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REAL-TIME PARAMETER ESTIMATION AND OUTPUT PREDICTION FOR ARMA-TYPE SYSTEM MODELS*

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A multi-input multi-output dynamic stochastic system, describable by a model of ARMA-type and possibly controlled in closed adaptive control loop, is considered. The paper solves the problem of real-time parameter estimation and output prediction for the case of known covariances of the stochastic moving-average (MA) term of the model. The presented bayesian solution, based on LD factorization of the covariance matrix, shows that for the given purpose the C-parameters of the MA term have to be considered as time varying even when the covariances are time-invariant.

1. INTRODUCTION

The model ARMA (Autoregressive - Moving Average) or ARMAX (X indicates the external manipulated input) is a very general input-output model of linear stochastic systems and therefore the problem of estimation of its parameters has attracted much attention in system identification literature. The maximum-likelihood method of Åström and Bohlin [2] became popular for off-line estimation in single-output case. The practical requirement to simplify the calculation and/or to track the model parameters in real time motivated the development of a number of recursive algorithms with reduced demand on the memory of the computing device. Their comparison and further references can be found in [9, 8, 3]. The most recent contribution to the topic seems to be [4, 7].

The present paper deals with the multivariate model of ARMAX type with unknown matrix parameters of the deterministic part but with known covariances of the stochastic part. Due to the assumed knowledge of the covariances a sufficient statistic of fixed (nongrowing) dimension can be found both for the estimated parameters and the predicted output. This makes it possiple to give an exact bayesian

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solution of the problem of real-time parameter estimation and output prediction in closed adative control loop with no loss of information. This optimal solution for the case of known covariances yields a better understanding of existing recursive suboptimal algorithms for the case of unknown covariances and may help to suggest their improvement and extension.

2. PROBLEM STATEMENT

It is assumed that the input-output relation for the observed system with v-dimensional output $y_{(t)}$ can be described by the model

$$f_{(t)} = Pz_{(t)} + v_{(t)} \tag{1}$$

where $f_{(t)}$ is a known vector-valued function of the output $y_{(t)}$ (and contingently also of the recent input $u_{(t)}$ and the past observed input-output data $\mathscr{D}^{(t-1)}$) such that the mapping $y_{(t)} \leftrightarrow f_{(t)}$ (for any fixed $u_{(t)}$ and $\mathscr{D}^{(t-1)}$) is one-to-one, e.g. $f_{(t)} = y_{(t)}$, or $f_{(t)} = y_{(t)} - y_{(t-1)}$, or $f_{(t)} = \ln(y_{(t)}/y_{(t-1)})$ with $y_{(k)} > 0$, etc. The vector $z_{(t)}$ of dimension ϱ is a known function of the recent $u_{(t)}$ and the past input-output data $\mathscr{D}^{(t-1)}$, P is the $(v \times \varrho)$-matrix of unknown parameters and $v_{(t)}$ is a colored stationary gaussian noise with zero mean and with a finite correlation span

$$\begin{aligned} \mathsf{E}[v_{(t)}v_{(t-i)}^{\mathsf{T}}] &= R_i \quad \text{for} \quad i = 0, 1, \ldots, n \\ &= 0 \quad \text{for} \quad |i| > n \end{aligned} \tag{2}$$

which can be modelled as a moving average defined on the sequence of mutually uncorrelated normally distributed random variables $\{e_{(t)}\}$.

$$v_{(t)} = e_{(t)} + \sum_{i=1}^{n} C_i e_{(t-i)} \tag{3}$$

$$\mathsf{E}[e_{(t)}] = 0\,, \quad \mathsf{E}[e_{(t)}e_{(t)}^{\mathsf{T}}] = R_e \tag{4}$$

Apparently

$$R_i = \sum_{k=i}^{n} C_k R_e C_{k-i}^{\mathsf{T}} \quad \text{with} \quad C_0 = I \tag{5}$$

It is assumed that R_e and C_i $(i = 1, 2, \ldots, n)$, or directly R_i $(i = 0, 1, \ldots, n)$ are known.

The time indexing is chosen in such a way that $u_{(t)}$ precedes $y_{(t)}$ (i.e. when $u_{(t)}$ is decided $y_{(t)}$ is not yet known) and $t = 1$ indicates the first time instant for which $z_{(t)}$ and $f_{(t)}$ are defined. Hence

$$\mathscr{D}^{(t)} = \{y_{(t)}, u_{(t)}, y_{(t-1)}, u_{(t-1)}, \ldots, y_{(1)}, u_{(1)}, \mathscr{D}^{(0)}\}$$

where $\mathscr{D}^{(0)}$ are initial input-output data which are required for $z_{(1)}$ and $f_{(1)}$. In the standard case of a linear ARMAX model we have $f_{(t)} = y_{(t)}$ and the matrix P of

unknown parameters and the corresponding vector $z_{(t)}$ can be chosen, for instance, as follows

$$P = [B_0, A_1, B_1, \ldots, A_n, B_n], \quad z_{(t)}^{\mathrm{T}} = [u_{(t)}^{\mathrm{T}}, y_{(t-1)}^{\mathrm{T}}, u_{(t-1)}^{\mathrm{T}}, \ldots, y_{(t-n)}^{\mathrm{T}}, u_{(t-n)}^{\mathrm{T}}]$$

Clearly, in this case

$$\mathscr{D}^{(0)} = \{y_{(0)}, u_{(0)}, \ldots, y_{(-n+1)}, u_{(-n+1)}\}$$

where $u_{(-n+1)}, y_{(-n+1)}$ is the first observed input-output pair.

To define the input-output relation by the model (1) uniquely it is necessary to assume that there does not exist any hidden channel from the input signal $\{u_{(t)}\}$ to the noise $\{v_{(t)}\}$. This condition can be formally expressed as the equality of the following conditional probability densities (c.p.d.)

$$(6) \qquad \begin{aligned} &\mathscr{A}(v_{(t)} \mid v_{(t-1)}, \ldots, v_{(1)}, u_{(t)}, u_{(t-1)}, \ldots, u_{(1)}, \mathscr{D}^{(0)}, P) = \\ &\qquad = \mathscr{A}(v_{(t)} \mid v_{(t-1)}, \ldots, v_{(1)}) . \end{aligned}$$

The problem can be formulated as follows. Given the prior c.p.d. $\mathscr{A}(P \mid \mathscr{D}^{(0)})$ calculate recursively, for growing t, the c.p.d.

$$(7) \qquad \mathscr{A}(y_{(t)} \mid u_{(t)}, \mathscr{D}^{(t-1)})$$

which does not contain the unknown parameters P, and update, when a new input-output pair $\{y_{(t)}, u_{(t)}\}$ is observed, the c.p.d. characterizing the uncertainty of the unknown parameters

$$(8) \qquad \mathscr{A}(P \mid \mathscr{D}^{(t-1)})$$

Bearing in mind the possible application in adaptive control systems the input $u_{(t)}$ is allowed to be generated in closed control loop, but under the "natural conditions of control" [6] which means that the input $u_{(t)}$ may depend on the unknown parameters only through the past observed data and it holds

$$\mathscr{A}(u_{(t)} \mid \mathscr{D}^{(t-1)}, P) = \mathscr{A}(u_{(t)} \mid \mathscr{D}^{(t-1)})$$

or equivalently

$$(9) \qquad \mathscr{A}(P \mid u_{(t)}, \mathscr{D}^{(t-1)}) = \mathscr{A}(P \mid \mathscr{D}^{(t-1)}) .$$

3. GENERAL FUNCTIONAL RECURSION

Since the mapping $y_{(t)} \leftrightarrow f_{(t)}$, for any given $\{u_{(t)}, \mathscr{D}^{(t-1)}\}$, is assumed to be one-to-one, it is sufficient, and more convenient to consider, instead of the c.p.d. (7) for $y_{(t)}$ the c.p.d. for $f_{(t)}$. Given the c.p.d. (8) and employing the natural conditions of control (9) the predictive c.p.d. can be determined as follows.

$$(10) \qquad \mathscr{A}(f_{(t)} \mid u_{(t)}, \mathscr{D}^{(t-1)}) = \int \mathscr{A}(f_{(t)} \mid u_{(t)}, \mathscr{D}^{(t-1)}, P)\, \mathscr{A}(P \mid \mathscr{D}^{(t-1)})\, \mathrm{d}P .$$

When the new input $u_{(t)}$ is applied, the true value of the new output $y_{(t)}$ observed and corresponding $f_{(t)}$ determined, the new piece of information can be built into (8) using the Bayes formula which, together with (9), gives

$$(11) \qquad \mathcal{p}(P \mid \mathcal{D}^{(t)}) = \frac{\mathcal{p}(f_{(t)} \mid u_{(t)}, \mathcal{D}^{(t-1)}, P)}{\mathcal{p}(f_{(t)} \mid u_{(t)}, \mathcal{D}^{(t-1)})} \mathcal{p}(P \mid \mathcal{D}^{(t-1)}) .$$

The functional recursion (10) and (11) describes the evolution of the c.p.d.'s (7) and (8) which are of interest. However, to be able to exploit this recursion the c.p.d.

$$(12) \qquad \mathcal{p}(f_{(t)} \mid u_{(t)}, \mathcal{D}^{(t-1)}, P) , \quad t \geqq 1$$

entering both (10) and (11) must be determined as a function of unknown parameters P.

Considering (6) and the fact that the model (1) defines, for any fixed P and $\{u_{(t)}, \mathcal{D}^{(t-1)}\}$, the one-to-one mappings $v_{(t)} \leftrightarrow f_{(t)} \leftrightarrow y_{(t)}$ (the first one with the jacobian equal to one) it is seen that the c.p.d. (12) can be determined as

$$(13) \qquad \mathcal{p}(v_{(t)} \mid v_{(t-1)}, v_{(t-2)}, \ldots, v_{(1)}) = \frac{\mathcal{p}(v_{(t)}, v_{(t-1)}, \ldots, v_{(1)})}{\mathcal{p}(v_{(t-1)}, v_{(t-2)}, \ldots, v_{(1)})}$$

where each $v_{(k)}$ is expressed through the model (1).

If the set of noise vectors $\{v_{(k)};\ k = 1, 2, \ldots, t\}$ is ordered into a single column vector

$$v^{\mathrm{T}} = [v_{(1)}^{\mathrm{T}}, v_{(2)}^{\mathrm{T}}, \ldots, v_{(t)}^{\mathrm{T}}]$$

then the joint p.d. in the numerator on the right-hand side of (13) can be written as follows

$$(14) \qquad \mathcal{p}(v_{(t)}, v_{(t-1)}, \ldots, v_{(1)}) = (2\pi)^{-vt/2} |V|^{-1/2} \exp\left(-\tfrac{1}{2} v^{\mathrm{T}} V^{-1} v\right)$$

where V is the covariance matrix with the block entries of dimension $(v \times v)$

$$V_{(\tau,s)} = \mathsf{E}[v_{(\tau)} v_{(s)}^{\mathrm{T}}]$$

$$V_{(\tau,\tau-i)} = R_i , \quad V_{(\tau,\tau+i)} = R_i^{\mathrm{T}} , \quad \text{for} \quad i = 0, 1, \ldots, n$$

$$V_{(\tau,\tau\pm i)} = 0 , \quad \text{for} \quad i > n .$$

4. LD FACTORIZATION OF THE COVARIANCE MATRIX V

In order to determine the c.p.d. (13) the block-LD factorization of the covariance matrix $V = LDL^{\mathrm{T}}$ will appear to be convenient, where L is a lower triangular matrix

of the form

$$L = \begin{bmatrix} I & & & & \\ C_{1(2)} & I & & \mathbf{0} & \\ \vdots & & \ddots & & \\ C_{n(n-1)} & C_{n-1(n-1)} & \cdots & C_{1(n-1)} & I \\ \mathbf{0} & & C_{n(t)} & C_{n-1(t)} \cdots & C_{1(t)} \quad I \end{bmatrix} \begin{matrix} \}v \\ \}v \\ \\ \\ \}v \end{matrix}$$

and $D = \operatorname{diag}[D_{(1)}, D_{(2)}, \ldots, D_{(t)}]$ is a $(v \times v)$-block-diagonal matrix.

It is easy to verify that it holds

$$D_{(1)} = R_0\,, \quad C_{1(2)} = R_1 D_{(1)}^{-1}\,, \quad D_{(2)} = R_0 - C_{1(2)} D_{(1)} C_{1(2)}^{\mathrm{T}}$$

and further on, for $\tau = 3, 4, \ldots$ and $i = n, n-1, \ldots, 1$ with $n_\tau = \min(n, \tau)$,

$$C_{i(\tau)} = \Big(R_i - \sum_{k=i+1}^{n_\tau} C_{k(\tau)} D_{(\tau-k)} C_{k-i(\tau-i)}^{\mathrm{T}}\Big) D_{(\tau-i)}^{-1} \tag{15}$$

$$D_{(\tau)} = R_0 - \sum_{k=1}^{n_\tau} C_{k(\tau)} D_{(\tau-k)} C_{k(\tau)}^{\mathrm{T}}\,. \tag{16}$$

As the matrix L is lower triangular its inverse $F = L^{-1}$ is also lower triangular (with $F_{(\tau,\tau)} = I$) and the quadratic form in the exponent of (14) can be given the following form

$$v^{\mathrm{T}} V^{-1} v = v^{\mathrm{T}} F^{\mathrm{T}} D^{-1} F v = \sum_{\tau=1}^{t} \varepsilon_{(\tau)}^{\mathrm{T}} D_{(\tau)}^{-1} \varepsilon_{(\tau)}$$

where $\varepsilon_{(\tau)}$ is the τ-th block-row of the product Fv

$$\varepsilon_{(\tau)} = \sum_{k=1}^{\tau} F_{(\tau,k)} v_{(k)} = v_{(\tau)} + \sum_{k=1}^{\tau-1} F_{(\tau,k)} v_{(k)}\,. \tag{17}$$

The fact that the weighted sum (17) does not contain $v_{(k)}$ for $k > \tau$ makes it possible to determine the conditional probability density (13) in a straightforward way

$$f\big(v_{(t)} \mid v_{(t-1)}, \ldots, v_{(1)}\big) = \tag{18}$$

$$= (2\pi)^{-v/2} \big|D_{(t)}\big|^{-1/2} \exp\big(-\tfrac{1}{2} \varepsilon_{(t)}^{\mathrm{T}} D_{(t)}^{-1} \varepsilon_{(t)}\big)\,.$$

To derive the required c.p.d. (12) it now remains to express all $v_{(\tau)}$ in (18) through the data and unknown parameters using the model (1). It appears suitable to perform this in the following way

$$v_{(\tau)} = f_{(\tau)} - P z_{(\tau)} = f_{(\tau)} - Z_{(\tau)} \underset{\sim}{p}$$

where $Z_{(\tau)}$ is the quasidiagonal $(v \times v\varrho)$-matrix

$$Z_{(\tau)} = \begin{bmatrix} z_{(\tau)}^{\mathrm{T}} & 0 & \ldots & 0 \\ 0 & z_{(\tau)}^{\mathrm{T}} & \ldots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & & z_{(\tau)}^{\mathrm{T}} \end{bmatrix}$$

and p is a (vq)-vector of the unknown parameters constructed from the matrix P by "stacking" its elements in the row-by-row way, i.e. if P_i is the i-th row of P then

$$p = \begin{bmatrix} P_1^{\mathrm{T}} \\ P_2^{\mathrm{T}} \\ \vdots \\ P_n^{\mathrm{T}} \end{bmatrix}$$

After this substitution $\varepsilon_{(\tau)}$ defined by (17) can be expressed as follows

$$\varepsilon_{(\tau)} = \tilde{f}_{(\tau)} - \tilde{Z}_{(\tau)} p \tag{19}$$

where

$$\tilde{f}_{(\tau)} = \sum_{k=1}^{\tau} F_{(\tau,k)} f_{(k)}$$

$$\tilde{Z}_{(\tau)} = \sum_{k=1}^{\tau} F_{(\tau,k)} Z_{(k)}$$

To see that this filtering of data can be performed recursively consider the following equalities

$$\begin{bmatrix} \tilde{f}_{(1)} \\ \tilde{f}_{(2)} \\ \vdots \\ \tilde{f}_{(t)} \end{bmatrix} = F \begin{bmatrix} f_{(1)} \\ f_{(2)} \\ \vdots \\ f_{(t)} \end{bmatrix}, \qquad L \begin{bmatrix} \tilde{f}_{(1)} \\ \tilde{f}_{(2)} \\ \vdots \\ \tilde{f}_{(t)} \end{bmatrix} = \begin{bmatrix} f_{(1)} \\ f_{(2)} \\ \vdots \\ f_{(t)} \end{bmatrix}$$

The last row in the second of these equalities gives the recursion relation

$$\tilde{f}_{(t)} + \sum_{i=1}^{n_t} C_{i(t)} \tilde{f}_{(t-i)} = f_{(t)}, \quad \tilde{f}_{(1)} = f_{(1)} \tag{20}$$

where $n_t = \min(n, t)$. Similarly for $\tilde{Z}_{(t)}$,

$$\tilde{Z}_{(t)} + \sum_{i=1}^{n_t} C_{i(t)} \tilde{Z}_{(t-i)} = Z_{(t)}, \quad \tilde{Z}_{(1)} = Z_{(1)} \tag{21}$$

Note that, while $Z_{(t)}$ is quasidiagonal, the filtered matrix $\tilde{Z}_{(t)}$ $(t > 1)$ is, in general, a full $(v \times \varrho v)$-matrix.

Hence, the c.p.d. (12) has a normal form

$$\mathcal{N}(f_{(t)} \mid u_{(t)}, \mathscr{D}^{(t-1)}, P) = \tag{22}$$

$$= (2\pi)^{-v/2} |D_{(t)}|^{-1/2} \exp\{-\tfrac{1}{2}\varepsilon_{(t)}^{\mathrm{T}} D_{(t)}^{-1} \varepsilon_{(t)}\} \sim \mathcal{N}(\tilde{f}_{(t)}, D_{(t)})$$

where the conditional mean $\tilde{f}_{(t)}$, expressed as a function of any given parameters p, using (19) and (20), is

$$\tilde{f}_{(t)} = \tilde{Z}_{(t)} p + \sum_{i=1}^{n_t} C_{i(t)} \tilde{f}_{(t-i)} \tag{23}$$

or, when $\tilde{Z}_{(t)}$ is substituted from (21),

$$\tilde{f}_{(t)} = Z_{(t)} p + \sum_{i=1}^{n_t} C_{i(t)} (\tilde{f}_{(t-i)} - \tilde{Z}_{(t-i)} p) \tag{24}$$

5. FINAL ALGORITHM

A straightforward calculation of the functional recursion (10) and (11) using (22) proves the following

Theorem. If the c.p.d. (8) is normal

$$\mathcal{N}(p \mid \mathcal{D}^{(t-1)}) \sim \mathcal{N}(\hat{p}_{(t-1)}, R_{p(t-1)})$$

then, for the model (1) and the conditions defined in Section 2, the normality is reproduced

$$\mathcal{N}(f_{(t)} \mid \mathcal{D}^{(t-1)}) \sim \mathcal{N}(\hat{f}_{(t)}, R_{f(t)})\,;$$

$$\mathcal{N}(p \mid \mathcal{D}^{(t)}) \sim \mathcal{N}(\hat{p}_{(t)}, R_{p(t)})$$

and the following algebraic recursion holds for the conditional mean values and covariance matrices

$$\hat{f}_{(t)} = \tilde{Z}_{(t)}\hat{p}_{(t-1)} + \sum_{i=1}^{n_t} C_{i(t)}\hat{f}_{(t-i)} \tag{25}$$

$$R_{f(t)} = D_{(t)} + \tilde{Z}_{(t)}R_{p(t-1)}\tilde{Z}_{(t)}^{\mathrm{T}} \tag{26}$$

$$\hat{p}_{(t)} = \hat{p}_{(t-1)} + K_{(t)}(\hat{f}_{(t)} - \tilde{Z}_{(t)}\hat{p}_{(t-1)}) \tag{27}$$

$$K_{(t)} = R_{p(t-1)}\tilde{Z}_{(t)}^{\mathrm{T}}R_{f(t)}^{-1} = R_{p(t)}\tilde{Z}_{(t)}^{\mathrm{T}}D_{(t)}^{-1} \tag{28}$$

$$R_{p(t)} = R_{p(t-1)} - R_{p(t-1)}\tilde{Z}_{(t)}^{\mathrm{T}}R_{f(t)}^{-1}\tilde{Z}_{(t)}R_{p(t-1)} \tag{29}$$

The overall algorithm for the real-time estimation of the parameters p and for the one-step-ahead prediction of $f_{(t)}$ is obtained when the recursion (25) to (29) is supplemented with the recursive calculation of $C_{i(t)}$ $(i = n_t, n_t - 1, \ldots, 1)$, $D_{(t)}$ (factorization) according to (15) and (16), and with the data filtering (20) and (21). If little is known a priori about the possible values of the unknown parameters p then it can be recommended to start the recursion with $\hat{p}_{(0)} = 0$ and $R_{f(0)} = \delta^{-1}I$ where δ^{-1} is a large number the influence of which rapidly disappears if the data do carry the information about p.

6. CONCLUDING REMARKS

The recursive formulae (15) and (16) can be regarded as an algorithm for the factorization of the matrix-polynomial product $C(z)\,R_e\,C^{\mathrm{T}}(z^{-1})$ where $C(z) = I + C_1 z + \ldots + C_n z^n$, [10]. However, it should be stressed that the optimality of prediction and parameter estimation requires that the factorization be performed in real-time even when the result of factorization is a priori known, i.e. when all roots of $|C(z)|$ lie outside or on the unit circle. For the case of prediction with the known

parameters p this has been shown by Aasnaes and Kailath [1], who also showed that the often used predictor with $C_{i(t)} = C_{i(\infty)}$ does not produce optimal predictions (not even asymptotically) if the roots of $|C(z)|$ lie on the unit circle and that the convergence to the optimality may be very slow if the roots are close to the unit circle.

The presented optimal solution for the case of known noise covariances clearly shows the principal difficulties which are encountered when this knowledge cannot be assumed. If the uncertainty of the covariances can be reduced so that only a finite number N of sets of values can be considered as possible then the optimal solution can be found by combining the result of this paper with the bayesian approach to system classification developed in [5, 6]. However, in such a case N filters have to run in parallel.

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