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VARIETIES WITH MODULAR AND DISTRIBUTIVE LATTICES OF SYMMETRIC OR REFLEXIVE RELATIONS

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A binary relation R on an algebra (A, F) is called *compatible* if R satisfies the *Substitution Property* with respect to F, i.e. if for each n-ary $f \in F$, $\langle a_i, b_i \rangle \in R$ for $i = 1, \ldots, n$ imply $\langle f(a_1, \ldots, a_n), f(b_1, \ldots, b_n) \rangle \in R$. It was shown in [1] that for any subcollection C of the properties: *reflexivity*, *symmetry*, *transitivity*, the set of all compatible relations on (A, F) satisfying C forms an algebraic lattice (with respect to set inclusion). The modularity or distributivity of such lattices were characterized by some authors, especially for varieties of algebras. For *congruences* (i.e. reflexive, symmetric and transitive compatible relations), it was done by A. Day [5] and B. Jónsson [6]. For *tolerances* (i.e. reflexive and symmetric compatible relations), it was solved in [2]. For *quasiorders* (i.e. reflexive and transitive compatible relations), the distributivity was characterized in [4]. For *weak congruences* (symmetric and transitive compatible relations), the answer has been given recently by G. Vojvodić and B. Šešelja in [8]. For general compatible relations, the solution is contained in [3].

The aim of this paper is to characterize varieties whose members have distributive or modular lattices of symmetric or reflexive compatible relations.

N o t a t i o n . An algebra and its support will be denoted by the same letter. Let A be an algebra. Denote by $\mathrm{Sym}(A)$ the lattice of all symmetric compatible relations on A. Clearly, the empty relation is the least and A^2 is the greatest element of $\mathrm{Sym}(A)$. The operation $\wedge$ (meet) in $\mathrm{Sym}(A)$ coincides with set intersection. Denote by $\vee$ the join in $\mathrm{Sym}(A)$. For $a, b \in A$ denote by $S(a, b)$ the least element of $\mathrm{Sym}(A)$ containing the pair $\langle a, b \rangle$. If $x_1, \ldots, x_n$ are elements of A, denote by $\mathbf{x}$ the sequence $x_1, \ldots, x_n$.

Lemma 1. *Let a, b, c, d, x, y, a_i, b_i $(i = 1, \ldots, n)$ be elements of an algebra A and let $S_j \in \mathrm{Sym}(A)$ for $j \in J$. Then*

(a) $\langle c,d\rangle \in S(a,b)$ *if and only if* $c=t(a,b), d=t(b,a)$ *for some binary term* $t(x,y)$ *over* A;

(b) $\langle x,y\rangle \in \vee\{S_j\,;\, j\in J\}$ *if and only if there exist an* m*-ary term* p *and elements* x_k, y_k *of* A ($k=1,\ldots,m$) *such that* $\langle x_k,y_k\rangle \in S_{j_k}$ *for some* $j_k\in J$ *and* $x=p(x_1,\ldots,x_m)$, $y=p(y_1,\ldots,y_m)$;

(c) $\langle x,y\rangle \in \bigvee\{S(a_i,b_i)\,;\, i=1,\ldots,n\}$ *if and only if there exists a* $2n$*-ary term* q *with* $x=q(a_1,\ldots,a_n,b_1,\ldots,b_n)$, $y=q(b_1,\ldots,b_n,a_1,\ldots,a_n)$.

The proof is elementary, for details see e.g. [1].

Theorem 1. *For a variety* V, *the following conditions are equivalent:*

(1) $\mathrm{Sym}(A)$ *is distributive for each* $A\in V$;

(2) *For every* n*-ary term* p *there exist an* m*-ary term* q *and binary terms* r_j, s_j ($j=1,\ldots,m$) *such that* $p(\mathbf{x})=q(r_1(p(\mathbf{x}),p(\mathbf{y})),\ldots,r_m(p(\mathbf{x}),p(\mathbf{y})))$, *and for each* $j\in\{1,\ldots,m\}$ *there exists* $i\in\{1,\ldots,n\}$ *with* $r_j(p(\mathbf{x}),p(\mathbf{y}))=s_j(x_i,y_i)$.

Proof. (1) $\Rightarrow$ (2): Let p be an n-ary term and let $A=F_v(x_1,\ldots,x_n,y_1,\ldots,y_n)$ be a free algebra of V with $2n$ free generators $x_1,\ldots,x_n,y_1,\ldots,y_n$. Denote $x=p(\mathbf{x})$, $y=p(\mathbf{y})$. By Lemma 1 we have

$$\langle x,y\rangle \in S(x,y)\wedge \bigvee\{S(x_i,y_i)\,;\, i=1,\ldots,n\}.$$

Distributivity of $\mathrm{Sym}(A)$ implies

$$\langle x,y\rangle \in \bigvee\{S(x,y)\wedge S(x_i,y_i)\,;\, i=1,\ldots,n\},$$

thus, by Lemma 1, there exist an m-ary term q and elements $u_j,v_j\in A$ ($j=1,\ldots,m$) such that $x=q(u_1,\ldots,u_m)$, $y=q(v_1,\ldots,v_m)$, where for each $j\in\{1,\ldots,m\}$,

$$\langle u_j,v_j\rangle \in S(x,y)\wedge S(x_i,y_i) \text{ for some } i\in\{1,\cdots,n\}.$$

By Lemma 1, there exist binary terms r_j,s_j with

$$u_j=r_j(x,y)=s_j(x_i,y_i),\;\; v_j=r_j(y,x)=s_j(y_i,x_i),$$

whence (2) is evident.

(2) $\Rightarrow$ (1): Let $A\in V$ and $R,S,Q\in\mathrm{Sym}(A)$. Suppose $\langle a,b\rangle\in R\wedge(S\vee Q)$. By Lemma 1, there exist an n-ary term p and elements $a_1,\ldots,a_n,b_1,\ldots,b_n$ of A such that

$$a=p(a_1,\ldots,a_n),\;\; b=p(b_1,\ldots,b_n)$$

and $\langle a,b\rangle \in R$, thus $S(a,b) \subseteq R$, and either $\langle a_i,b_i\rangle \in S$ or $\langle a_i,b_i\rangle \in Q$ for $i=1,\dots,n$. By (2), there exist terms q, s_j, r_j such that

$$a = q(r_1(a,b),\dots,r_m(a,b)), \qquad b = q(r_1(b,a),\dots,r_m(b,a))$$

and, for each j,

$$r_j(a,b) = s_j(a_i,b_i) \quad \text{and} \quad r_j(b,a) = s_j(b_i,a_i)$$

for some $i \in \{1,\dots,n\}$. Hence, if $\langle a_i,b_i\rangle \in S$, then $\langle r_j(a,b), r_j(b,a)\rangle \in R \wedge S$, and $\langle r_j(a,b), r_j(b,a)\rangle \in R \wedge Q$ provided $\langle a_i,b_i\rangle \in Q$. By Lemma 1, we conclude $\langle a,b\rangle \in (R\wedge S)\vee(R\wedge Q)$. □

Example 1. Every unary variety V has distributive Sym(A) for each $A \in V$.

Evidently, every n-ary term in a unary variety V is properly unary. Without loss of generality, suppose $p(x_1,\dots,x_n) = p_0(x_1)$. We can put $m=1$, $q(x)=x$, $r_1(x,y)=x$, $s_1(x,y)=p_0(x)$. Then (2) of Theorem 1 is satisfied:

$$p(\mathbf{x}) = p_0(x_1) = q(r_1(p(\mathbf{x}),p(\mathbf{y}))) \text{ and } r_1(p(\mathbf{x}),p(\mathbf{y})) = p(\mathbf{x}) = p_0(x_1) = s_1(x_1,y_1).$$

Now, we turn to the modularity of Sym(A).

Theorem 2. *For a variety V, the following conditions are equivalent:*

(1) Sym(A) *is modular for each $A \in V$;*

(2) *for every n-ary term p and each $k \in \{1,\dots,n\}$ there exist an m-ary term q, $(2+2k)$-ary terms w_j, $(2n-2k)$-ary terms g_j and $2k$-ary terms t_j ($j=1,\dots,m$) such that $p(\mathbf{x}) = q(z_1,\dots,z_m)$, where for each j either*

$$\begin{aligned} z_j &= w_j(p(\mathbf{x}),p(\mathbf{y}),x_1,\dots,x_k,y_1,\dots,y_k) \\ &= g_j(x_{k+1},\dots,x_n,y_{k+1},\dots,y_n) \ \text{ or} \\ z_j &= t_j(x_1,\dots,x_k,y_1,\dots,y_k). \end{aligned}$$

Proof. (1) ⇒ (2): Let p be an n-ary term over V, $k \in \{1,\dots,n\}$, and let $A = F_v(x_1,\dots,x_n,y_1,\dots,y_n)$ be a free algebra of V. Denote $x = p(\mathbf{x})$, $y = p(\mathbf{y})$ and put

$$Q = \bigvee\{S(x_i,y_i);\, i=1,\dots,k\}, \quad T = \bigvee\{S(x_i,y_i);\, i=k+1,\dots,n\},$$
$$R = S(x,y)\vee Q.$$

Then $\langle x,y\rangle \in T \vee Q$ and $\langle x,y\rangle \in R$, thus $\langle x,y\rangle \in R \wedge (T \vee Q)$. Since $Q \subseteq R$, the modularity of Sym (A) implies $\langle x,y\rangle \in (R \wedge T) \vee Q$. By Lemma 1, there exist an m-ary term q and elements $z_j, u_j \in A$ such that $x = q(z_1, \ldots, z_m)$, $y = q(u_1, \ldots, u_m)$, where for each $j \in \{1, \ldots, m\}$ either

$$\langle z_j, u_j\rangle \in R \wedge T \quad \text{or} \quad \langle z_j, u_j\rangle \in Q.$$

By an argument similar to that of the proof of Theorem 1, we obtain (2).

(2) $\Rightarrow$ (1): Let $A \in V$ and $R, Q, T \in \mathrm{Sym}(A)$. Let

$$\langle a,b\rangle \in R \wedge (T \vee (R \wedge Q)).$$

Then $\langle a,b\rangle \in R$ and, by Lemma 1, there exist an n-ary term p and elements a_i, b_i of A ($i = 1, \ldots, n$) such that $a = p(a_1, \ldots, a_n)$, $b = p(b_1, \ldots, b_n)$, where $\langle a_i, b_i\rangle \in R \wedge Q$ for $i \leqslant k$ and $\langle a_i, b_i\rangle \in T$ for $i > k$ for some $k \in \{1, \ldots, n\}$. By (2), we have

$$a = q(z_1, \ldots, z_m) \quad \text{and} \quad b = q(u_1, \ldots, u_m),$$

where either

$$\begin{aligned}
z_j &= w_j(a, b, a_1, \ldots, a_k, b_1, \ldots, b_k) = g_j(a_{k+1}, \ldots, a_n, b_{k+1}, \ldots, b_n),\\
u_j &= w_j(b, a, b_1, \ldots, b_k, a_1, \ldots, a_k) = g_j(b_{k+1}, \ldots, b_n, a_{k+1}, \ldots, a_n),\\
&\quad \text{i.e. } \langle z_j, u_j\rangle \in (R \vee (R \wedge Q)) \wedge T = R \wedge T, \text{ or}\\
z_j &= t_j(a_1, \ldots, a_k, b_1, \ldots, b_k), u_j = t_j(b_1, \ldots, b_k, a_1, \ldots, a_k),\\
&\quad \text{i.e. } \langle z_j, u_j\rangle \in R \wedge Q.
\end{aligned}$$

By Lemma 1, $\langle a,b\rangle \in (R \wedge T) \vee (R \wedge Q)$. □

Example 2. The variety $\mathscr{A}$ of all abelian groups has modular $\mathrm{Sym}(A)$ for each $A \in \mathscr{A}$.

Evidently, every n-ary term $p(x_1, \ldots, x_n)$ of A is of the form $x_1^{\alpha_1} \ldots x_n^{\alpha_n}$, where $\alpha_1, \ldots, \alpha_n$ are integers. Put $m = 2$, $q(z,v) = v \circ z$,

$$\begin{aligned}
w_1(a, b, x_1, \ldots, x_k, y_1, \ldots, y_k) &= a \circ x_1^{-\alpha_1} \ldots x_k^{-\alpha_k},\\
g_1(x_{k+1}, \ldots, x_m, y_{k+1}, \ldots, y_n) &= x_{k+1}^{\alpha_{k+1}} \ldots x_n^{\alpha_n},\\
t_2(x_1, \ldots, x_k, y_1, \ldots, y_k) &= x_1^{\alpha_1} \ldots x_k^{\alpha_k}.
\end{aligned}$$

Then

$$\begin{aligned}
z_1 &= w_1(p(\mathbf{x}), p(\mathbf{y}), x_1, \ldots, x_k, y_1, \ldots, y_k) = x_1^{\alpha_1} \ldots x_n^{\alpha_n} \circ x_1^{-\alpha_1} \ldots x_k^{-\alpha_k}\\
&= x_{k+1}^{\alpha_{k+1}} \ldots x_n^{\alpha_n} = g_1(x_{k+1}, \ldots, x_n, y_{k+1}, \ldots, y_n),\\
z_2 &= t_2(x_1, \ldots, x_k, y_1, \ldots, y_k) = x_1^{\alpha_1} \ldots x_k^{\alpha_k},
\end{aligned}$$

and

$$q(z_1, z_2) = z_2 \circ z_1 = x_1^{\alpha_1} \dots x_k^{\alpha_k} \circ x_{k+1}^{\alpha_{k+1}} \dots x_n^{\alpha_n} = p(\mathbf{x}),$$

proving (2) of Theorem 2.

Similarly as in Example 2, we can show that Sym(B) is modular for every Boolean algebra B. In this case $p(\mathbf{x})$ is expressed in the form of the canonical disjunction and the proof is rather tedious.

Now we turn to *reflexive relations*. For an algebra A, denote by Ref(A) the lattice of all reflexive compatible relations on A; denote by $\vee$ or $\wedge$ the operation join or meet in Ref(A), respectively. Evidently, $\wedge$ coincides with set intersection and the identity relation ω is the least and A^2 is the greatest element of Ref(A). Denote by $R(a, b)$ the least relation of Ref(A) containing the given pair $\langle a, b\rangle$ of elements $a, b \in A$. The following elementary assertion has been proved in [1] (Theorems 4 and 6):

Lemma 2. *Let A be an algebra, let a, b, c, d, x, y, a_1, ..., a_n, b_1, ..., b_n be elements of A and $R_j \in$ Ref(A) for $j \in J$.*

Then (a) $\langle c, d\rangle \in R(a, b)$ *iff there exists an $(n+1)$-ary term t and elements e_1, ..., $e_n \in A$ such that $c = t(a, e_1, \dots, e_n)$, $d = t(b, e_1, \dots, e_n)$;*

(b) $\langle x, y\rangle \in \bigvee\{R_j, j \in J\}$ *iff there exist an m-ary term p and elements $x_k, y_k \in A$ ($k = 1$, ..., m) such that $\langle x_k, y_k\rangle \in R_{j_k}$ for some $j_k \in J$ and $x = q(x_1, \dots, x_m)$, $y = q(y_1, \dots, y_m)$;*

(c) $\langle x, y\rangle \in \bigvee\{R(a_i, b_i); i = 1, \dots, n\}$ *iff there exist an $(n+m)$-ary term q and elements e_1, ..., $e_m \in A$ such that*

$$x = q(a_1, \dots, a_n, e_1, \dots, e_m), \quad y = q(b_1, \dots, b_n, e_1, \dots, e_m).$$

Theorem 3. *For a variety V the following conditions are equivalent:*

(1) Ref(A) *is distributive for each $A \in V$;*

(2) *For every n-ary term p there exist an m-ary term q and $(2n+1)$-ary terms r_j, s_j ($j = 1$, ..., m) such that*

$$p(\mathbf{x}) = q(r_1(p(\mathbf{x}), \mathbf{x}, \mathbf{y}), \dots, r_m(p(\mathbf{x}), \mathbf{x}, \mathbf{y})),$$
$$p(\mathbf{y}) = q(r_1(p(\mathbf{y}), \mathbf{x}, \mathbf{y}), \dots, r_m(p(\mathbf{y}), \mathbf{x}, \mathbf{y}))$$

and for each $j \in \{1, \dots, m\}$ there exists $i \in \{1, \dots, n\}$ with

$$r_j(p(\mathbf{x}), \mathbf{x}, \mathbf{y}) = s_j(x_i, \mathbf{x}, \mathbf{y}) \quad \textit{and} \quad r_j(p(\mathbf{y}), \mathbf{x}, \mathbf{y}) = s_j(y_i, \mathbf{x}, \mathbf{y}).$$

The proof is word for word analogous to that of Theorem 1 only Lemma 2 is applied instead of Lemma 1.

E x a m p l e 3. Every variety of unary algebras has distributive $\mathrm{Ref}(A)$ for each $A \in V$.

Without loss of generality, $p(\mathbf{x}) = p_0(x_1)$ for some unary term p_0 and $s_1(\mathbf{x}) = p_0(x)$.

R e m a r k . If a variety V is congruence-permutable, then $\mathrm{Con}\, A = \mathrm{Ref}(A)$ for each $A \in V$, see [9] ($\mathrm{Con}\, A$ denotes the congruence lattice of A). Therefore, $\mathrm{Ref}(A)$ is distributive e.g. for every Boolean algebra A. However, we can give also the explicit boolean terms satisfying (2) of Theorem 3:

Let p be an n-ary boolean term. We can put $m = n$, $q = p$ and for every $j = 1, \ldots, n$, $s_j(z, \mathbf{x}, \mathbf{y}) = [(x \vee y) \wedge z] \vee (x \wedge y)$ and $r_j(z, \mathbf{x}, \mathbf{y}) = \{[(x \vee y) \wedge x_j] \vee (y \wedge z) \vee [x \wedge (x \oplus y \oplus z)]\} \wedge \{[(x \vee y) \wedge y_j] \vee [(y \vee z) \wedge (x \vee (x \oplus y \oplus z))]\}$, where $a \oplus b = (a \wedge b') \vee (a' \wedge b)$ and $x = p(\mathbf{x})$, $y = p(\mathbf{y})$. It is an easy exercise to verify (2) of Theorem 3.

However, we are able to give a more general example:

E x a m p l e 4. For every distributive lattice L, $\mathrm{Ref}(L)$ is distributive.

Denote by D the variety of all distributive lattices. Since every n-ary term p over D arises by a finite number of lattice operations $\vee$ and $\wedge$, we can prove the existence of q, r_i, s_i satisfying (2) of Theorem 3 by induction over the rank of the term p. Hence, it suffices to show it for two cases, namely $p(x_1, x_2) = x_1 \vee x_2$ and $p(x_1, x_2) = x_1 \wedge x_2$.

(a) Let $p(x_1, x_2) = x_1 \vee x_2$. Put $n = 2$, $q = p$, $s_i(z, \mathbf{x}, \mathbf{y}) = [(x \vee y) \wedge z] \vee (x \wedge y)$, $r_i(z, \mathbf{x}, \mathbf{y}) = \{x \wedge z \wedge [((x \vee y) \wedge x_i) \vee (x \wedge y)]\} \vee \{y \wedge z \wedge [((x \vee y) \wedge y_i) \vee (x \wedge y)]\}$, where $x = p(x_1, x_2)$, $y = p(y_1, y_2)$. Hence $x_i \leqslant x, y_i \leqslant y$, thus also $(x \wedge x_i) \vee (x \wedge y) = ((x \vee y) \wedge x_i) \vee (x \wedge y)$, i.e. $r_i(x, \mathbf{x}, \mathbf{y}) = s_i(x_i, \mathbf{x}, \mathbf{y})$, analogously $r_i(y, \mathbf{x}, \mathbf{y}) = s_i(y_i, \mathbf{x}, \mathbf{y})$. It is easy to show that

$$x_1 \vee x_2 = s_1(x_1, \mathbf{x}, \mathbf{y}) \vee s_2(x_2, \mathbf{x}, \mathbf{y}),$$
$$y_1 \vee y_2 = s_1(y_1, \mathbf{x}, \mathbf{y}) \vee s_2(y_2, \mathbf{x}, \mathbf{y}),$$

thus (2) of Theorem 3 is satisfied.

(b) If $p(x_1, x_2) = x_1 \wedge x_2$, then we can choose r_i dually to the case (a); s_i is clearly self-dual. Moreover, $x = x_1 \wedge x_2$, $y = y_1 \wedge y_2$ gives $x \leqslant x_i$, $y \leqslant y_i$, thus (2) of Theorem 3 can be shown dually to (a). By induction over the rank of the term p, it can be generalized for any term p over D.

Corollary. *Let V be a non-trivial variety of lattices. The following conditions are equivalent:*

(1) Ref(L) *is distributive for each* $L \in V$;

(2) V *is a variety of all distributive lattices.*

Proof. (2) $\Rightarrow$ (1) By Example 4. Conversely, let V be a nontrivial lattice variety which is not a variety of distributive lattices. Then V contains at least one of the lattices N_5 or M_3, i.e. V contains at least one of the lattices L_1, L_2 in Fig. 1:

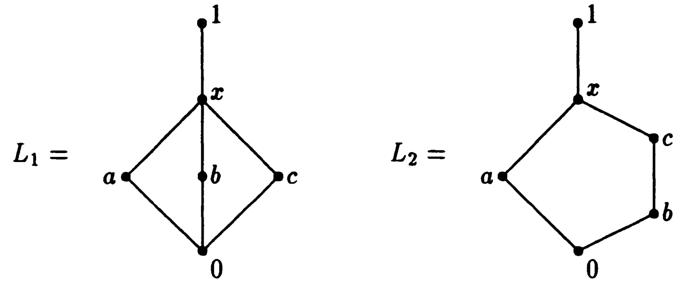

Fig. 1

Denote by B the subset $\{0, a, b, c, x\}$ and put

$$R_1 = \{1, x, a\}^2 \cup B^2, \; R_2 = \{1, x, b\}^2 \cup B^2, \; R_3 = \{1, x, c\}^2 \cup B^2.$$

It is an easy exercise to show that for L_1, $\{R_1 \wedge R_2 \wedge R_3, R_1, R_2, R_3, R_1 \vee R_2 \vee R_3\}$ forms a sublattice of Ref(L_1) isomorphic to M_3. In the case of L_2, $R_3 \subseteq R_2$ and $\{R_3 \wedge R_1, R_1, R_2, R_3, R_1 \vee R_2\}$ forms a sublattice of Ref(L_2) isomorphic to N_5. Hence neither Ref(L_1) nor Ref(L_2) are distributive. □

We can proceed to characterize the varieties with modular lattices of reflexive relations.

Applying Lemma 2 instead of Lemma 1, we can also prove similarly as in the case of Theorem 2:

Theorem 4. *For a variety* V, *the following conditions are equivalent:*

(1) Ref(A) *is modular for each* $A \in V$;

(2) *For every* n*-ary term* p *and each* $k \in \{1, \dots, n\}$ *there exist an* m*-ary term* q, $(2n + 1 + k)$*-ary terms* w_j, $(2n + k)$*-ary* t_j *and* $(3n - k)$*-ary* g_j $(j = 1, \dots, m)$ *such that* $p(\mathbf{x}) = q(u_1, \dots, u_m)$, $p(\mathbf{y}) = q(v_1, \dots, v_m)$ *where for each* j *either*

$$u_j = w_j(p(\mathbf{x}), x_1, \dots, x_k, \mathbf{x}, \mathbf{y}) = g_j(x_{k+1}, \dots, x_n, \mathbf{x}, \mathbf{y}),$$
$$v_j = w_j(p(\mathbf{y}), y_1, \dots, y_k, \mathbf{x}, \mathbf{y}) = g_j(y_{k+1}, \dots, y_n, \mathbf{x}, \mathbf{y})$$
$$\text{or} \qquad u_j = t_j(x_1, \dots, x_k, \mathbf{x}, \mathbf{y}), \qquad v_j = t_j(y_1, \dots, y_k, \mathbf{x}, \mathbf{y}).$$

It can be shown that every variety of groups (or quasigroups) has modular Ref(A) for each $A \in V$. However, this is an easy corollary of [5] since groups are congruence-permutable, thus Ref(A) = Con A, see [9].

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