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Antinomy of "Liar" and Antimony of Synonymous Names

LADISLAV TONDL

The present paper gives a critical analysis of Tarski's reconstruction of the antinomy of "Liar". A new reconstruction is given on the basis of semantic conception of synonymous names and on the basis of semantic analysis of context.

The purpose of this paper is not to give a historical recapitulation of this paradox in the development of logical thinking, of the incorporation of this paradox into a scheme of logical and logic-semantical paradoxes.* It has as its proper scope the analysis of A. Tarski's procedure in his classical work "Der Wahrheitsbegriff in den formalisierten Sprachen"*** which was used for proving the inconsistency of the so called semantically closed language and for a definition of the term "true". The result of this analysis is the ascertainment that Tarski's procedure takes into account the extensional aspect of the meaning only and that, in fact, it is based only upon the analysis of the one of three suppositions, i.e. the supposition of semantically closed language. Simultaneously a conception is presented that explains the origin of semantical antimony on base of the analysis of the so called empirical premise and on base of semantical analysis of the context.

* A survey and analysis of paradoxes may be found for instance in the work [5] and other ones. In Czech literature it is especially the work [18].

** The original version of A. Tarski's work was presented on March 21, 1931, at the Session of Warsaw's Scientific Society and was published in an abbreviated form under the headings "O pojęciu prawdy w odniesieniu do sformalizowanych nauk dedukcyjnych" *Ruch Filozoficzny*, XII, 1931. One year later the first concise German version was published under headings "Der Wahrheitsbegriff in den Sprachen der deduktiven Disziplinen", *Akad. der Wiss. in Wien, Mathematisch-naturwissenschaftliche Klasse*, 1932. The original work was published in *Studia philosophica*, Vol. I, 1935, pp. 261—405. This work served then as a base for Woodger's translation into English [14, pp. 152—278]. Another paper, headed "The semantic Conception of Truth" [15], published for the first time in 1944, is an abbreviated and partly completed version of this work.

1. RECONSTRUCTION OF ANTINOMY OF LIAR AND TARSKI'S
ANALYSIS

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A. Tarski starts out from a simple formulation of antinomy which was presented by J. Lukasiewicz. For the sake of a greater lucidity, he introduces the sign "S" as a "typographical abbreviation" of the sentence: "The sentence written in line 8 from above of page 15 of this article". Then it is possible to examine the following sentence:

S is not a true sentence.

With regard to the meaning of the sign "S" it is possible to ascertain empirically, that

(α) "S is not a true sentence" is identical with S.

A. Tarski introduced for the sake of explanation of the term "true" the following scheme:*

(1) True is such a sentence that says, that the things stand so and the things stand just so and so.

A. Tarski formulates this general scheme in a simpler form.

(2) x is a true sentence then and only then, if p .

In accordance with the scheme (2) it is possible to formulate the second premiss

(β) "S is not a true sentence" is a true sentence if and only if S is not a true sentence.

The following antinomy results evidently from premisses (α) and (β):

S is a true sentence if and only if S is not a true sentence.

This reconstruction of the antinomy is based on the scheme (1) or scheme (2) for the sake of explanation of the term "true". It is possible, for the purpose of a higher clarity and conclusionness, to reconstruct the antinomy so, that we shall make no direct reference to this scheme:**

Let us suppose that the following sentence is in the text of our explanation:

This sentence written on the page 15 of this text in the oblong is not true.

* This scheme originates from A. Katarbiński [6, pp. 127—136].

** A. Tarski proceeded thus in his later work [15]. Further mentioned version is a somewhat corrected procedure of the reconstruction, given by Tarski in [15].

If we replace (as A. Tarski says) – for the sake of abbreviation – this sentence, written on the page 15 in the oblong with the sign “S”, we shall obtain:

- (a) “S” is true then and only then, if this sentence, written on the page 15 of this text in the oblong is not true.

On the basis of the meaning of “S”, we may, as A. Tarski says, establish empirically the following fact.

- (b) “S” is identical with this sentence, written on the page 15 of this text in the oblong.

As we suppose, the identical terms to be mutually interchangeable (in the sense of Leibnitz’s principle of identity: *eadem sunt quae sibi mutuo substitui possunt salva veritate*), we may substitute the part in (a) reading “this sentence, written on the page 15 of this text in the oblong” with the sign “S”. We obtain then:

“S” is true then and only then, if “S” is not true.

In principle, this reconstruction of the antinomy of “Liar” does not differ substantially from the reconstruction previously shown. In this manner other analogic formulations could be reconstructed too.

A. Tarski analysed the suppositions under which this antinomy may originate. There are three such suppositions:

(I) We suppose that the language which the antinomy may arise in, contains besides the terms, that appear in this language, the names of these terms and the predicate “true” too. A. Tarski characterizes such a language as a semantically closed language.*

(II) We suppose “the ordinary laws of logic” to be valid in this language.**

(III) We suppose to be possible to formulate a so called empirical premiss, i.e. for instance a sentence (α) in the first or a sentence (β) in the second version. This empirical premiss implies, as it is obvious, a formulation of the so called ostensive definition or of whatsoever ostensive procedure.***

* Semantically closed language is whatever one, which involves the names of expressions it comprises, and semantical terms such as e.g. “fulfills”, “designates”, “is a name for” etc. We generally suppose by intuition a natural language to be a semantically closed language. Hence, in question are predicates of which at least one of their arguments is an expression of the given language.

** A. Tarski does not explain in details that term. It is possible to understand from the whole context, that it is a question of the classical logics and that it is, hence, necessary, to respect the principle of excluded middle. It equally means, that the nonclassical logic systems, e.g. intuitionist logic, are not taken into consideration for the reconstruction of the antinomy.

*** We use here the term “ostensive definition” and “ostensive procedure” in the sense of B. Russell (see [10], [11]). The ostensive procedure concerns the semantization of an linguistic expression by other than linguistic means.

A. Tarski examined only the first two of above given suppositions. The conclusion he reached, is a simple one. If we take into consideration the first two suppositions, we shall come to the conclusion, that the language, which fulfils simultaneously the supposition (I) and (II) is inconsistent. As we are not likely to give up the supposition (II), there remains but one way out: to give up the supposition of semantically closed language. If we decide not to use the semantically closed language, we have to introduce the semantical notions, the notion "true" including, no sooner than in meta-language. In other words, we must come over to a hierarchical classification of languages, i.e. to the differentiation of the object-language and meta-language, eventually of the meta-language and the meta-meta-language etc. Main efforts of A. Tarski was oriented, therefore, to the analysis of presuppositions of defining the semantical notions in the meta-language.*

A. Tarski considers the supposition (III) as unsubstantial, because, as he explicitly points out [15 page 349], it is possible to reconstruct a semantical antinomy without this supposition. In a remark in appendix [15 page 371] he states, as an example a reconstruction of antinomy, which does not repose on a so called empirical premiss, and the possibility of its formulation in the given language, a sentence, which is introduced by a universal quantifier.**

It results from the above stated, that A. Tarski sees a source of antinomy in the assumption (I). In further exposition we shall show, that it is not possible to consider this standpoint as satisfactory. The analysis of origin of antinomy, which will be submitted, will show, that it is not possible to trifle even with further suppositions relating to origin of antinomy.

2. PROBLEM OF EMPIRICAL PREMISE AND OF IDENTIFICATION OF LINGUISTIC EXPRESSIONS

When searching the sources of antinomy, the assumption (II) can not be omitted. It may seem that the assumption (II) may be modified in such a way, that we shall

* A. Tarski has proved that semantical notions may be defined for formalized languages of final order under the supposition, that the meta-language relativised with regard to these languages, is richer than the object-language in the sense, that it contains variables of a higher type. In contradiction of this Tarski's procedure, that supposes an introduction of semantical notions in such a way, that they are explicitly defined, it is possible to consider still another introduction of these notions (as shown especially by R. M. Martin [7]) in the form of primitive terms of the meta-language assuming that the meaning of these notions is contained in axioms of this language.

** The example given here by A. Tarski, relates in reality to an other semantical notion, namely to the notion "it relates to it itself." On the basis of the sentence

"Every sentence does not relate to it itself"

it is easy possible to reconstruct an antinomy that is analogical to the antinomy of "Liar" without the need to rely on the supposition (III). It is, however, necessary to take sentences with universal quantifier, so that the interpretation of the quantifier equally decides.

renounce on principles of classical logic, e.g. so, that we shall assume the intuitionist logic. In this way it is possible, indeed, to suspend the principle of excluded middle, but this approach is limited to languages only, the universum of which is infinite. In the case of finistic approach, the same problems remain here as in case of classical logic. For these reasons, we are leaving these questions apart and shall analyse before all the supposition (III). The formulation of the so called empirical premise is based upon the circumstance, that in the given language it is possible to formulate an expression, that states the identity of the sign "S" and of the given sentence, that it is possible to read or otherwise empirically establish in the accurately determined line or oblong. Already here it is possible to point at some inaccuracies of Tarski's reconstruction.

(1) Before all it holds, that it is only possible empirically to establish, whether the given sentence is written in the stated line or oblong or not. It is, however, not possible to verify empirically whatever statement about identity especially if we assume the criterion "salva veritate". It means, that it is not possible to consider sentences (α) in the first version or (b) in the second version of the reconstruction as empirically verifiable or verifiable sentences so that it appears disputable whether it is possible to characterize these sentences as empirical premisses at all.

(2) A. Tarski uses the expression "typographical abbreviation" [13], [14]. If "S" is an typographical abbreviation of the mentioned sentence, then "S" is a name of the said sentence (whereas we assume — as per Frege's conception — this sentence to be a name sui generis). It is, of course, evident that it is not possible, salva veritate, to interchange a certain linguistic expression with his name in an arbitrary context. In other words: If we accept Tarski's explanation, that "S" is an abbreviation of the given sentence, we have already obtained in premisses what Tarski characterizes as semantically closed language. If we know to make in a given language a distinction between linguistic expressions and names of these expressions, it is then easy to formulate a prohibition of their mutual interchangeability "salva veritate". In addition, the reasons, this prohibition is based upon, may be of various nature: It is possible to point out at the necessity of distinction between a so called autonomous and heteronymous use of name, eventually at analogical differentiation of cases, when the name is used, and cases, when we mention the name.* When autonomously used, the name relates to it itself. It is analogically so in cases, when we mention the name. If we interchange two names that we consider as identical, difficulties may arise, if in one case an autonomous, in other case an heteronymous usage is made. Let us have as example two sentences:

- (1) Lenin consists of five letters.
- (2) Lenin = Vladimir Ilyich Ulyanov.

* The conception of autonomous or heteronymous usage of names was explained especially by R. Carnap [2]. The differentiation between "use" and "mention" is due to W. Quine [9].

If we keep (2) for a sentence, which states the identity of two names and if we substitute then the name Vladimir Ilyich Ulyanov for the name Lenin in (1) we shall obtain the sentence:

(3) Vladimir Ilyich Ulyanov consists of five letters.

It is, of course, evident, that the name of "Lenin" was used in (1) in an autonomous and in (2) in a heteronomous way, so that the principle of preservation of the same form of using a name was injured. By nonobserving this principle it is possible to come from true sentences to untrue conclusions in a manner, which is correct by itself.

Hence, it may be seen, that Tarski's differentiation of object-language and of meta-language is but one of methods how to solve difficulties resulting from injuring the principle of preservation of the same form of using the name. If A. Tarski considers the supposition (1), i.e. the supposition of a semantically closed language, as a main source of semantical antinomies, we may add, that it is possible to imagine semantically closed languages, which the antinomies do not originate in, only so far, as the principle of preservation of the same form of using name is respected.

Hence, it may be inferred, that Tarski's formulations and interpretations of the so called empirical premiss lead necessarily to difficulties, that may gain resemblance of semantical antinomies, especially then, if the identity of two linguistic expressions is stated, from which one is the name or abbreviation of the other and if their mutual interchangeability is admitted without respecting the principle of preservation of the same form of using the name.

Let us now examine the question, in as far it is possible to avoid these difficulties, if we leave the supposition, that "S" is an abbreviation or name of the given sentence. Let us suppose only, that a demand expressed, that "S" should be identical with the given sentence. In order to fulfil this demand, it is necessary to examine such requirements, put forward by the identification of objects.*

Let us, in the meantime, disregard the fact, that the expressions of a certain language are these objects. Then the application of a method, which is characterized as a method of identification of indiscernables offers itself. This method may be used in two somewhat different versions:

If x and y are variables of objects forming the given universe, it is possible to make use of system of distinguishing criteria R , which may be understood as a finite set of two-placed (two-arguments) predicates A_1, A_2, \dots, A_n . Further on it is important, that these predicates express properties, which enable the differentiation of objects of the given universe (e.g. "to be lighter", "to be longer", "to have a greater specific weight). In other words, it is desirable, that these predicates express relations which are irreflexive and asymmetrical.

As we assume that real differentiation and real identification may be realisable

*) More detailed analysis of identification of objects in the nominalistic system is given in [16].

20 only on the basis of finite number of distinguishing criteria (we shall call this requirement a requirement of finitism for identification), it is desirable, that in

$$R = \{A_1, A_2, \dots, A_n\}$$

n be less than or equal to number, that is compatible with claims on "capacities", "memories" and "delays", we may dispose with for solution of the given task of identification.

If we denote the identity of objects by the sign \equiv_{id} (in contradiction to the sign $=$, which we reserve for the numerical equality, i.e. the identity of classes), we may record the procedure of identification of the indiscernibles as follows:

$$\begin{aligned} (\forall x) (\forall y) [& \sim A_1 x, y . \sim A_1 y, x . \sim A_2 x, y . \sim A_2 y, x . \dots \\ & \dots \sim A_n x, y . \sim A_n y, x \rightarrow (x \equiv_{\text{id}} y)]. \end{aligned}$$

An other version of the method of identification of indiscernibles takes into consideration, besides the system of distinguishing criteria R and in addition to it, a system of "measures". This approach starts out from intuitive deliberation, according to which we use, when comparing and distinguishing two or more objects, an other object, which serves as a measure (for instance set of weights, meter or whatever measuring device). Further on, we consider measures as constants of objects, that are selected so as to be in accordance with requirements of distinguishing criteria. In other words, the measure a_1 corresponds to the criterion A_1 , a_2 to the criterion A_2 and so on. If we take into account not only the system R ($R = \{A_1, A_2, \dots, A_n\}$) but also the system R' ($R' = \{a_1, a_2, \dots, a_n\}$) we may write down the procedure of identification of discernibles as follows:

$$\begin{aligned} (\forall x) (\forall y) [& [(\sim A_1 x, a_1 \rightarrow \sim A_1 y, a_1) . (\sim A_1 a_1, x \rightarrow \sim A_1 a_1, y)] . \\ & . [(\sim A_2 x, a_2 \rightarrow \sim A_2 y, a_2) . (\sim A_2 a_2, x \rightarrow \sim A_2 a_2, y)] . \dots \\ & . [(\sim A_n x, a_n \rightarrow \sim A_n y, a_n) . (\sim A_n a_n, x \rightarrow \sim A_n a_n, y)] \rightarrow \\ & \rightarrow (x \equiv_{\text{id}} y)]. \end{aligned}$$

These procedures may be generalized so, that we shall arrive at the known formalized formula of Leibnitz's principle of identity. We suppose that, if two objects are identical all that may be said about one object, may equally be said about the other object and vice versa.

In the formalized manner:

$$(\forall x) (\forall y) [(x \equiv_{\text{id}} y) \rightarrow (Fx \equiv Fy)].$$

In this formula, respecting the requirements of finitism for identification, we may understand F as a finite set of properties, may be expressed as a system of one-placed predicates $\{F_1, F_2, \dots, F_n\}$.

The requirement of finitisme for whatever real identification must not be understood abstractly. The above mentioned script, in which $F = \{F_1, F_2, \dots, F_n\}$ n being finite, may be explained as follows: If two objects are, in the frame of a certain class of tasks, considered as identical, then the complete set of properties, which may be assigned to one object in the frame of the given class of tasks, may also be assigned to another object and vice versa. In addition, it is natural, that the definition of this analysis depends on circumstances of two kinds:

(a) on the possibilities of the observer, experimentator, measuring equipment or of whosoever or of whatsoever to operate with means, defining means, delays, etc. which are, within the frame of the given class of tasks, at disposal;

(b) on requirements resulting from the tasks itselfs, defined e.g. by the demands on the quality of solution, extent of scopes aimed in the given tasks at and so on.

The elucidation of these circumstances in case of any real identification also means that the authority, which this identification is based on, is substantially of a semantic and pragmatic nature, i.e. it means that it is relativized with regard to the given universe and with regard to properties, possibilities and scopes of that who operates with elements of this universe. If we abandon the requirement of finitisme, it means if we presuppose $F = \{F_1, F_2, \dots, F_\infty\}$ then whatever real identification is not possible. It does not remain but to acknowledge, that no such two objects exist, which could not be distinguished each from the other supposing infinite set of properties, which may be ascribed to them or supposing the infinite system of distinguishing criteria, which it is necessary to use for the identification.

In our essay, which is oriented on the identification of linguistic expressions, it is necessary to presuppose, that the elements of the given universe, the identity of which has to be proved on the basis of shown methods of identification of indiscernibles are linguistic expressions of a single one or of more languages.

If we judge now from these points of view of the formulation of the so called empirical premiss (i.e. the sentence: "S" is identical with the sentence written on page 15 of this text in the oblong) we have evidently two possibilities:

Either there is an identification of two linguistic expressions of different languages and in this case it is not warranted, that we have anything to do with a semantically closed language, or the expressions of a single language are the question but they differ with their outer form. In such a case it is indispensable to determine all that the two expressions may have of common. In other words, it is necessary to fix the extent $F = \{F_1, F_2, \dots, F_n\}$ namely all that what, when it may be said about one expression, may be also said about the other of the two expressions and vice versa. It is evident that the two expressions, i.e. "S" and "This sentence, written on page 15 of this text in the oblong, is not true" are not identical in that sense, that all which may be said about one of them, may also be said about the other and contrarily, but only

in that sense, that some properties can exist, which may be common to both of them. Hence, the question arises, what are these properties or what we expect from these properties in order that the two expressions may be interchangeable.

From the intuitive point of view, the following solution presents itself: In order that the two expressions may be interchangeable, it is necessary to demand them to have the same meaning.*

This requirement may also be formulated in the following way: In order that the two linguistical expressions may be interchangeable, they must be synonymous. (The expression "to have the same meaning" or "to be synonymous" is to be considered as a "more-placed meta-linguistic predicate, which the names of expressions of the object-language are arguments of.

If we compare now this point of view based on synonymy with the point of view of Tarski, we may say as follows: A. Tarski gets satisfied by stating the identity of two expressions, which may then be interchangeable in the sense of Leibnitz's principle "salva veritate". Nevertheless, he does not determine more closely the extent $F = \{F_1, F_2, \dots, F_n\}$. As he, however, characterizes the given statement about identity as an empirical premiss, it results that this extent could involve empiric predicates only. In contradiction to that the point of view, which is based on synonymy, gets satisfied by a single predicate, namely by predicate "to have the same meaning" or "to be synonymous".

It is, in the total, easily evident, that the point of view of Tarski, based on statement of identity of the two expressions, the interchangeability of which is under consideration, cannot be realized especially then, if the extent $F = \{F_1, F_2, \dots, F_n\}$ is not accurately determined.

Nor is the point of view, based on synonymy, free of some difficulties, either. The basic difficulty lies in that the predicate "to have the same meaning" is itself semantically indefinite. Its more accurate definition commands to take into consideration important results of logical semantics starting with works of Frege, Russell, Lewis, Carnap and others, which argue for distinction of two modes of meaning: of sense and denotation, eventually of intension and of extension (according to terminology of Carnap).

3. ANTINOMY OF SYNONYMOUS NAMES

The problem of synonymy has already been practically for more than a half century the object of intensive attention in the logico-semantical literature. The difficulties arising through interchangeability of synonymous names are one of

* This requirement may be considered as the minimal presupposition for interchangeability. It is possible to ask further on that the two expressions be of the same language that they be written in the same manner of script and so on. In our further explanation we shall take into consideration this minimal requirement only.

important causes of this attention. If we, namely, content ourselves with indefinite characteristics, that synonyms are names "having the same meaning" or which are identical by their meaning, we may arrive, when interchanging synonymous names, at antinomies, which may be characterized as antinomies of synonymous names.

It results from following explanation, that it is possible to consider the antinomy of Liar in Tarski's reconstruction as a special case of antinomy of synonymous names.* In order to secure the interchangeability of synonymous names without difficulties and equally with exclusion of origination of antinomies of synonymous names, it is necessary to solve two basic problems of synonymous names. These problems may be summarized into the following questions:

- (A) What are the conditions of interchangeability of synonymous names?
- (B) If two expressions of the given language are synonymous, then "salvo quo"?

The first question is connected before all with semantic nature of context. By means of the analysis of this question it may be shown, that the traditional answer, that was earlier considered as selfevident, i.e. interchangeability within the sentential context does not satisfy for some situations. The up-to-date solutions of conditions of interchangeability of synonymous names, for example Carnap's conception of the so called intensional isomorfisme [2], are not able to remove all difficulties, that are originating here. These difficulties appear before all in context which are not extensional, especially in belief sentences (i.e. "XY believes that ...") in propositional attitudes and so on. We shall return once more to the problematics of semantic nature of context in the analysis of the antinomy of Liar in our subsequent explanation.

The other question concerns a more accurate definition of what is maintained in interchangeability. If we content ourselves with maintaining only what the name indicates, namely with maintaining the denotation (or extension in the terminology of Carnap)** we may easily arrive at antinomies. A number of examples of such antinomies were analysed before all by W. Quine [9]. It is easy to construct antinomies of synonymous names with sentences beginning with modal terms. Let us take for example following sentences (example is taken over from Quine [9]):

- (1) $9 > 7$.

* This is so, because the expressions "S" and "This sentence written on page 15 of this text in the oblong is not true" are presupposed" to be identical, as far as their meaning is concerned, that "they have the same meaning".

** We presuppose in doing so that the truth value of a sentence is, in spirits of Frege's results, its denotation. Leibnitz's "salva veritate" corresponds equally to that.

24 This sentence may also be formulated, as it is an analytical sentence, as follows:*

9 is necessarily greater than 7.

(2) The number of planets = 9.

By introducing (2) into (1) we obtain:

(3) The number of planets is greater than 7.

As the sentence about the number of planets is not analytical, but is a result of empirical ascertainment, it holds, that

(4) The number of planets is not necessarily greater than 7.

The expressions "number of planets" and "9" are, of course, not supposed to be synonymous. Only then, if we reduce the meaning of name to a denotation or extension, it holds, that classes designated by expressions "number of planets" and "9" have equal number of elements. If we do not consider from an intuitive standpoint the expressions "9" and the number of planets" as synonymous names, then we usually consider as synonymous the expressions "morning star" and "evening star" (example of G. Frege), "Walter Scott" and "the author of Waverley" (example of B. Russell). These, however, are only apparent synonymous, as these expressions have only equal denotation but a different sense. Hence for the question "salvo quo" is insufficient the answer: to maintain "the same meaning" but it is also necessary to take into consideration that mode of meaning, which may be characterized as sense or intension of the expression.

That may be expressed more accurately — if we regard results submitted in [2] — in the following way: Let A_i and A_j be names of two expressions of a object language, such expressions that it makes sense to deliberate of a relatively independent semantic analysis** and \cap is symbol for contextual concatenation. Then it is possible in the expression $A_i \cap A_j$ to replace A_j with A'_j (or in other words A_j and A'_j are synonymous) then and only then if their equivalence is always true (i.e. when $A_j \equiv_L A'_j$, where \equiv_L is symbol for L-equivalence).

It is, hence, possible to conclude, that to the question "salvo quo" the answer "salva extensione" is not sufficient (because "salva extensione" is only a partial

* This reformulation of the original sentence is naturally bound to accepting in advance certain presuppositions, i.e. presupposition of corresponding conception of theory of numbers and presupposition of a certain interpretation of the modal term "necessary", as it corresponds to Carnap's interpretation of modality in [2].

** It is evident that A corresponds to Carnap's concept of "designator" [2]. A more detailed analysis is contained in [17].

answer as we consider the truth value as an extension or denotation of the sentence). It is necessary to secure more than maintaining the extension; therefore it is necessary to require the maintaining of the intension.

If we revert now to Tarski's formulation of the so called empirical premiss, we can ascertain the following: the expressions "S" and "This sentence written on page 15 in the oblong is not true" may have the same extension (i.e. may be equivalent), but this does by itself not grant their troublefree interchangeability in a generally conceived context. It is necessary to require, these expressions to have the same intension (i.e. to be L-equivalent). This requirement cannot be warranted empirically, for example through the shown ostensive procedure.* Hence, if no reliable warranties for synonymy of the given expressions can be submitted, than difficulties including the known antinomies may arise from their being interchanged.

Our up-to now results may be summarized into the following conclusions:

(1) The so called empirical premiss of Tarski is incorrect, it is necessary to substitute it with synonymy of the said expressions. Such requirement, however, cannot be warranted empirically, for instance by means of the stated ostensive procedure.

(2) The antinomy of Liar may be considered as a special case of antinomy of synonymous names.

(3) In the discussed reconstruction of the antinomy of "Liar", it is necessary not only to refuse the presupposition of a semantically closed language, but also the presumption based on the so called empirical premiss.

4. ANTINOMY OF "LIAR" AND SEMANTIC PROBLEMS OF THE CONTEXT

We have already pointed out, that A. Tarski considered the presupposition, based on the so called empirical premise, as not substantial as it was possible to reconstruct semantic antinomy without this presupposition. Let us examine more closely this possibility.

The known antic form of antinomy of "Liars" offers for realization of this possibility. "Cretian Epimenides said that all Cretian were liars." Did he told the truth or a lie? Analysing the origination of this antinomy, we may proceed in a double manner.

The first manner** consists in it that the expression "all" in here used, which from the point of view of logics approaches to an universal quantifier. In reality, however,

* This conclusion also corresponds to conception of W. Quine [9] which characterizes the concept of "synonymy" as a concept of so called "theory of meaning", in which empirical warranties, in contradiction to the so called "theory of reference" are not sufficient, but analytical warranties are required.

** This manner is introduced by A. Mostowski [8], pp. 319.

26 it is here not an universal quantifier in logical sense at the issue as universal quantifier presupposes variables to be in the formula and to be bound by the quantifier. It is evident, that the expression "all" is used here in the sense, which does not exclude the existence of at least a single contrary instance. For this reason, too, A. Mostowski observes, that out of purely logical reasons it is necessary to presuppose existence of a Cretian, who at least once tells the truth.

The second manner consists in semantical analysis of the predicate "to be a liar". This predicate may have a double interpretation: We shall call a "liar" such a person who is not telling the truth, which we have been able to persuade ourselves of. In question is, hence, a "liar hic et nunc". We also consider as a "liar" a man not only with regard to single utterance, but a man, who as a rule does not tell the truth although it is not excluded that he now and then can tell the truth*. If we ask, whether Cretians are liars, we have, evidently, the second interpretation in mind. If we inquire whether Epimenides did or did not told the truth, i.e. whether he was or was not a liar, we have in mind a liar hic et nunc.

In order to avoid consequence, which result as from the expression "all" so from double interpretation of the expression "liar" we shall hold only to the first interpretation i.e. of "liar hic et nunc". In the same time we shall select such interpretation that does not include expressions "all", "everybody" and so on. The formulation satisfying these requirements is very simple: "I lie". If I ask then, whether I tell the truth, I shall get following answers: If I tell the truth, so I tell a lie. If I do not tell the truth, so I do not tell a lie. If we suppose then, that the expression "not to tell the truth" is synonymous with the expression "to tell lie" (of to be "liar hic et nunc") we shall obtain:

I am telling lie then and only then if I am not telling lie.

To these formulations do, hence, not relate considerations that were pronounced in the connection with the so called empirical premiss and in connection with the interpretation of such expressions as "all", "everybody" and so on. In the same time these formulations, however, make it possible to have due regard to semantic aspects of context and to the task of contextual concatenation in the origin of antinomy.

Let us suppose, that the notation "S" is a sign of a object-language \mathcal{L} for an atomic sentence of this language. Among the logical terms of this language belongs \sim (negation). It is known that by means of negation an untrue sentence will arise from a true sentence and a true sentence from a untrue one. Substantial is, however, semantic nature of contextual concatenation of the sign of negation and of the sign

* B. Bolzano has already pointed out at the possibility of the following interpretation: If a liar is who always lies and never speaks the truth, then a liar can never confess that he is a liar. If such a confession is possible, we must suppose to be also a liar who sometimes, be it only in exceptional cases, speaks the truth. See B. Bolzano, *Wissenschaftslehre*, III. Band, Sulzbach 1837, S. 488.

of atomic sentence. If NON is a metalanguage notation for negation in \mathcal{L} , Σ is a metalanguage sign for atomic sentence in \mathcal{L} , then the resulting concatenation may be put down as follows:

$$\text{NON} \overset{\circ}{\cap} \Sigma$$

This concatenation as it is evident, has an extensional character. In other words, the negation is used here as an extensional connective. We shall obtain a completely different situation, if we analyse the sentence "I tell lies". This sentence involves a double statement: (1) a statement about myself, that I tell anything, state or speak anything. (2) Statement about what I am telling. This means, that we shall interpret the sentence "I tell lie" as follows: "I say (event. "I tell the truth") that I tell lie". This interpretation also corresponds to above shown question, whether I speak the truth.

Let us now examine the nature of the contextual concatenation in the sentence: "I tell the truth, that ...". This sentence is but another formulation of what is characterized in the literature as propositional attitudes (B. Russell) or belief sentences.* The sentences of this kind imply two components: (1) Statement about a certain activity of the stating person, that relates to his attitude towards what he states (this is usually expressed by words as he "judges", "believes", "is convinced", "assumes" "tells the truth", followed by the word "that" and any further component, (2) that what is the content proper of the statement. Let us have as an instance the following sentence:

Mr. Novák is convinced that living creatures live on Mars.

The truth value of this sentence does not depend upon the truthvalue of the sentential component "living creatures live on Mars". This sentence is true, if it is a real conviction of Mr. Novák, it is untrue, if it isn't. The judgement of a specialist, which is able to verify the sentential component "living creatures live on Mars", is, hence, irrelevant with regard to the verification of the whole sentence. The contextual concatenation of the sentential components "Mr. Novák is convinced" and "living creatures live on Mars" is, hence, not extensional.**

Let us now return to the sentence: "I tell lie". If we interpret now the sentence in the following way:

"I tell the truth, that I tell lie"

* About different possibilities of interpretation of the so called propositional attitudes or of belief sentences see [16].

** In system of Principia Mathematica, the concept "extensional function" is used in the same sense: Function of another function is extensional if its truth value is with whatsoever argument the same as its truth value with arbitrary argument, which is equivalent with the original argument [17, p. 72]. If, using symbolics of the system of "Principia Mathematica" $f(\varphi\hat{x})$ is an extensional function with regard to $\varphi\hat{x}$, then, when we replace $\varphi\hat{x}$ with equivalent $\psi\hat{x}$, then $f(\varphi\hat{x})$ will also be equivalent to $f(\psi\hat{x})$.

we shall obtain contextual concatenation of two sentential components, that, if taken isolately, may be considered as negation of each by the other. (This means, that the expression "to tell the truth" is the negation of the expression "to tell lie" and vice versa). We shall try now to write down this situation so, that we shall substitute the expression "I tell the truth" with the sign S_i and the expression "to tell lie" with the sign $\sim S_i$.

The expression "I tell lie" appears as a negation of the expression "I tell the truth". In reality, if we take into consideration the above given interpretation, i.e. "I tell the truth, that I tell lie", this negation is connected by nonextensional context with the component "I tell the truth"*. If we make use of the introduced signs (as well as the brackets in the same sense as used in the precedent reference) we may write the whole expression as follows:

$$S_i(\sim S_i).$$

In this formula (in object-language), the nature of the contextual concatenation is, of course, not obvious.** As we register the contextual concatenation in meta-language notation, we shall introduce the sign $\overset{c-ext}{\cap}$ for the extensional contextual concatenation and the notation $\overset{c-non-ext}{\cap}$ for the nonextensional contextual concatenation. Then the meta-language name of the expression $S_i(\sim S_i)$ is:***

$$\Sigma_i \overset{c-non-ext}{\cap} \text{NON} \overset{c-ext}{\cap} \Sigma_i.$$

If we even suppose, that Σ_i before the sign of nonextensional contextual concatenation and Σ_i which appears after this sign, are names of the same expressions in object-language, it means, if we pass by their difference of types, supposed by A. N. Whitehead and B. Russell, it is not possible to come at the usually stated form of antinomy which could be put down as follows:

$$\begin{aligned} S_i &\rightarrow \sim S_i, \\ \sim S_i &\rightarrow S_i, \end{aligned}$$

* A somewhat different interpretation is given in the introduction to [17]: The sentence "I am lying" is interpreted in the following manner: "There is a proposition which I am affirming and which is false". The term "false" is ambiguous in this case. In order to remove this ambiguity, it is necessary to indicate the type. As the expressions "I am affirming" and "I am lying" are not of the same type, it is not possible to judge from the value of the expression of one type on the value of the equally worded analogical expression of the other type. That means, that A. N. Whitehead and B. Russell solve the given situation by means, which the theory of types offers.

** In the system of "Principia Mathematica" this formula corresponds to so called hierarchy of functions or to hierarchy of propositions. In order to enable to determine the function of first order, the sign "P" is introduced for it. Only individual may be, hence, values of the variable in the function $\phi! \hat{x}$. The whole formula may be then rewritten as follows: $f(\phi! \hat{x})$.

*** In this formula, there are not meta-language signs for brackets in object-language, as for instance R. M. Martin applies them in meta-language notations [7].

hence

$$S_i \equiv \sim S_j.$$

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If two sentential components are connected with extensional context, for instance in the form $S_j \cdot S_k$ (that means in meta-language $\Sigma_j \overset{c-ext}{\cap} ET \overset{c-ext}{\cap} \Sigma_k$ where ET is a meta-language sign for conjunction in object-language, then by substitution of Σ_k with Σ'_k under supposition, that $\Sigma_k \equiv \Sigma'_k$ the value of the whole expression will not change; it may then change under supposition, that $\Sigma_k \not\equiv \Sigma'_k$. In contradiction to it this dependence does not hold in the case of nonextensional contextual concatenation.

Hence, it is possible to pronounce the following conclusion: The expression "I tell lie" involves implicite contextual concatenation that is not extensional. From this point of view it is not possible to consider this expression as a simple negation (where in the negation would be a connective of extensional language) of the expression "I tell the truth". If we take into consideration the nature of the context, the antinomy will not arise.

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VÝTAH

Antinomie „lháře“ a antinomie synonymních jmen

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Práce podává kritický rozbor Tarského rekonstrukce antinomie „lháře“, především tzv. empirické premisy jakožto jednoho z předpokladů, za nichž antinomie vznikne. Rozbor této premisy vyžaduje přihlídnout k metodám identifikace jazykových výrazů. Je použito přitom metody identifikace nerozlišitelného, která je vyložena v souvislosti s požadavky finitismu. Výsledkem je zjištění, že sémantická antinomie „lháře“ je zvláštním případem antinomie synonymních jmen. Další část práce se týká úlohy kontextu a kontextového zřetězení při vzniku antinomie. Je zjištěno, že antinomie vzniká záměnou dvou sémanticky odlišných druhů kontextového zřetězení, tj. extenzionálního a neextenzionálního kontextového zřetězení.

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