(\hat{x}, t)$ is an arbitrary symmetric, positive semidefinite matrix for all $t \in [0, \infty)$ or,

(ii) $T(x, t)$ is an arbitrary skew-symmetric matrix for all $t \in [0, \infty)$, and $S(\hat{x}, t)$ is an arbitrary symmetric matrix for all $t \in [0, \infty)$, so that

$$\sum_{i=1}^{k} C_i^T F_i^T R_i F_i C_i + Q + 2\alpha P + 2 P S(\hat{x}, t) P \geqq 0 \tag{26}$$

for all $t \in [0, \infty)$;

then the perturbed system (8) and (9) is asymptotically stable.

Proof. The proof runs along identical lines to that of Theorem 1. Choosing $V(\hat{x}) = \hat{x}(t) P \hat{x}(t)$, where P is the positive definite solution of (6), as a Lyapunov weighting by a scalar 37·5. Based on the model-decomposition concept, the state weighting matrix Q is selected (see [5]) to penalize the complex modes associated with inter-area oscillations. For this example the optimal feedback gain matrix is

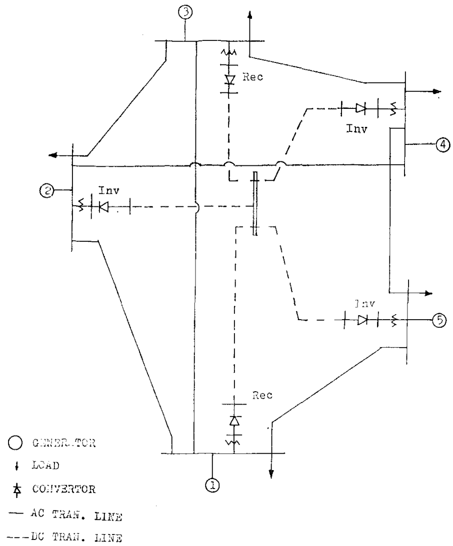

Fig. 1. A power system with five DC terminals and five AC modes.

given with

$$F = \begin{bmatrix} 0{\cdot}2728\ \mathrm{D}{+}3 & -0{\cdot}1379\ \mathrm{D}{+}0 & -0{\cdot}4046\ \mathrm{D}{+}3 & 0{\cdot}3716\ \mathrm{D}{+}0 & 0{\cdot}3489\ \mathrm{D}{+}2 \\ 0{\cdot}2862\ \mathrm{D}{+}3 & -0{\cdot}4398\ \mathrm{D}{+}0 & 0{\cdot}5988\ \mathrm{D}{+}1 & 0{\cdot}2226\ \mathrm{D}{+}0 & -0{\cdot}3400\ \mathrm{D}{+}3 \\ 0{\cdot}2116\ \mathrm{D}{+}3 & 0{\cdot}2356\ \mathrm{D}{-}2 & -0{\cdot}3677\ \mathrm{D}{+}2 & 0{\cdot}2097\ \mathrm{D}{+}0 & -0{\cdot}2123\ \mathrm{D}{+}2 \\ 0{\cdot}1474\ \mathrm{D}{+}3 & -0{\cdot}1674\ \mathrm{D}{-}1 & 0{\cdot}3952\ \mathrm{D}{+}2 & -0{\cdot}1010\ \mathrm{D}{+}0 & 0{\cdot}5175\ \mathrm{D}{+}2 \end{bmatrix}$$

$$\begin{bmatrix} -0{\cdot}1516\ \mathrm{D}{+}0 & 0{\cdot}5404\ \mathrm{D}{+}2 & 0{\cdot}5745\ \mathrm{D}{-}1 & 0{\cdot}4245\ \mathrm{D}{+}2 \\ -0{\cdot}5506\ \mathrm{D}{-}4 & 0{\cdot}1908\ \mathrm{D}{+}2 & 0{\cdot}1013\ \mathrm{D}{+}0 & 0{\cdot}2776\ \mathrm{D}{+}2 \\ -0{\cdot}7648\ \mathrm{D}{-}1 & -0{\cdot}1568\ \mathrm{D}{+}3 & -0{\cdot}8022\ \mathrm{D}{-}1 & 0{\cdot}2858\ \mathrm{D}{+}1 \\ 0{\cdot}1166\ \mathrm{D}{+}0 & -0{\cdot}6033\ \mathrm{D}{+}1 & -0{\cdot}2525\ \mathrm{D}{-}1 & -0{\cdot}2325\ \mathrm{D}{+}3 \end{bmatrix}$$

The feedback law given by the linear quadratic methodology requires a knowledge of all the state variables at each actuator. However, the angles of the areas cannot be measured easily, thus only the frequency information is available to the controllers. As alternatives to the fill-state feedback structure, we consider the following information pattern: feedback using local frequency and frequency of the original voltage-setting terminal (terminal 1). This decentralized information pattern corresponds to:

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$C_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Using the least-square optimization algorithm [6] the centralized state feedback controller is reduced to the decentralized control scheme, eqn. (2),

$$F_1C_1 = [0{\cdot}2728 \text{ D}+3 \;\; 0 \;\; -0{\cdot}4046 \text{ D}+3 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]$$

$$F_2C_2 = [0{\cdot}2863 \text{ D}+3 \;\; 0 \;\; 0 \;\; 0 \;\; -0{\cdot}3400 \text{ D}+3 \;\; 0 \;\; 0 \;\; 0 \;\; 0]$$

$$F_3C_3 = [0{\cdot}2161 \text{ D}+3 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; -0{\cdot}1568 \text{ D}+3 \;\; 0 \;\; 0 \;\; 0]$$

$$F_4C_4 = [0{\cdot}1474 \text{ D}+3 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; -0{\cdot}2325 \text{ D}+3]$$

As known, a power system can be viewed as a nonlinear system whose dynamics are governed by a vector nonlinear function. However, the MTDC control system design has been studied using linear system model. Therefore, the actual system can be defined by ($\hat{S}$), eqns. (8) and (9), i.e., all the parameter variations, nonlinearities in the open-loop system dynamics and the design linearization are lumped into the vector function $\boldsymbol{f}(\hat{\boldsymbol{x}}, \hat{\boldsymbol{u}}, t)$.

To realistically evaluate the robustness of the MTDC control system design, modelling uncertainties described by the function $\boldsymbol{f}(\hat{\boldsymbol{x}}, \hat{\boldsymbol{u}}, t)$ must be known a priori. This characterization can be based on experimental measurement. If the time domain data are used in this purpose then the results of this paper can be employed for robustness characterization. For the purpose of illustration, let us examine the robustness of the perturbed system using the condition (17), Corollary 1. Let us assume that

$$d_{\mathrm{I}} = \sum_{i=0}^{4} d_{2i+1}$$

and

$$d_{\mathrm{II}} = \sum_{i=0}^{4} d_{2i}$$

where d_{2i+1}, $i = 0, 1, 2, 3, 4$ are nonnegative numbers which correspond to the states which describe the changes in electrical angles, and d_{2i}, $i = 1, 2, 3, 4$ are nonnegative numbers which correspond to the states which describe the changes in frequences. The visualization of the d_I and d_{II}, for different values of the prescribed degree of stability parameter α, for which the perturbed system remains stable, is shown in Figure 2.

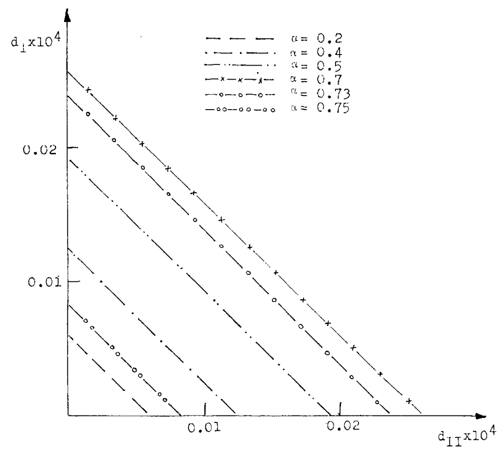

Fig. 2. Visualization of the values of d_I and d_{II} for which the perturbed system remains stable, for different values of the prescribed degree of stability parameter *J*.

4. CONCLUSIONS

The robustness of decentralized control systems subject to nonlinear perturbations in the system dynamics has been considered. It has been shown that there exist finite nonlinear perturbations, which admit certain norm bounds, so that the perturbed system remains stable. A new relationship has been established between perturbation bounds and dominant poles of the nominal, unperturbed closed loop system. Alternative conditions for stability, which do not take into account the norm bounds on the nonlinear perturbations but rather constrain the structural arrangement of the perturbations, have also been given.

(Received November 2, 1983.)

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