Alena Vanžurová

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ON REDUCIBILITY OF DOUBLE LINEAR CONNECTIONS ON A DOUBLE VECTOR FIBRATION WITH SOLDERING

ALENA VANŽUROVÁ, Olomouc

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In this paper we will answer some questions about reducibility of connections on the principal fibrations of double linear frames corresponding to TTM and TT^*M using the terminology introduced in [10]–[11]. The original concept of the category of double vector fibrations and morphisms is due to J. Pradines, [6], [7], and was developed by I. Kolář, [2]. Double linear connections were studied in [11], the isomorphisms called solderings were introduced in [2], [11], [12].

Under a (generalized) connection on a fibred manifold $\pi\colon Y \to M$ we understand a smooth section $\Gamma\colon Y \to J^1 Y$ of the natural projection $\varrho_0^1\colon J^1 Y \to Y$ on a target, $\varrho_0^1 \circ \Gamma = \mathrm{id}$. If $\mathscr{C}$, $p\colon \mathscr{C} \to M$ is a double linear ($\mathscr{DL}-$) fibration with the underlying vector fibrations $\mathscr{A}, \mathscr{B}, \mathscr{V}$, then $J^1\mathscr{C}$ (and more generally, $J^r\mathscr{C}$ for $r \geqq 0$) is also endowed with a structure of a $\mathscr{DL}$-fibration, the natural projection $\varrho_0^1\colon J^1\mathscr{C} \to \mathscr{C}$ (or $\varrho_r^s\colon J^s\mathscr{C} \to J^r\mathscr{C}$) being a morphism of $\mathscr{DL}$-fibrations. A connection $\Gamma\colon \mathscr{C} \to J^1\mathscr{C}$ which is at the same time a double linear morphism of $\mathscr{DL}$-fibrations, will be called a $\mathscr{DL}$-connection. Any $\mathscr{DL}$-connection, as a $\mathscr{DLF}$-morphism, induces three underlying linear connections $\Gamma_1\colon \mathscr{A} \to J^1\mathscr{A}$, $\Gamma_2\, \mathscr{B} \to J^1\mathscr{B}$, and $\Gamma_3\colon \mathscr{V} \to J^1\mathscr{V}$. Similarly to the linear case, any $\mathscr{DL}$-fibration is associated with a principal fibration of all double linear ($\mathscr{DL}-$) frames, denoted here by $\mathscr{F}$. A $\mathscr{DL}$-frame on $\mathscr{C}$, at a point x, is a $\mathscr{DL}$-isomorphism $f\colon K(n, s, t) \to \mathscr{C}_x$ of the trivial $\mathscr{DL}$-space $K(n, s, t) = \mathbb{R}^n \times \mathbb{R}^s \times \mathbb{R}^t$ onto the fibre $\mathscr{C}_x$ through $x \in M$. The structure group $\mathrm{Aut}(n, s, t)$ of $\mathscr{F}$ is the group of all $\mathscr{DL}$-automorphisms of the trivial $\mathscr{DL}$-space $K(n, s, t)$. The associated fibration $\mathscr{F}(K(n, s, t))$ is $\mathscr{DLF}$-isomorphic to $\mathscr{C}$.

On the principal fibration $\mathscr{F}$, we admit „principal" connections only, i.e. connections Δ satisfying the right invariant property $\Delta(f\,.\,g) = \Delta(f)\,.\,g$ for any frame $f \in \mathscr{F}$ and any element g of the structure group.

The results obtained here are motivated by the following consideration. The second tangent and cotangent spaces TTM, TT^*M, T^*TM, and T^*T^*M can be regarded as soldered $\mathscr{DL}$-fibrations, all associated with the principal fibration H^2M of second order frames on M, its structure group being L_m^2. Since the functors T^*T and T^*T^* are naturally equivalent to TT^*, we can omit the last two cases.

The $\mathscr{DL}$-fibration TTM has the underlying vector fibrations $\mathscr{A} = \mathscr{B} = \mathscr{V} =$

$= (TM, p_M, M)$, the TT-solderings X_1, X_2: $TM \to TM$ being $X_1 = X_2 = \mathrm{id}_{TM}$. Any principal invariant connection Δ on H^2M induces a generalized connection on TTM, denoted by $\Gamma = TT(\Delta)$: $TTM \to J^1TTM$. Now we can ask when a connection Γ on TTM is of the form $TT(\Delta)$ for any invariant connection Δ on H^2M. This problem was, in a slightly modified version, solved in [8].

It can be verified that $TT(\Delta)$ is a $\mathscr{DL}$-connection. Hence double-linearity is a necessary condition for Γ to be of the above form. Further, we will describe a monomorphism h of H^2M into the principal subfibration $\mathscr{F}_s$ of $\mathscr{F}$, containing so called *soldered frames*, and characterize the image $\mathscr{F}_{ss} = h(H^2M)$ by vanishing of the „structure function" introduced on $\mathscr{F}_s$. Now any connection on H^2M is, in fact, a connection on $\mathscr{F}_{ss}$, and can be extended to a connection Δ' on $\mathscr{F}_s$, and to Δ'' on $\mathscr{F}$. Since there is an isomorphism ι associating any $\mathscr{DL}$-connection Γ on a $\mathscr{DL}$-fibration with an invariant connection $\iota\Gamma$ on the principal fibration of $\mathscr{DL}$-frames, we can write $\Delta'' = \iota\Lambda$ for a unique $\mathscr{DL}$-connection Λ on TTM. By Theorem 1, the underlying linear connections of $\iota\Lambda$ satisfy $\iota\Lambda_1 = \iota\Lambda_2 = \iota\Lambda_3$, since $\iota\Lambda = \Delta''$ is reducible to $\mathscr{F}_s$, and the maps tangent to solderings are $TX_1 = TX_2 = \mathrm{id}_{TTM}$. Consequently, we obtain a condition

$$\Lambda_1 = \Lambda_2 = \Lambda_3$$

for the underlying connections of Λ. Finally, by Theorem 6, the reducibility of Δ'' to the principal subfibration $\mathscr{F}_{ss}$ is equivalent to the i-invariance of Λ with respect to the canonical involution i on TTM, $J^1(i^{-1}) \circ \Lambda \circ i = \Lambda$. Together, we can give the following answer: $\Gamma = TT(\Delta)$ if and only if Γ is double linear, i-invariant, and the underlying linear connections coincide.

In the paper, similar statements for TT^*-soldered $\mathscr{DL}$-spaces are deduced. Similarly, $TT^*(\Delta)$ is a $\mathscr{DL}$-connection, and Theorems 2, 10 describe the situation. Let us remark that there is no "canonical" involution on TT^*M. To characterize reducibility, we use an isomorphism between $\mathscr{F}_s$ and the principal fibration $\tilde{\mathscr{F}}_s$ of TT^*-soldered $\mathscr{DL}$-frames on TT^*M.

1. PRELIMINARIES

Let C denote a double vector space ($\mathscr{DL}$-space) over reals with the natural projection π: $C \to A \times B$ and with the centre (kernel) V, [10]. If $\dim A = n$, $\dim B = s$, $\dim V = t$ we set $\dim C = (n, s, t)$. Any two double vector spaces are $\mathscr{DL}$-isomorphic iff they have the same dimension. Hence C is isomorphic to the trivial $\mathscr{DL}$-space $K(n, s, t) = \mathbb{R}^n \times \mathbb{R}^s \times \mathbb{R}^t$ with the natural projection $K \to \mathbb{R}^n \times \mathbb{R}^s$ and centre $\mathbb{R}^t$. A $\mathscr{DL}$-frame in C is a $\mathscr{DL}$-isomorphism f: $K(n, s, t) \to C$, the set $F(C)$ of all frames in C forms a Lie group diffeomorphic with the Lie group $\mathrm{Aut}(n, s, t)$ of all $\mathscr{DL}$-automorphisms of $K(n, s, t)$, [11]. Any frame f in C determines linear isomorphisms $\tau_1 f = f_1$: $\mathbb{R}^n \to A$, $\tau_2 f = f_2$: $\mathbb{R}^s \to B$, and $\tau_3 f = f/\mathbb{R}^t \to V$, i.e. frames in A, B, and V, respectively.

Let $(\mathscr{C}, p, M)$ be a double vector fibration, [11], with the underlying vector fibrations $(\mathscr{A}, p_1, M), (\mathscr{B}, p_2, M), (\mathscr{V}, p_3, M)$. As in the case of vector fibrations (bundles), there is a principal fibration of double linear frames associated with $\mathscr{C}$. The union $\mathscr{F} = \bigcup_{x\in M} F(\mathscr{C}_x)$ of all $\mathscr{DL}$-frames (on fibres $\mathscr{C}_x$ of $\mathscr{C}$ over $x \in M$) forms a principal fibration $(\mathscr{F}, q, M)$ over M with the structure group $\mathrm{Aut}(n, s, t)$ and projection $q: \mathscr{F} \to M$, $q(f) = x$ where x is such an element of M that $f \in F(\mathscr{C}_x)$. Any frame $f \in \mathscr{C}_x$ determines elements $\tau_1 f, \tau_2 f, \tau_3 f$ which can be regarded as elements of the fibres $\mathscr{F}_{1,x}, \mathscr{F}_{2,x}, \mathscr{F}_{3,x}$ of the principal fibrations $(\mathscr{F}_1, q_1, M, \mathrm{Aut}(n))$, $(\mathscr{F}_2, q_2, M, \mathrm{Aut}(s))$, $(\mathscr{F}_3, q_3, M, \mathrm{Aut}(t))$ corresponding to the underlying vector fibrations $\mathscr{A}, \mathscr{B}, \mathscr{V}$ of $\mathscr{C}$. In this way, we obtain smooth morphisms of principal fibrations over homomorphisms of structure groups

$$\tau_1: (\mathscr{F}, q, M, \mathrm{Aut}(n, s, t)) \to (\mathscr{F}_1, q_1, M, \mathrm{Aut}(n)) \quad \text{over}$$
$$\mathrm{Aut}(n, s, t) \to \mathrm{Aut}(n)\,,$$

and similarly for $\tau_2: \mathscr{F} \to \mathscr{F}_2$ and $\tau_3: \mathscr{F} \to \mathscr{F}_3$. The morphisms τ_1, τ_2, τ_3 determine a morphism of principal fibrations

$$\tau = (\tau_1, \tau_2, \tau_3): \mathscr{F} \to (\tilde{\mathscr{F}}, q, M, \mathrm{Aut}(n) \times \mathrm{Aut}(s) \times \mathrm{Aut}(t))$$

where $\tilde{\mathscr{F}} = \mathscr{F}_1 \times_M \mathscr{F}_2 \times_M \mathscr{F}_3$ denotes the Whitney sum.

A TT-soldering (or TT^*-soldering) on the $\mathscr{DL}$-space C is a couple of linear isomorphisms

$$\chi_1: V \to A\,, \quad \chi_2: V \to B$$

(or $\chi_1: V \to A$, $\chi_2: V \to B^*$, respectively), [12].

A double linear morphism $\varphi = (\varphi_1, \varphi_2, \varphi_3, \sigma): C \to C'$ of two TT-soldered (or TT^*-soldered) $\mathscr{DL}$-spaces is called TT- (TT^*-) *soldered*, [11], [12], if the underlying linear morphisms $\varphi_1: A \to A'$, $\varphi_2: B \to B'$, $\varphi_3: V \to V'$ satisfy

$$\chi_1'\varphi_3 = \varphi_1\chi_1$$

and

$$\chi_2'\varphi_3 = \varphi_2\chi_2 \quad (\text{or } \varphi_2^*\chi_2'\varphi_3 = \chi_2, \text{ respectively})\,.$$

A frame f in the TT-soldered (or TT^*-soldered) $\mathscr{DL}$-space C is TT- (or TT^*-)soldered if

$$\chi_1\tau_3 f = \tau_1 f$$

and

$$\chi_2\tau_3 f = \tau_2 f \quad (\text{or } \chi_2\tau_3 f = (\tau_2 f)^*, \text{ respectively})\,.$$

A $\mathscr{DL}$-fibration $(\mathscr{C}, p, M)$ is TT- (or TT^*-) soldered if there exists a $\mathscr{DL}$-space C with TT- (TT^*-) soldering such that any point x of M has a neighborhood U such that the restriction $(\mathscr{C}_U, p_U, M)$ of $\mathscr{C}$ to U is isomorphic with $(U \times C, \mathrm{pr}_1, U)$ over identity. Any TT- (or TT^*-) soldering on $\mathscr{C}$ induces, via linear isomorphisms

$$\chi_{1,x}: \mathscr{V}_x \to \mathscr{A}_x\,,$$
$$\chi_{2,x}: \mathscr{V}_x \to \mathscr{B}_x \quad (\text{or } \chi_{2,x}: \mathscr{V}_x \to \mathscr{B}_x^*)\,,$$

the isomorphisms of the underlying fibrations, [11],

$$X_1: (\mathscr{V}, p_3, M) \to (\mathscr{A}, p_1, M),$$
$$X_2: (\mathscr{V}, p_3, M) \to (\mathscr{B}, p_2, M) \quad (\text{or } X_2: (\mathscr{V}, p_3, M) \to (\mathscr{B}^*, p_2^*, M)).$$

2. THE CONNECTIONS ON TT- AND TT^*-SOLDERED $\mathscr{DL}$-FIBRATIONS

Consider a $\mathscr{DL}$-fibration $\mathscr{C}$ with a TT-soldering, and assume a double linear connection $\Gamma: \mathscr{C} \to J^1\mathscr{C}$ on $\mathscr{C}$, [11], with the underlying linear connections Γ_1 on $\mathscr{A}$, Γ_2 on $\mathscr{B}$, and Γ_3 on $\mathscr{V}$. The set of all TT-soldered frames on $\mathscr{C}$ forms a principal fibration $(\mathscr{F}_s, q_s, M)$, $q_s = q/\mathscr{F}_s$, a subfibration of $(\mathscr{F}, q, M)$. The structure group of $\mathscr{F}_s$ is the group $\mathrm{Aut}_s(\mathbb{R}^m \times \mathbb{R}^m \times \mathbb{R}^m)$ of all TT-soldered $\mathscr{DL}$-automorphisms of the trivial $\mathscr{DL}$-space $\mathbb{R}^{3m}$ with the canonical TT-soldering $\chi_1 = \chi_2 = \mathrm{id}$, $m = \dim M$.

Denote by $\mathscr{T}$ the set

$$\mathscr{T} = \{(f_1, f_2, f_3) \in \tilde{\mathscr{F}};\ X_1 f_3 = f_1,\ X_2 f_3 = f_2\}.$$

$\mathscr{T}$ is a closed submanifold in $\tilde{\mathscr{F}}$, and the following is satisfied:

Lemma 1. *$f \in \mathscr{F}_s$ if and only if $\tau f = \mathscr{T}$.*

Similarly as in the linear case, there is a one-to-one map between the set of double linear connections on $\mathscr{C}$ and the set of right invariant connections on the principal fibration $\mathscr{F}$. In both linear and double linear cases, let us denote this map by ι. Now a natural question arises under what conditions the invariant connection $\iota\Gamma$ on $\mathscr{F}$ corresponding to Γ on $\mathscr{C}$ can be reduced to $\mathscr{F}_s$.

Theorem 1. *The invariant connection $\iota\Gamma$ is reducible to the principal subfibration $(\mathscr{F}_s, q_s, M)$ if and only if the horizontal subspaces H_1, H_2, and H_3 of connections $\iota\Gamma_1$, $\iota\Gamma_2$, and $\iota\Gamma_3$ satisfy*

(1) $$(TX_1) H_3 = H_1, \quad (TX_2) H_3 = H_2.$$

Proof. (a) Suppose that $\iota\Gamma$ is reducible to $\mathscr{F}_s$. Let $f_3 \in \mathscr{F}_3$, and let $v_3 \in (H_3)_{f_3}$ be any element of the horizontal space of $\iota\Gamma_3$ at the point f_3. Define $f_1 = X_1 f_3$, $f_2 = X_2 f_3$, and choose $f \in \mathscr{F}$ so that $f = (f_1, f_2, f_3)$. Then $f \in \mathscr{F}_s$. In the horizontal space H_f with respect to Γ, assume any vector $v \in H_f$ with the property $(T\tau_3)\, v = v_3$. Choose an $\iota\Gamma$-horizontal curve $\gamma: (-\varepsilon, \varepsilon) \to \mathscr{F}$ such that

$$\gamma(0) = f, \quad \frac{d\gamma(0)}{dt} = v.$$

Since $\gamma(0) = f \in \mathscr{F}_s$ and $\iota\Gamma$ is reducible to $\mathscr{F}_s$ we have $\gamma(t) \in \mathscr{F}_s$ for all $t \in (-\varepsilon, \varepsilon)$. By Lemma 1, $\tau\gamma(t) \in \mathscr{T}$ for $t \in (-\varepsilon, \varepsilon)$, which means

$$X_1 \tau_3\, \gamma(t) = \tau_1\, \gamma(t), \quad X_2 \tau_3\, \gamma(t) = \tau_2\, \gamma(t)$$

for $t \in (-\varepsilon, \varepsilon)$. This implies

$$(TX_1)\, v_3 = v_1, \quad (TX_2)\, v_3 = v_2$$

where

$$v_1 = \frac{\mathrm{d}(\tau_1\,\gamma(0))}{\mathrm{d}t} \in H_1\,, \quad v_2 = \frac{\mathrm{d}(\tau_2\,\gamma(0))}{\mathrm{d}t} \in H_2\,.$$

This proves (1).

(b) Conversely, let (1) be fulfilled. Let $f \in \mathscr{F}_s$, and let $\gamma: (-\varepsilon, \varepsilon) \to \mathscr{F}$ be a horizontal curve of $\iota\Gamma$. Denote $f_i = \tau_i f$, $i = 1, 2, 3$. The curve $\tau_1\gamma$ is a horizontal lift of the curve $q\gamma$ with respect to $\iota\Gamma_1$ through the point $\tau_1\,\gamma(0) = f_1$. Similarly, $\tau_3 f$ is a horizontal lift of $q\gamma$ with respect to $\iota\Gamma_3$ through the point $\tau_3\,\gamma(0) = f_3$. But $X_1 f_3 = f_1$, and $(TX_1)\,H_3 = H_1$. That is why $X_1\tau_3$ is also a horizontal lift of $q\gamma$ through f_1 with respect to $\iota\Gamma_1$. We obtain $X_1\tau_3\,\gamma(t) = \tau_1\,\gamma(t)$ for $t \in (-\varepsilon, \varepsilon)$. In a similar way, $X_2\tau_3\,\gamma(t) = \tau_2\,\gamma(t)$ for $t \in (-\varepsilon, \varepsilon)$. Thus $\tau\,\gamma(t) \in \mathscr{T}$ for all $t \in (-\varepsilon, \varepsilon)$, and $\gamma(t) \in \mathscr{F}_s$ for $t \in (-\varepsilon, \varepsilon)$. Therefore $\iota\Gamma$ is reducible to $\mathscr{F}_s$.

Now consider a $\mathscr{DL}$-fibration $\mathscr{C}$ on M with TT^*-soldering given by

$$X_1: (\mathscr{V}, p_3, M) \to (\mathscr{A}, p_1, M)\,, \quad X_2: (\mathscr{V}, p_3, M) \to (\mathscr{B}^*, p_2^*, M)\,.$$

All TT^*-soldered frames in $\mathscr{F}$ corresponding to $\mathscr{C}$ constitute again a principal fibration $\mathscr{F}_s$, a subfibration of $\mathscr{F}$. Its structure group is the group $\mathrm{Aut}_s(\mathbb{R}^{m*} \times \mathbb{R}^m \times \mathbb{R}^{m*})$ of all TT^*-soldered $\mathscr{DL}$-automorphisms of the space $\mathbb{R}^{m*} \times \mathbb{R}^m \times \mathbb{R}^{m*}$ with its canonical TT^*-soldering $\chi_1 = \mathrm{id}$, $\chi_2 = \mathrm{id}$.

The map $\tau: \mathscr{F} \to \tilde{\mathscr{F}} = \mathscr{F}_1 \times \mathscr{F}_2 \times \mathscr{F}_3$ is again a surjective submersion. Let us define a closed submanifold $\mathscr{T}^*$ in $\tilde{\mathscr{F}}$ by

$$\mathscr{T}^* = \{(f_1, f_2, f_3) \in \tilde{\mathscr{F}}\,, \quad X_1 f_3 = f_1\,, \quad X_2 f_3 = f_2^*\}\,.$$

The frame f in $\mathscr{F}$ is soldered iff τf belongs to $\mathscr{T}^*$. Let Γ, Γ_i, $\mathscr{F}_i$, $i = 1, 2, 3$, and $\iota\Gamma$ be as above. Denote by $\mathscr{F}_2^*$ the principal fibration corresponding to the vector fibration $\mathscr{B}^*$. A map associating any frame with its dual coframe gives an isomorphism $\mathscr{F}_2 \to \mathscr{F}_2^*$ of principal fibrations over M. This isomorphism maps the invariant connection $\iota\Gamma_2$ on $\mathscr{F}_2$ onto an invariant connection $\iota\Gamma_2^*$ on $\mathscr{F}_2^*$.

Theorem 2. *The right invariant connection $\iota\Gamma$ on the TT^*-soldered $\mathscr{DL}$-fibration $\mathscr{C}$ is reducible to the subfibration $\mathscr{F}_s$ of TT^*-soldered frames if and only if the horizontal spaces H_1, H_2^*, H_3 of the connections $\iota\Gamma_1$, $\iota\Gamma_2^*$, $\iota\Gamma_3$ satisfy*

$$(TX_1)\,H_3 = H_1\,, \quad (TX_2)\,H_3 = H_2\,.$$

The proof is similar as in the case of Theorem 1.

3. THE STRUCTURE FUNCTION AND REDUCTIONS

Given an m-dimensional manifold M, let $H^2M = \mathrm{inv}\,J_0^2(\mathbb{R}^m, M)$ denote the principal fibration of second order frames on M with the structure group L_m^2, the group of all invertible 2-jets on $\mathbb{R}^m$ with source and target 0, $L_m^2 = \mathrm{inv}\,J_0^2(\mathbb{R}^m, \mathbb{R}^m)_0$. This structure group can be regarded as a semidirect product of the linear group

$L^1_m = GL(m, \mathbb{R})$ and the abelian group of all symmetric bilinear maps $\mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}^m$, $L^2_m = L^1_m \times \mathrm{Hom}_{\mathrm{sym}}(\mathbb{R}^m \times \mathbb{R}^m, \mathbb{R}^m)$, [10]. So we can write its elements as couples (φ, σ) with $\varphi \in L^1_m$ and σ a symmetric bilinear homomorphism on $\mathbb{R}^m$. Furthermore, L^2_m is isomorphic with the group $G_{ss} = \mathrm{Aut}_{ss}(TT_0\mathbb{R}^m)$ of all strongly soldered automorphisms of the $\mathscr{DL}$-space $TT_0\mathbb{R}^m$, [12], via the map

$$\varkappa\colon L^2_m \to \mathrm{Aut}_{ss}(TT_0\mathbb{R}^m)\,, \quad \varkappa(\varphi, \sigma) = (\varphi, \varphi, \varphi, \sigma)\,.$$

We shall identify the both groups.

Consider now the principal fibration $\mathscr{F}$ or $\mathscr{F}_s$, of frames or TT-soldered frames, respectively, on TTM. We shall construct a morphism

$$h\colon H^2M \to \mathscr{F}$$

of principal fibrations as follows. An element $\varrho \in H^2_xM$ is of the form $\varrho = j^2_0\alpha$ where $\alpha\colon U \subset \mathbb{R}^m \to M$ is a local diffeomorphism

$$TT_0\alpha\colon TT_0\mathbb{R}^m \to TT_xM\,,$$

a restriction of the map $TT\alpha$ to the fibre of $TT\mathbb{R}^m$ through the origin $0 \in \mathbb{R}^m$. This definition is independent of the choice of a diffeomorphism α with the property $\varrho = j^2_0\alpha$. Since $TT_0\alpha$ respects the natural TT-solderings on $TT_0\mathbb{R}^m$ and TT_xM we have $h(\varrho) \in \mathscr{F}_s$ for any $\varrho \in H^2M$. Hence we obtain a monomorphism of principal fibrations

$$h\colon H^2M \to \mathscr{F}_s\,.$$

For any $g \in L^2_m$ and $\varrho \in H^2M$, $h(\varrho g) = h(\varrho)\,.\,\varkappa(g)$.

Now we introduce a structure function Θ on $\mathscr{F}_s$ which enables us to characterize the frames belonging to $h(H^2M)$. For simplicity we use the notation G_s for the group $\mathrm{Aut}_s(TT_0\mathbb{R}^m)$ of all TT-soldered $\mathscr{DL}$-automorphisms of $TT_0\mathbb{R}^m$. Any element $(\varphi, \varphi, \varphi, \sigma) \in G_s$ is uniquely expressible in the form $(\varphi, \varphi, \varphi, \sigma) = (\varphi, \varphi, \varphi, b)\,.\,(1, 1, 1, a)$ where $b \in \mathrm{Hom}_{\mathrm{sym}}(\mathbb{R}^m \times \mathbb{R}^m, \mathbb{R}^m)$, and a is an element of the vector space $\mathrm{Hom}_{\mathrm{ant}}(\mathbb{R}^m \times \mathbb{R}^m, \mathbb{R}^m)$ of all antisymmetric bilinear maps on $\mathbb{R}^m$. Any frame $f \in \mathscr{F}_{s,x}$ is a soldered $\mathscr{DL}$-isomorphism

$$f\colon TT_0\mathbb{R}^m \to TT_xM\,. \tag{2}$$

Let $\alpha, \alpha'\colon U \subset \mathbb{R}^m \to M$ be local diffeomorphisms with $\alpha(0) = \alpha'(0) = x$. The element $(TT_0\alpha)^{-1} f \in G_s$ has a unique decomposition

$$(TT_0\alpha)^{-1} f = g\,.\,(1, 1, 1, a)\,, \quad g \in G_{ss}\,, \quad a \in \mathrm{Hom}_{\mathrm{ant}}(\mathbb{R}^m \times \mathbb{R}^m, \mathbb{R}^m)\,.$$

Similarly for $(TT_0\alpha')^{-1} f = g'\,.\,(1, 1, 1, a')$. A simple evaluation shows that $g' = (TT_0(\alpha'^{-1} \circ \alpha))\, g$, and $a = a'$. So we can define a structure function

$\Theta\colon \mathscr{F}_s \to \mathrm{Hom}_{\mathrm{ant}}(\mathbb{R}^m \times \mathbb{R}^m, \mathbb{R}^m)$ on the principal fibration $\mathscr{F}_s$ of TT-soldered frames on TTM by

$$\Theta(f) = a.$$

Theorem 3. *The structure function Θ has the following properties*:

(3) Θ *is differentiable map.*

(4) *If $\tilde{g} \in G_s$ with the decomposition $\tilde{g} = \tilde{\varphi}, \tilde{\varphi}, \tilde{\varphi}, \tilde{b}) . (1, 1, 1, \tilde{a})$ then $\Theta(f\tilde{g}) = \tilde{\varphi}^{-1}\, \Theta(f)\,(\tilde{\varphi}, \tilde{\varphi}) + \tilde{a}$.*

Proof. The verification of (3) is standard, (4) follows by a direct evaluation.

The frames from $\mathscr{F}_{ss} = h(H^2M)$ can be now characterized as follows:

Theorem 4. *The frame f belongs to $\mathscr{F}_{ss}$ if and only if $\Theta(f) = 0$.*

Now consider the second tangent bundle on M as a double vector fibration $\mathscr{C} = TTM$ with $\mathscr{A} = \mathscr{B} = \mathscr{V} = TM$, and with projections $p: TTM \to M$, $\pi_1: TTM \to TM$, $\pi_2: TTM \to TM$, [11]. Let $i: TTM \to TTM$ denote the canonical involution, and denote by q^2 the projection $q^2: \mathscr{F}_{ss} \to M$.

Lemma 2. *Let z, z' be elements of TTM, let λ be a real number. Then the following is satisfied*:

(4) $\quad$ *If* $\pi_1 z = \pi_1 z'$ *then* $i(z +_1 z') = (iz) +_2 (iz')$.

(5) $\quad$ *If* $\pi_2 z = \pi_2 z'$ *then* $i(z +_2 z') = (iz) +_1 (iz')$.

(6) $\quad i(\lambda \cdot_1 z) = \lambda \cdot_2 (iz)$,

(7) $\quad i(\lambda \cdot_2 z) = \lambda \cdot_1 (iz)$.

Proof. We shall prove (4). Choose a frame $f \in \mathscr{F}_{ss} = h(H^2M)$ with $q^2(f) = p(z) = p(z')$. Since f is of the form (2) and $TT_0\mathbb{R}^m$ is isomorphic to $\mathbb{R}^{3m}$, there are uniquely determined elements (a, b, v), (a', b', v') from $\mathbb{R}^{3m}$ such that

$$z = f(a, b, v)\,, \quad z' = f(a', b', v')\,.$$

Let i be an element of $Z_{ss}(\mathbb{R}^m \times \mathbb{R}^m \times \mathbb{R}^m)$ determining the canonical involution i on TTM. Here Z_{ss} denotes the set of all differentiable maps of the given $\mathscr{D}\mathscr{L}$-space into itself commuting with all its strongly TT-soldered $\mathscr{D}\mathscr{L}$-automorphisms, [12]. We have

$$i(z +_1 z') = if(a, b + b', v + v') = f(\bar{\imath}(a, b + b', v + v')) =$$
$$= f(b + b', a, v + v') = f(\bar{\imath}(a, b, v)) +_2 f(\bar{\imath}(a, b', v')) =$$
$$= (iz) +_2 (iz')\,.$$

Similarly in the other cases.

Assume a frame $f \in \mathscr{F}$, i.e. a $\mathscr{D}\mathscr{L}$-isomorphism $f: \mathbb{R}^{3m} \to TT_xM$. It is easily checked that $if\bar{\imath}: \mathbb{R}^{3m} \to TT_xM$ is again a $\mathscr{D}\mathscr{L}$-isomorphism. The map

$$f \to If = if\bar{\imath}\,, \quad I: \mathscr{F} \to \mathscr{F}$$

will be called the *canonical involution* on $\mathscr{F}$.

Theorem 5. *The canonical involution I on $\mathscr{F}$ satisfies*

(8) $\qquad I^2 = \mathrm{id}\,,$

(9) $\qquad I(\mathscr{F}_s) = \mathscr{F}_s\,,$

(10) $\qquad If = f \quad \text{for} \quad f \in \mathscr{F}_{ss}\,.$

(11) $\qquad$ *If* $f \in \mathscr{F}_s$ *then* $\Theta(If) = -\Theta f\,.$

Proof. (8) is clear. Let $f \in \mathscr{F}_s$. Since $\mathscr{F}_s$ corresponding to TTM has the group $\mathrm{Aut}_s(\mathbb{R}^{3m}) = \mathrm{Aut}_s(TT_0\mathbb{R}^m) = G_s$ as its structure group we can choose $\tilde{f} \in \mathscr{F}_{ss}$ and $g = (\varphi, \varphi, \varphi, \sigma) \in G_s$ such that $f = \tilde{f}g$. Evaluation of If on an arbitrary $(a, b, v) \in \mathbb{R}^{3m}$ shows that

$$If = \tilde{f}(\varphi, \varphi, \varphi, \bar{\sigma})$$

where $\bar{\sigma}\colon \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}^m$ is a bilinear map given by $\bar{\sigma}(x, y) = \sigma(y, x)$. Hence $(\varphi, \varphi, \varphi, \bar{\sigma}) \in G_s$, and $If \in \mathscr{F}_s$ which proves (9). (10) follows immediately by the equality $if\tilde{\imath}(a, b, v) = f\tilde{\imath}^2(a, b, v)$ for $f \in \mathscr{F}_{ss}$. Further, given $f \in \mathscr{F}_s$ let $\tilde{f} \in \mathscr{F}_{ss}$ be a unique form with the property $f = \tilde{f}(1, 1, 1, a)$ where $a \in \mathrm{Hom}_{\mathrm{ant}}(\mathbb{R}^m \times \mathbb{R}^m, \mathbb{R}^m)$. Evaluation shows that $If = \tilde{f}(1, 1, 1, -a)$. By (4) and Theorem 4, $\Theta(f) = \Theta(\tilde{f}) + a = a$, $\Theta(If) = \Theta(\tilde{f}) - a = -a$.

Remark. The frames from $\mathscr{F}$ satisfying (10) are also „soldered" in some sense, but it can be verified that $\mathscr{F}_{ss} \neq \{f \in \mathscr{F}, If = f\}$.

Theorem 6. *Let Γ be a $\mathscr{DL}$-connection on TTM such that the invariant connection $\iota\Gamma$ on $\mathscr{F}$ is reducible to $\mathscr{F}_s$. Then $\iota\Gamma$ is reducible to $\mathscr{F}_{ss}$ if and only if Γ is invariant with respect to the canonical involution i on TTM.*

Proof. (a) First suppose that $\iota\Gamma$ is reducible to $\mathscr{F}_s = h(H^2M)$. Let $z \in TT_xM$. Choose a vector $Z \in H_z$ from the horizontal space with respect to Γ and a frame $f \in h(H_x^2M)$. Clearly there exists a unique element $c \in \mathbb{R}^{3m}$ such that $z = fc$, a value of the map f on c which is an element of the associated fibration TTM determined by an element f of the principal fibration $\mathscr{F}_{ss} \subset \mathscr{F}_s$, and an element c of the standard fibre $\mathbb{R}^{3m}$. Choose a curve $\delta\colon (-\varepsilon, \varepsilon) \to M$ such that

$$\delta(0) = x\,, \quad \left(\frac{\mathrm{d}}{\mathrm{d}t}\right)_{t=0} \delta(t) = (Tp)\,Z$$

where $p\colon TTM \to M$ is a projection, and consider its horizontal lift with respect to $\iota\Gamma$, $\gamma\colon (-\varepsilon, \varepsilon) \to \mathscr{F}$ with $\gamma(0) = f$. Then

$$\gamma(0)\,c = z\,, \quad \left(\frac{\mathrm{d}}{\mathrm{d}t}\right)_{t=0} (\gamma(t)\,c) = Z\,,$$

and $\gamma(t) \in \mathscr{F}_{ss}$ for $t \in (-\varepsilon, \varepsilon)$ because of the reducibility of $\iota\Gamma$ to $\mathscr{F}_{ss}$. Using again an element $\tilde{\imath} \in Z_{ss}(\mathbb{R}^{3m})$ and a horizontal curve (with respect to Γ) $\gamma(t)\,(\tilde{\imath}c)$ we obtain $(Ti)\,Z = (\mathrm{d}/\mathrm{d}t)_{t=0}\,(\gamma(t)\,(\tilde{\imath}c)) \in H_{iz}$, which proves the i-invariance.

(b) Now suppose the i-invariance of Γ on TTM. Let $f \in \mathscr{F}_{ss}$ and $Z \in H_f$ where H_f

is horizontal to $\iota\Gamma$. Assume a horizontal curve $\gamma: (-\varepsilon, \varepsilon) \to \mathscr{F}$ with $\gamma(0) = f$, $(\mathrm{d}/\mathrm{d}t)_{t=0}\, \gamma(t) = Z$, and choose $\tilde{\gamma}: (-\varepsilon, \varepsilon) \to \mathscr{F}_{ss}$ such that $q^2\tilde{\gamma} = q_s\gamma$, $\tilde{\gamma}(0) = f$. There is a uniquely determined curve $g: (-\varepsilon, \varepsilon) \to G_s$ such that $\gamma(t) = \tilde{\gamma}(t)\, g(t)$ for $t \in (-\varepsilon, \varepsilon)$, $g(0) = \mathrm{id}_{\mathbb{R}^{3m}}$. For any $c \in \mathbb{R}^{3m}$, the curve $\gamma c: (-\varepsilon, \varepsilon) \to TTM$ is horizontal to Γ. Since Γ is i-invariant, $i(\gamma c)$ is a horizontal curve, and we have

$$(12) \qquad i(\gamma(t)\, c) = \tilde{\gamma}(t)\,(\tilde{\imath}(g(t)\, c))\,.$$

Further, $\gamma(\tilde{\imath}c): (-\varepsilon, \varepsilon) \to TTM$ is another horizontal curve satisfying $\gamma(t)\,(\tilde{\imath}c) = \tilde{\gamma}(t)\,(g(t)\,(\tilde{\imath}c))$. We have

$$p(i(\gamma c)) = p(\gamma c) = q^2\gamma = p(\gamma(\tilde{\imath}c))\,,$$

$$i(\gamma(0)\, c) = \gamma(0)\,(\tilde{\imath}c)\,.$$

By the unicity of a horizontal lift, $i(\gamma c) = \gamma(\tilde{\imath}c)$. By (12), (13) we obtain

$$\tilde{\imath}(g(t)\, c) = g(t)\,(\tilde{\imath}c) \quad \text{for} \quad t \in (-\varepsilon, \varepsilon)\,.$$

Rewriting this equality for components of $g(t) = (\varphi_t, \varphi_t, \varphi_t, \sigma_t)$ and $c = (v_1, v_2, v_3)$ and comparing them yields $\sigma_t(v_1, v_2) = \sigma_t(v_2, v_1)$. Since c was arbitrary we obtain $\sigma_t \in \mathrm{Hom}_{\mathrm{sym}}(\mathbb{R}^m \times \mathbb{R}^m, \mathbb{R}^m)$ for $t \in (-\varepsilon, \varepsilon)$. Consequently, $g(t) \in G_{ss}$ for $t \in (-\varepsilon, \varepsilon)$. Hence $\gamma(t) = \tilde{\gamma}(t)\, g(t) \in h(H^2M)$ and

$$Z = \left(\frac{\mathrm{d}}{\mathrm{d}t}\right)_{t=0} \gamma(t) \in T_f(\mathscr{F}_{ss})\,,$$

which proves the reducibility of $\iota\Gamma$ to $\mathscr{F}_{ss}$.

In the case of the functor TT^*, similar statements can be proved. Let $\tilde{\mathscr{F}}$ or $\tilde{\mathscr{F}}_s$ denote the fibration of frames or of TT^*-soldered frames, respectively, on TT^*M. We introduce a morphism

$$\tilde{h}: H^2M \to \tilde{\mathscr{F}}$$

similarly as above. For $\varrho = j_0^2\alpha \in H_x^2M$, $\alpha(0) = x$, define $\tilde{h}(\varrho) = TT_0^*\alpha^{-1}: TT_0^*\mathbb{R}^m \to TT_x^*M$. This definition depends only on the 2-jet of the local diffeomorphism $\alpha: U \subset \mathbb{R}^m \to M$, and since $TT_0^*\alpha$ respects the natural soldering, $\tilde{h}$ takes its values in $\tilde{\mathscr{F}}_s$. Again, $\tilde{h}: H^2M \to \tilde{\mathscr{F}}_s$ is a monomorphism of principal fibrations, $\tilde{h}(\varrho g) = \tilde{h}(\varrho)\, \tilde{\varkappa}(g)$ for $\varrho \in H^2M$, $g \in L_m^2$. Here $\tilde{\varkappa}: L_m^2 \to \mathrm{Aut}_s(TT_0^*\mathbb{R}^m)$ is an isomorphism identifying this both groups given by

$$\tilde{\varkappa}(\varphi, \sigma) = (\varphi^{*-1}, \varphi, \varphi^{*-1}, \psi)\,,$$

where a bilinear map ψ is defined by the equality

$$\langle \varphi^{-1}\sigma(v_1, \varphi^{-1}(v_2), a\rangle = -\langle v_2, \psi(a, v_1)\rangle\,.$$

Let $\tilde{G}_s$ (or $\tilde{G}_{ss}$) denote the group of all TT^*-soldered (or strongly TT^*-soldered, respectively) $\mathscr{D}\mathscr{L}$-automorphisms of $TT_0^*\mathbb{R}^m$, [13].

Lemma 3. *Any element* $(\varphi^{*-1}, \varphi, \varphi^{*-1}, \psi) \in \tilde{G}_s$ *has a unique decomposition*

$$(\varphi^{*-1}, \varphi, \varphi^{*-1}, \psi) = (\varphi^{*-1}, \varphi, \varphi^{*-1}, \varepsilon)\,(\mathrm{id}^{*-1}, \mathrm{id}, \mathrm{id}^{*-1}, \beta)$$

where $\varepsilon\colon \mathbb{R}^{m*} \times \mathbb{R}^m \to \mathbb{R}^{m*}$ *is a* φ*-symmetric bilinear map, and* $\beta\colon \mathbb{R}^{m*} \times \mathbb{R}^m \to$ $\to \mathbb{R}^{m*}$ *is an id-antisymmetric bilinear map.*

Remark. In our case, ε is φ-symmetric (φ-antisymmetric) if

$$\langle v, \varepsilon(a, \varphi^{-1}(w))\rangle = \langle w, \varepsilon(a, \varphi^{-1}(v))\rangle \text{ (or } \langle v, \varepsilon(a, \varphi^{-1}(w)) = $$
$$= -\langle w, \varepsilon(a, \varphi^{-1}(v))\rangle) \quad \textit{for} \quad v, w \in \mathbb{R}^m, \quad a \in \mathbb{R}^{m*}.$$

Proof. The unicity of the decomposition is based on the fact that both the φ-symmetric and φ-antisymmetric parts of the bilinear map $\psi = \varepsilon + \varphi^{*-1}\beta$ are determined uniquely. Let us prove the existence. Define a bilinear $\sigma\colon \mathbb{R}^m \times \mathbb{R}^m \to$ $\to \mathbb{R}^m$ by

$$\langle \varphi^{-1}\sigma(v_1, \varphi^{-1}(v_2)), a\rangle = -\langle v_2, \psi(a, v_1)\rangle,$$

and denote by σ', σ'' its symmetric or antisymmetric parts, tespectively. Further, let bilinear maps $\varepsilon, \tilde{\beta}\colon \mathbb{R}^{m*} \times \mathbb{R}^m \to \mathbb{R}^{m*}$ be introduced by

$$\langle \varphi^{-1}\sigma'(v_1, \varphi^{-1}(v_2), a\rangle = -\langle v_2, \varepsilon(a, v_1)\rangle,$$
$$\langle \varphi^{-1}\sigma''(v_1, \varphi^{-1}(v_2)), a\rangle = -\langle v_2, \tilde{\beta}(a, v_1)\rangle.$$

It can be checked that ε is φ-symmetric, $\tilde{\beta}$ is φ-antisymmetric, $\beta = \varphi^*\tilde{\beta}$ is id-antisymmetric, and

$$(\varphi^{*-1}, \varphi, \varphi^{*-1}, \varepsilon)(\mathrm{id}^{*-1}, \mathrm{id}, \mathrm{id}^{*-1}, \beta) = (\varphi^{*-1}, \varphi, \varphi^{*-1}, \varepsilon + \tilde{\beta}).$$

Further,

$$-\langle v_2, (\varepsilon + \beta)(a, v_1)\rangle =$$
$$= \langle \varphi^{-1}\sigma'(v_1, \varphi^{-1}(v_2)), a\rangle + \langle \varphi^{-1}\sigma''(v_1, \varphi^{-1}(v_2), a\rangle =$$
$$= -\langle v_2, \psi(a, v_1)\rangle.$$

Therefore $\varepsilon + \beta = \psi$, which proves the existence.

Now we shall describe frames from $\tilde{h}(H^2M) = \tilde{\mathscr{F}}_{ss}$ by means of a structure function

$$\tilde{\Theta}\colon \tilde{\mathscr{F}}_s \to \mathrm{Hom}_{\mathrm{ant}}(\mathbb{R}^{m*} \times \mathbb{R}^m, \mathbb{R}^{m*}).$$

Let $f \in \tilde{\mathscr{F}}_{s,x}$, and let $\alpha \in U \subset \mathbb{R}^m \to M$ be a local diffeomorphism with $\alpha(0) = x$. Then $(TT_0^*\alpha^{-1})f \in \tilde{G}_s$, and there is a unique decomposition $(TT_0^*\alpha^{-1})f =$ $= g(\mathrm{id}^{*-1}, \mathrm{id}, \mathrm{id}^{*-1}, \beta)$ where $g \in \tilde{G}_{ss}$, and an id-antisymmetric $\beta \in \mathrm{Hom}_{\mathrm{ant}}(\mathbb{R}^{m*} \times$ $\times \mathbb{R}^m, \mathbb{R}^{m*})$ is independent of the choice of α with the above property. Hence we can put

$$\tilde{\Theta}(f) = \beta.$$

Theorem 7. *The structure function* $\tilde{\Theta}$ *has the properties*

(14) $\tilde{\Theta}$ *is differentiable.*

(15) *If* $\tilde{g} \in \tilde{G}_s$ *with the decomposition* $\tilde{g} = (\tilde{\varphi}^{*-1}, \tilde{\varphi}, \tilde{\varphi}^{*-1}, \tilde{b})\,.$ $.\,(\mathrm{id}^{*-1}, \mathrm{id}, \mathrm{id}^{*-1}, \tilde{\beta})$ *then*

$$\tilde{\Theta}(f\tilde{g}) = \tilde{\varphi}^*\,\tilde{\Theta}(f)\,(\tilde{\varphi}^{*-1}, \tilde{\varphi}) + \tilde{\beta}.$$

The proof uses similar arguments as the proof of Theorem 3.

Theorem 8. *$f \in \tilde{h}(H^2M)$ if and only if $\tilde{\Theta}(f) = 0$.*

The following lemma can be proved.

Lemma 4. *A map μ: $G_s \to \tilde{G}_s$ given by the formula*

$$\mu(\varphi, \varphi, \varphi, \sigma) = (\varphi^{*-1}, \varphi, \varphi^{*-1}, \varepsilon)$$

where ε: $\mathbb{R}^{m*} \times \mathbb{R}^m \to \mathbb{R}^{m*}$ is a bilinear map given by

$$\langle \varphi^{-1}\sigma(v_1, \varphi^{-1}(v_2)), a \rangle = -\langle v_2, \varepsilon(a, v_1) \rangle \quad \textit{for} \quad v_1, v_2 \in \mathbb{R}^m, \quad a \in \mathbb{R}^{m*}$$

is a group isomorphism. Moreover, μ maps the subgroup $G_{ss} \subset G_s$ isomorphically onto $\tilde{G}_{ss} \subset \tilde{G}_s$.

A map $\tilde{h}h^{-1}$: $\mathscr{F}_{ss} \to \tilde{\mathscr{F}}_{ss}$ is an isomorphism of principal fibrations over a structure group morphism $\mu \mid G_{ss}$: $G_{ss} \to \tilde{G}_{ss}$. We will construct an extension of $\tilde{h}h^{-1}$ as follows. Let $f \in \mathscr{F}_{s,x}$, and choose $f_0 \in \mathscr{F}_{ss,x}$. Then there is a single element $\hat{g} \in G_s$ such that $f = f_0\hat{g}$. Define

$$\varkappa(f) = (\tilde{h}h^{-1}(f_0))\,\mu(\hat{g})\,.$$

It can be verified that this definition is independent of the choice of f_0. For $f \in \mathscr{F}_{ss}$, we have $\varkappa(f) = \tilde{h}h^{-1}(f)$. So we have proved

Theorem 9. *The map $\varkappa$: $\mathscr{F}_s \to \tilde{\mathscr{F}}_s$ is an isomorphism of principal fibrations over the structure group isomorphism μ: $G_s \to \tilde{G}_s$, and $\varkappa$ maps a principal subfibration $\mathscr{F}_{ss}$ onto $\tilde{\mathscr{F}}_{ss}$.*

Invariant connections Γ on $\mathscr{F}_s$ and $\tilde{\Gamma}$ on $\tilde{\mathscr{F}}_s$ will be called *conjugated* if $(T\varkappa)\,\Gamma = \tilde{\Gamma}$. The existence of a conjugated connection to a given connection on $\mathscr{F}_s$ or $\tilde{\mathscr{F}}_s$, respectively, is clear.

Theorem 10. *Let $\tilde{\Gamma}$ be an invariant connection on $\tilde{\mathscr{F}}_s$. Then $\tilde{\Gamma}$ is reducible onto $\tilde{h}(H^2M)$ if and only if the corresponding conjugated connection Γ is reducible to $h(H^2M)$.*

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Author's address: 771 46 Svobody 26, Olomouc, Czechoslovakia (Katedra algebry a geometrie PU).