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ON THE WEAK-OPEN IMAGES OF METRIC SPACES

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Abstract. In this paper, we give characterizations of certain weak-open images of metric spaces.

Keywords: g -metrizable spaces, weak-bases, weak-open mappings, σ -mappings, π -mappings, cs -mappings

MSC 2000: 54E99, 54C10

1. INTRODUCTION

To find internal characterizations of certain images of metric spaces is one of the central problems in General Topology. Recently, S. Xia [4] introduced the concept of weak-open mappings. By using it, certain g -first countable spaces are characterized as images of metric spaces under various weak-open mappings. Papers [6], [8], [9], [10], [11], [20] have done some wonderful work on g -metrizable spaces, but have only investigated internal characterizations of g -metrizable spaces. The present paper establishes the relationships between g -metrizable spaces (spaces with compact-countable weak-bases) and metric spaces by means of weak-open mappings, π -mappings and σ -mappings (weak-open mappings and cs -mappings, respectively).

In this paper, all spaces are regular and T_1 , all mappings are continuous and surjective. \mathbb{N} denotes the set of all natural numbers, ω denotes $\mathbb{N} \cup \{0\}$. For a collection \mathcal{P} of subsets of a space X and a mapping $f: X \rightarrow Y$, denote $f(\mathcal{P}) = \{f(P): P \in \mathcal{P}\}$.

Definition 1.1. Let \mathcal{P} be a cover of a space X . \mathcal{P} is called compact-countable if for each compact subset K of Y , only countably many members of \mathcal{P} intersect K .

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Definition 1.2. Let $\mathcal{P} = \bigcup\{\mathcal{P}_x : x \in X\}$ be a collection of subsets of a space X satisfying that for each $x \in X$,

- (1) \mathcal{P}_x is a network of x in X ,
- (2) if $U, V \in \mathcal{P}_x$, then $W \subset U \cap V$ for some $W \in \mathcal{P}_x$.

\mathcal{P} is called a weak-base for X [2] if a subset G of X is open in X if and only if for each $x \in G$, there exists $P \in \mathcal{P}_x$ such that $P \subset G$.

A space X is called a g -metrizable space [3] if X has a σ -locally finite weak-base.

Definition 1.3. Let $f: X \rightarrow Y$ be a mapping.

- (1) f is a weak-open mapping [4] if there exists a weak-base $\mathcal{B} = \bigcup\{\mathcal{B}_y : y \in Y\}$ for Y , and for $y \in Y$ there exists $x(y) \in f^{-1}(y)$ satisfying condition (*): for each open neighbourhood U of $x(y)$, $B_y \subset f(U)$ for some $B_y \in \mathcal{B}_y$.
- (2) f is a cs-mapping [5] if for each compact subset K of Y , $f^{-1}(K)$ is separable in X .
- (3) f is a σ -mapping [1] if there exists a base \mathcal{B} for X such that $f(\mathcal{B})$ is a σ -locally finite collection of subsets of Y .
- (4) f is a π -mapping [19] if (X, d) is a metric space, and for each $y \in Y$ and its open neighbourhood V in Y , $d(f^{-1}(y), X \setminus f^{-1}(V)) > 0$.

It is easy to check that a weak-open mapping is a quotient mapping.

2. THE WEAK-OPEN σ -IMAGE OF A METRIC SPACE

Lemma 2.1 [6]. *Suppose (X, d) is a metric space and $f: X \rightarrow Y$ is a quotient mapping. Then Y is a symmetric space if and only if f is a π -mapping.*

Theorem 2.2. *The following are equivalent for a space X :*

- (1) Y is a g -metrizable space.
- (2) Y is a weak-open, π , σ -image of a metric space.
- (3) Y is a weak-open σ -image of a metric space.

Proof. (1) \Rightarrow (2) Suppose Y is a g -metrizable space, then Y has a σ -locally finite weak-base. Let $\mathcal{P} = \bigcup\{\mathcal{P}_i : i \in \mathbb{N}\}$ be a σ -locally finite weak-base for Y , where each $\mathcal{P}_i = \{P_\alpha : \alpha \in A_i\}$ is locally finite in Y which is closed under finite intersections and $Y \in \mathcal{P}_i \subset \mathcal{P}_{i+1}$. For each $i \in \mathbb{N}$, endow A_i with discrete topology. Then A_i is a metric space. Put

$$X = \left\{ \alpha = (\alpha_i) \in \prod_{i \in \mathbb{N}} A_i : \{P_{\alpha_i} : i \in \mathbb{N}\} \subset \mathcal{P} \text{ forms a network at some point } x(\alpha) \in Y \right\},$$

and endow X with the subspace topology induced from the usual product topology of the collection $\{A_i: i \in \mathbb{N}\}$ of metric spaces. Then X is a metric space. Since Y is Hausdorff, $x(\alpha)$ is unique in Y for each $\alpha \in X$. We define $f: X \rightarrow Y$ by $f(\alpha) = x(\alpha)$ for each $\alpha \in X$. Because \mathcal{P} is a σ -locally finite weak-base for Y , we conclude that f is surjective. For each $\alpha = (\alpha_i) \in X$, $f(\alpha) = x(\alpha)$. Suppose V is an open neighbourhood of $x(\alpha)$ in Y . Then there exists $n \in \mathbb{N}$ such that $x(\alpha) \in P_{\alpha_n} \subset V$. If we set $W = \{c \in X: \text{the } n\text{-th coordinate of } c \text{ is } \alpha_n\}$, then W is an open neighbourhood of α in X and $f(W) \subset P_{\alpha_n} \subset V$. Hence f is continuous. We will show that f is a weak-open σ -mapping.

(i) f is a σ -mapping.

For each $n \in \mathbb{N}$ and $\alpha_n \in A_n$, put

$$V(\alpha_1, \dots, \alpha_n) = \{\beta \in X: \text{for each } i \leq n, \text{ the } i\text{-th coordinate of } \beta \text{ is } \alpha_i\}.$$

It is easy to check that $\{V(\alpha_1, \dots, \alpha_n): n \in \mathbb{N}\}$ is a locally neighbourhood base of α in X .

Let $\mathcal{B} = \{V(\alpha_1, \dots, \alpha_n): \alpha_i \in A_i (i \leq n) \text{ and } n \in \mathbb{N}\}$; then \mathcal{B} is a base for X . To prove that f is a σ -mapping, we only need to check that $f(V(\alpha_1, \dots, \alpha_n)) = \bigcap_{i \leq n} P_{\alpha_i}$ for each $n \in \mathbb{N}$ and $\alpha_n \in A_n$ because $f(\mathcal{B})$ is σ -locally finite in Y by this result. For each $n \in \mathbb{N}$, $\alpha_n \in A_n$ and $i \leq n$ we have $f(V(\alpha_1, \dots, \alpha_n)) \subset P_{\alpha_i}$, hence $f(V(\alpha_1, \dots, \alpha_n)) \subset \bigcap_{i \leq n} P_{\alpha_i}$. On the other hand, for each $x \in \bigcap_{i \leq n} P_{\alpha_i}$ there is $\beta = (\beta_j) \in X$ such that $f(\beta) = x$. For each $j \in \mathbb{N}$, $P_{\beta_j} \in \mathcal{P}_j \subset \mathcal{P}_{j+n}$, hence there is $\alpha_{j+n} \in A_{j+n}$ such that $P_{\alpha_{j+n}} = P_{\beta_j}$. Set $\alpha = (\alpha_j)$, then $\alpha \in V(\alpha_1, \dots, \alpha_n)$ and $f(\alpha) = x$. Thus $\bigcap_{i \leq n} P_{\alpha_i} \subset f(V(\alpha_1, \dots, \alpha_n))$, hence $f(V(\alpha_1, \dots, \alpha_n)) = \bigcap_{i \leq n} P_{\alpha_i}$.

Therefore, f is a σ -mapping.

(ii) f is a weak-open mapping.

Denote $\mathcal{P}_y = \{P \in \mathcal{P}: y \in P\}$; then $\mathcal{P} = \bigcup \{\mathcal{P}_y: y \in Y\}$.

For each $y \in Y$, by the idea \mathcal{P} , there exists $(\alpha_i) \in \bigcap_{i \in \mathbb{N}} A_i$ such that $\{P_\alpha: i \in \mathbb{N}\} \subset \mathcal{P}$ is a network of y in Y , hence $\alpha = (\alpha_i) \in f^{-1}(y)$.

Suppose G is an open neighbourhood of α in X . Then there exists $j \in \mathbb{N}$ such that $V(\alpha_1, \dots, \alpha_j) \subset G$. Thus $f(V(\alpha_1, \dots, \alpha_j)) \subset f(G)$. By (i), $f(V(\alpha_1, \dots, \alpha_j)) = \bigcap_{i \leq j} P_{\alpha_i}$. So $P_y \subset \bigcap_{i \leq j} P_{\alpha_i}$ for some $P_y \in \mathcal{P}_y$. Hence $P_y \subset f(G)$.

Hence there exists a weak-base \mathcal{P} for Y and $\alpha \in f^{-1}(y)$ satisfying the condition (*) from Definition 1.3(1). Therefore f is a weak-open mapping.

(iii) f is a π -mapping.

By (ii), f is a quotient mapping. Since a g -metrizable space is symmetric, f is a π -mapping by Lemma 2.1.

(2) \Rightarrow (3) is clear.

(3) \Rightarrow (1). Suppose Y is the image of a metric space X under a weak-open σ -mapping f . Since f is a σ -mapping, there exists a base \mathcal{B} for X such that $f(\mathcal{B})$ is σ -locally finite in Y . And since f is a weak-open mapping, there exists a weak-base $\mathcal{P} = \bigcup\{\mathcal{P}_y: y \in Y\}$ for Y such that for each $y \in Y$ there exists $x(y) \in f^{-1}(y)$ satisfying the condition (*) from Definition 1.3(1). For each $y \in Y$, put

$$\mathcal{F}_y = \{f(B): x(y) \in B \in \mathcal{B}\},$$

$$\mathcal{F} = \bigcup\{\mathcal{F}_y: y \in Y\}.$$

Obviously, $\mathcal{F} \subset f(\mathcal{B})$, hence \mathcal{F} is σ -locally finite in Y . We will prove that \mathcal{F} is a weak-base for Y .

It is obvious that \mathcal{F} satisfies the condition (1) from Definition 1.2. For each $y \in Y$, suppose $U, V \in \mathcal{F}_y$; then there exist $B_1 \in \mathcal{B}$ and $B_2 \in \mathcal{B}$ such that $x(y) \in B_1 \cap B_2$ and $f(B_1) = U$, $f(B_2) = V$. Since \mathcal{B} is a base for X , there exists $B \in \mathcal{B}$ such that $x(y) \in B \subset B_1 \cap B_2$. Thus $f(B) \in \mathcal{F}_y$ and $f(B) \subset f(B_1 \cap B_2) \subset U \cap V$. Hence \mathcal{F} satisfies the condition (2) from Definition 1.2.

Suppose $G \subset Y$ is open in Y , then $x(y) \in f^{-1}(G)$ for each $y \in G$. Since \mathcal{B} is a base for X , we have $x(y) \in B \subset f^{-1}(G)$ for some $B \in \mathcal{B}$. Thus $f(B) \in \mathcal{F}_y$ and $f(B) \subset G$. On the other hand, suppose that $G \subset Y$ and for $y \in G$ there exists $F \in \mathcal{F}_y$ such that $F \subset G$. Then there exists $B \in \mathcal{B}$ such that $x(y) \in B$ and $F = f(B)$. Since B is an open neighbourhood of $x(y)$, there exists $P_y \in \mathcal{P}_y$ such that $P_y \subset f(B)$. Thus for each $y \in G$ there exists $P_y \in \mathcal{P}_y$ such that $P_y \subset G$. Hence G is open in Y because \mathcal{P} is a weak-base for Y . So \mathcal{F} is a weak-base for Y .

Therefore Y is a g -metrizable space. □

3. THE WEAK-OPEN cs -IMAGE OF A METRIC SPACE

Theorem 3.1. *A space Y has a compact-countable weak-base if and only if Y is a weak-open cs -image of a metric space.*

Proof. *Sufficiency.* Suppose Y is the image of a metric space X under a weak-open cs -mapping f . Since f is a weak-open mapping, there exists a weak-base $\mathcal{B} = \bigcup\{\mathcal{B}_y: y \in Y\}$ for Y such that for each $y \in Y$ there exists $x(y) \in f^{-1}(y)$ satisfying the condition (*) from Definition 1.3(1). Because X is a metric space, X has a σ -locally finite base. Let \mathcal{P} be a σ -locally finite base for X . For each $P \in \mathcal{P}$, put

$$\mathcal{B}_P = \{B \in \mathcal{B}: B \subset f(P)\},$$

$$\mathcal{B}_P = \bigcup \mathcal{B}_P,$$

then $B_P \subset f(P)$. For each compact subset K of Y , since f is a cs -mapping, $f^{-1}(K)$ is separable in X . So $f^{-1}(K)$ is a Lindelöf subspace of X . Because a locally finite collection of a Lindelöf space is countable, $\{P \in \mathcal{P} : P \cap f^{-1}(K) \neq \Phi\}$ is countable. Thus $f(\mathcal{P})$ is compact-countable. Hence $\mathcal{B}^* = \{B_P : P \in \mathcal{P}\}$ is compact-countable. For each $y \in Y$, put

$$\begin{aligned}\mathcal{B}'_y &= \{B_P \in \mathcal{B}^* : B_y \in \mathcal{B}_P \text{ for some } B_y \in \mathcal{B}_y\}, \\ \mathcal{B}''_y &= \left\{ \bigcap \mathcal{U} : \mathcal{U} \text{ is a finite subcollection of } \mathcal{B}'_y \right\}, \\ \mathcal{B}'' &= \bigcup \{ \mathcal{B}''_y : y \in Y \},\end{aligned}$$

then \mathcal{B}'' is compact-countable. We will prove that \mathcal{B}'' is a weak-base for Y . It is easy to check that \mathcal{B}'' satisfies the condition (1), (2) from Definition 1.2.

Suppose V is open in Y for each $y \in V$, since \mathcal{P} is a base for X , then $x(y) \in P \subset f^{-1}(V)$ for some $P \in \mathcal{P}$. Thus there exists $B_y \in \mathcal{B}_y$ such that $B_y \subset f(P)$, and so $B_y \in \mathcal{B}_P$. Hence $B_P \in \mathcal{B}'_y \subset \mathcal{B}''_y$ and $B_P \subset f(P) \subset V$. On the other hand, suppose $V \subset Y$ is such that for each $y \in V$, $B \subset V$ for some $B \in \mathcal{B}''_y$. By the properties of \mathcal{B}' and \mathcal{B}'' and the condition (2) from Definition 1.2, there exists $B_y \in \mathcal{B}_y$ such that $y \in B_y \subset B \subset V$. Because $\mathcal{B} = \bigcup \{ \mathcal{B}_y : y \in Y \}$ is a weak-base for Y , V is open in Y . Therefore \mathcal{B}'' is a weak-base for Y .

Necessity. Suppose \mathcal{P} is a compact-countable weak-base for Y . Endow \mathcal{P} with discrete topology, then \mathcal{P} is a metric space. Put $X = \{(P_n) \in \mathcal{P}^\omega : \{P_n : n \in \mathbb{N}\} \text{ is a network of some point } y \in Y\}$, and endow X with the subspace topology induced by the product topology of the usual product space \mathcal{P}^ω . Then X is a metric space. Since Y is Hausdorff, y is unique in Y (in fact, it is easy to check that $\{y\} = \bigcap_{n \in \mathbb{N}} P_n$). We define $f : X \rightarrow Y$ by $f((P_n)) = y$ for each $(P_n) \in X$. For each $y \in Y$, since \mathcal{P} is point-countable in Y , denoting $\{P \in \mathcal{P} : y \in P\}$ by (P_n) , we have $(P_n) \in X$ and $f((P_n)) = y$. Thus f is a surjection. It is obvious that f is continuous. We will prove that f is a weak-open cs -mapping.

(i) f is a weak-open mapping.

For each $y \in Y$, denote a collection of weak neighbourhoods of y in Y by \mathcal{P}_y ; then \mathcal{P}_y is countable. Set $\mathcal{P}_y = \{P_n : n \in \mathbb{N}\}$, then $f((P_n)) = y$ and $(P_n) \in f^{-1}(y)$. For each $n \in \mathbb{N}$, put

$$B(P_1, \dots, P_n) = \{(P'_n) \in X : P'_i = P_i \text{ for each } i \leq n\}.$$

It is easy to check that $\{B(P_1, \dots, P_n) : n \in \mathbb{N}\}$ is a locally neighbourhood base of the point (P_n) in X . □

Claim. $f(B(P_1, \dots, P_n)) = \bigcap_{i \leq n} P_i$ for each $n \in \mathbb{N}$.

Suppose $(P'_i) \in B(P_1, \dots, P_n)$, then $f((P'_i)) = \bigcap_{i \in \mathbb{N}} P'_i \subset \bigcap_{i \leq n} P_i$. Thus $f(B(P_1, \dots, P_n)) \subset \bigcap_{i \leq n} P_i$. On the other hand, suppose $z \in \bigcap_{i \leq n} P_i$ and set $\mathcal{P}_z = \{P''_{n+j} : j \in \mathbb{N}\}$.

Put

$$P_r^* = \begin{cases} P_r, & r \leq n, \\ P''_r, & r > n, \end{cases}$$

then $(P_r^*) \in B(P_1, \dots, P_n)$ and $f((P_r^*)) = z$. Thus $\bigcap_{i \leq n} P_i \subset f(B(P_1, \dots, P_n))$. Hence

$$f(B(P_1, \dots, P_n)) = \bigcap_{i \leq n} P_i.$$

Because \mathcal{P} is a weak-base for Y and $\{P_n : n \in \mathbb{N}\} = \mathcal{P}_y$, we obtain $f(B(P_1, \dots, P_n)) = \bigcap_{i \leq n} P_i \in \mathcal{P}_y$ for each $n \in \mathbb{N}$. Suppose G is a open neighbourhood of the point (P_n) in X ; then there exists $j \in \mathbb{N}$ such that $B(P_1, \dots, P_j) \subset G$. So $f(B(P_1, \dots, P_j)) \subset f(G)$. By the Claim, $f(B(P_1, \dots, P_j)) = \bigcap_{i \leq j} P_i \in \mathcal{P}_y$. Hence there exists a weak-base \mathcal{P} for Y and $(P_n) \in f^{-1}(y)$ satisfying the condition (*) from Definition 1.3(1). Therefore f is a weak-open mapping.

(ii) f is a *cs*-mapping.

For each compact subset K of Y , since \mathcal{P} is compact-countable, hence $\{P \in \mathcal{P} : P \cap K \neq \Phi\}$ is countable. Thus $\{P \in \mathcal{P} : P \cap K \neq \Phi\}^\omega \cap X$ is a hereditarily separable subspace of X . Because $f^{-1}(K) \subset \{P \in \mathcal{P} : P \cap K \neq \Phi\}^\omega \cap X$, thus $f^{-1}(K)$ is separable in X . Hence f is a *cs*-mapping.

Remark 3.2. A mapping $f : X \rightarrow Y$ is an *s*-mapping (*ss*-mapping [16]) if for each $y \in Y$, $f^{-1}(y)$ is separable in X (for each $y \in Y$, there exists an open neighbourhood V of y in Y such that $f^{-1}(V)$ is separable in X). A mapping $f : X \rightarrow Y$ is a 1-sequence-covering mapping [14] if for each $y \in Y$ there exists $x \in f^{-1}(y)$ satisfying the following condition: whenever $\{y_n\}$ is a sequence in Y converging to a point y in Y , there exists a sequence $\{x_n\}$ of X converging to a point x in X such that each $x_n \in f^{-1}(y_n)$. Obviously, if X is a metric space, then an *ss*-mapping \Rightarrow a *cs*-mapping \Rightarrow an *s*-mapping. However, we have the following facts.

Example 1. A weak-open *s*-image of a metric space is not a weak-open *cs*-image of a metric space.

Let

$$S = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\} \cup \{0\}, \quad X = [0, 1] \times S,$$

and let

$$Y = [0, 1] \times \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$$

have the usual Euclidean topology as a subspace of $[0, 1] \times S$. Define a typical neighbourhood of $(t, 0)$ in X to be of the form

$$\{(t, 0)\} \cup \left(\bigcup_{k \geq n} V(t, 1/k) \right), \quad n \in \mathbb{N},$$

where $V(t, 1/k)$ is a neighbourhood of $(t, 1/k)$ in $[0, 1] \times \{1/k\}$. Put

$$M = \left(\bigoplus_{n \in \mathbb{N}} [0, 1] \times \{1/n\} \right) \oplus \left(\bigoplus_{t \in [0, 1]} \{t\} \times S \right)$$

and define f from M onto X such that f is an obvious mapping.

Then f is a compact-covering, quotient, two-to-one mapping from the locally compact metric space M onto the separable, regular, non-Lindelöf, k -space X (see Example 2.8.16 of [13] or Example 9.3 of [18]). It is easy to check that f is a 1-sequence-covering mapping. By Theorem 2.5 of [14], X has a point-countable weak-base. Thus X is a weak-open s -image of a metric space by Theorem 2.5 of [4].

X has no compact-countable k -network. Indeed, suppose \mathcal{P} is a compact-countable k -network for X . Put

$$\mathcal{F} = \{\{(t, 0)\}: t \in [0, 1]\} \cup \{P \cap Y: P \in \mathcal{P}\}.$$

Since $[0, 1] \times \{0\}$ is a closed discrete subspace of X , \mathcal{F} is a k -network for X . But Y is a σ -compact subspace of X . Thus $\{P \cap Y: P \in \mathcal{P}\}$ is countable, and so \mathcal{F} is star-countable. Since a regular k -space with a star-countable k -network is an \aleph_0 -space (see [17]), hence X is a Lindelöf space, a contradiction. Thus X has no compact-countable k -network. By Lemma 7 of [15], X has no compact-countable weak-base. Hence X is not a weak-open cs -image of a metric space by Theorem 3.1.

Example 2. A weak-open cs -image of a metric space is not a weak-open ss -image of a metric space.

Let X be a paracompact space with a point-countable base and not metrizable. Then X has a compact-countable base, and so X has a compact-countable weak-base. By Theorem 3.1, X is a weak-open cs -image of a metric space. But X is not a 1-sequence-covering ss -image of a metric space because X is not a metric space. Thus X is not a weak-open ss -image of a metric space by Proposition 3.3 of [5].

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