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QUADRILATERAL EMBEDDINGS OF THE CONJUNCTION OF GRAPHS

VLADIMÍR ŽELEZNÍK

1. Introduction

Throughout the paper we are concerned only with finite graphs without loops or multiple edges. It is assumed that the reader is familiar with the fundamental results of thed theory of graph embeddings in particular with the combinatorial tools that describe cellular embeddings of graphs into surfaces as presented in Stahl [8]. For terms not defined here the reader is referred to any standard textbook of graph theory, e.g. Harary [4].

We will use the following notation: $v(G)$, $e(G)$, $c(G)$, $\gamma(G)$ and $\tilde{\gamma}(G)$ will denote the number of vertices, the number of edges, the number of components, the orientable and the nonorientable genus of the graph G.

The present paper deals with the conjunction of graphs called also the categorical product, the Kronecker product, the tensor product, the cardinal product.

Definition 1. *Let for the graphs $G_i = (V_i, E_i)$, $i = 1, 2$, the set V_1, V_2 be disjoint. The conjunction $G_1 \wedge G_2$ of the graphs G_1, G_2 is a graph having $V_1 \times V_2$ as its vertex set and $u = [u_1, u_2]$ is adjacent to $v = [v_1, v_2]$, $u_1, v_1 \in V_1$, $u_2, v_2 \in V_2$, whenever u_1 is adjacent to v_1 in G_1 and u_2 is adjacent to v_2 in G_2.*

It is clear that $v(G_1 \wedge G_2) = v(G_1) . v(G_2)$ and $e(G_1 \wedge G_2) = 2 . e(G_1) . e(G_2)$.

The conjunction of two connected graphs is connected if and only if at least one of them has an odd cycle ([9]). Thus it can occur that graph G, which will be embedded, is disconnected. In this case we will use a 2-cell embedding for each of the components of G in the separate manifolds.

Since we consider the orientable and nonorientable cases, we use the (generalized) embedding schemes of Stahl [8] to describe 2-cell embeddings.

In the second section we will prove that if a bipartite graph G has a diagonalizable quadrilateral embedding (cf. Definition 2 below), then the conjunction G with an arbitrary graph has the embedding of the same type.

In the last part it is shown that the complete bipartite graph with an even number of vertices in both partitions, the graph of an s-dimensional cube and the conjunction of an even circuit with an arbitrary circuit have diagonalizable quadrilateral embeddings. In consequence the orientable and nonorientable genera of some types of graphs are obtained.

2. The main result

To formulate it we introduce the following definition.

Definition 2. *Let* $G = (V, E)$ *be a connected bipartite graph such that* $V = V_1 \cup V_2$ *and each edge has one vertex from* V_1 *and the other from* V_2. *We say that* G *has a diagonalizable quadrilateral embedding (for abbrevation we shall often write* DQE) $G(S)$ *in some manifold* S *if*:

(DQE) (*i*) $G(S)$ *is a quadrilateral embedding and*

(*ii*) *there exists a graph* $G' = (V, E')$ *having an embedding* $G'(S)$ *and an* 1-*factor* F *such that both vertices of each edge of* F *are from* V_1 *or from* V_2 *and* $G'(S) - F = G(S)$.

If a graph G is disconnected, we say that G has DQE if each component of G has DQE.

The edges of F will be called the diagonals.

Example. The graph of the 3-dimensional cube Q_3 has a DQE. One of the possibilities how to choose the diagonals is shown in Figure 1.

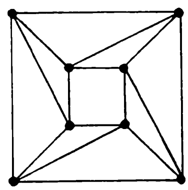

Fig. 1

Remark 1. A well-known consequence of Euler's equation (e.g. see Harary [4]) is that the orientable and nonorientable genus of a connected graph G having no triangles satisfies the inequality

$$\gamma(G) \geqq e(G)/4 - v(G)/2 + 1;$$
$$\tilde{\gamma}(G) \geqq e(G)/2 - v(G) + 1.$$

The equality holds if G admits an orientable or nonorientable quadrilateral embedding, respectively. Therefore for a graph G having DQE we get (using the additivity of the genus parameter over the components of G — [1])

$$\gamma(G) = e(G)/4 - v(G)/2 + c(G),$$
$$\tilde{\gamma}(G) = e(G)/2 - v(G) + c(G).$$

Theorem 1. *Let G, H be graphs, G bipartite. If G has a diagonalizable quadrilateral embedding (orientable or nonorientable), then the conjunction $G \wedge H$ has also a diagonalizable quadrilateral embedding (orientable or nonorientable, respectively).*

Proof. We shall prove a somewhat stronger result. If $f \in F$ is a diagonal joining vertices x, y from G, then there exists a diagonal f_u joining $[x, u]$, $[y, u]$ from $G \wedge H$ for each $u \in V(H)$.

For local rotations $P_1 = (p_1, \ldots, p_m)$, $P_2 = (q_1, \ldots, q_k)$ we denote $P_1 \cup P_2 = (p_1, \ldots, p_m, q_1, \ldots, q_k)$ and $P_1 \times \{u\} = ([p_1, u], \ldots, [p_m, u])$. We will write (a, b) instead of $([x, y], [a, b])$ in P_{xy} and analogously b instead of (x, b) in Q_x for abbrevation.

The distributive law $G_1 \wedge (G_2 + G_3) = (G_1 \wedge G_2) + (G_1 \wedge G_3)$ shows that it is sufficient to consider the connected graphs G and H, without loss of generality. We proceed by induction on the number $e(H)$.

1. For $e(H) = 1$ we have $H = K_2$ and the statement follows from the fact that if G is a bipartite graph, then $G \wedge K_2$ has two components and both are isomorphic to G. For every vertex x and every edge (y, z) of G the vertex $[x, u]$ is from one component and $[x, v]$ is from the other one (and analagously for the edges $([y, u], [z, v])$ and $([y, v], [z, u])$) in $G \wedge K_2$.

Let (Q, s) be an embedding scheme which describes a DQE of G in some manifold S. We define rotations and the labelling of the edges of $G \wedge K_2$ in the following way (let $V(K_2) = \{u, v\}$): For each $x \in V(G)$ let Q_x be a local rotation; then

$$Q'_{xu} = Q_x \times \{v\},$$
$$Q'_{xv} = Q_x \times \{u\},$$

and $s_1([x, u][y, v]) = s_1([x, v][y, u]) = s(x, y)$, for each $(x, y) \in E(G)$.
It is obvious that (Q', s_1) where $Q' = \{Q'_{xz}, x \in V(G), z \in V(K_2)\}$ represents a quadrilateral embedding of $G \wedge K_2$. Moreover it is easy to see that if a diagonal f joins the vertices $x, y \in V(G)$, we can choose a set of diagonals F' in $G \wedge K_2$ in such a way that f_z joins $[x, z]$ and $[y, z]$ for $z \in V(K_2)$. Therefore (Q', s_1) represents a DQE.

2. Let us suppose that the statement is valid for any graph having n edges and consider the graph H with $n + 1$ edges. Let $e = (u, v)$ be an edge of H such that the graph $H' = H - e$ is connected and $\deg_H(\mathrm{v}) > 1$.

The graph $G \wedge H'$ has a DQE in some manifold M. This embedding can be represented by the embedding scheme (P, s_2) with $P = \{P_{rs}, r \in V(G), s \in V(H')\}$. We also take a scheme (Q, s) characterizing a DQE of G. Let the vertices x, y be endpoints of the diagonal f_1, let the vertices $x, a, y, b \in V(G)$ form a quadrilateral face and f_1 be situated inside of this quadrangle. Then $[x, v]$ and $[y, v]$ are the endpoints of an analogue diagonal f'_{1v} in $G \wedge H'$ and also the endpoints of a diagonal f_{1v} in $G \wedge K_2$, where $E(K_2) = e = (u, v)$. Then the rotations mentioned above have the following forms:

$$(1) \qquad \begin{aligned} Q_x &= (a, c, \dots, b), \\ Q_y &= (b, d, \dots, a), \\ Q_w &= (w_1, w_2, \dots, w_{m(w)}) \quad \text{for} \quad w \in V(G) - \{x, y\} \end{aligned}$$

and

$$\begin{aligned} P_{xv} &= ((x_1, v_1), (x_2, v_2), \dots, (x_n, v_n)), \\ P_{yv} &= ((y_1, v_1'), (y_2, v_2'), \dots, (y_m, v_m')), \\ P_{rs} &= ((r_1, s_1), \dots, (r_k, s_k)) \quad \text{otherwise.} \end{aligned}$$

Define the new rotations as follows

$$(2) \qquad \begin{aligned} P'_{xv} &= P_{xv} \cup Q_x \times \{u, \\ P'_{yv} &= P_{yv} \cup Q_y \times \{u\}, \\ P'_{xu} &= Q_x \times \{u\}, \\ P'_{yu} &= Q_y \times \{v\}, \\ P'_{wv} &= Q_w \times \{u\}, \\ P'_{wu} &= Q_w \times \{v\} \quad \text{for} \quad w \in V(G) - \{x, y\}, \\ P'_{rs} &= P_{rs} \quad \text{otherwise.} \end{aligned}$$

The labelling s' of the edges of $G \wedge H$ by 0 or 1 is defined as follows. If the edge e of $G \wedge H'$, then $s'(e) = s_2(e)$. On the other hand there must be $e = ([x, v], [y, u])$ for some $e' = (x, y) \in E(G)$. Then $s'(e) = s(e')$.

If $\deg_H(u) = 1$, we have finished otherwise we proceed analogously (we only interchange u and v).

By repeating this process for each of the diagonals $f_2, \dots, f_m$ of the graph G' ("old" rotations (1) and "old" voltage map s_1 in every step are "new" rotations (2) and a map s' from the preceding step) we obtain a quadrilateral embedding of the graph $G \wedge H$ represented by (P', s').

It is obvious that this final embedding is orientable or nonorientable if the embedding (Q, s) of G is orientable or nonorientable, respectively. In the first case, we can take the labelling s to be constantly equal to 0. In the second case there exists an even cycle $C = (x_1, x_2, \dots, x_{2k})$ (all cycles are even) in G for which the odd number of edges are labelled by 1. It follows from the construction of (P', s') that in the first case s' is constantly equal to 0, too. In the second case there exists also in $G \wedge H$ a cycle $[x_1, u][x_2, v][x_3, u], \dots, [x_{2k}, v]$ for any $(u, v) \in$ $\in E(H)$ with the same labelling as C. This completes the proof.

Remark 2. The definition of "new" rotations and labelling (2) from the "old" ones (1) in the proff of Theorem 1 corresponds to the following construction.

Let C_1 be a simple closed curve in S' such that f_{1v}, $[x, v]$ and $[y, v]$ are inside

C_1 and all the other vertices of $G \wedge K_2$ are outside C_1. Moreover, if $e_1, \dots, e_k$ or $e'_1, \dots, e'_m$ are edges incident with vertex $[x, v]$ or $[y, v]$, respectively, then

$$C_1 \cap e_i = b_i, \quad C_1 \cap e'_j = b'_j \quad \text{for} \quad i = 1, \dots, k, \quad j = 1, \dots, m \quad \text{and}$$

$$G(S) \cap C_1 = \{b_1, \dots, b_k, b'_1, \dots, b'_m\}.$$

Let C_2 be a curve and $\{c_1, \dots, c_i, c'_1, \dots, c'_j\}$ be a set of points in M, having the same properties as C_1 and $\{b_1, \dots, b'_m\}$. In S', resp. in M, remove the open disk having C_1, resp. C_2 as its boundary. Then adjoin a topological cylinder K with bases C_1 and C_2 such that $K \cap (M \cup S) = C_1 \cup C_2$. Let X', $Y' \in K - (C_1 \cup C_2)$ be two different points. Join X' with each of the points $b_1, \dots, b_k$, $c_1, \dots, c_i$ and Y' with each of the points $b'_1, \dots, b'_m$, $c'_1, \dots, c'_j$ by mutually disjoint arcs. Points X', Y' correspond to the vertices $[x, v]$, $[y, v]$ in the graph $G \wedge H$, therefore we rename them $[x, v]$ and $[y, v]$.

After this construction two quadrangles $[a, u][x, v][b, u][y, v]$ and $a'[x, v]b'[y, v]$ change into the quadrangles $[a, u][x, v]a'[y, v]$ and $[b, u][x, v] \times b'[y, v]$ (see Figure 2). From this it is clear that we can join $[x, v]$ and $[y, v]$ by the diagonal f_{1v}.

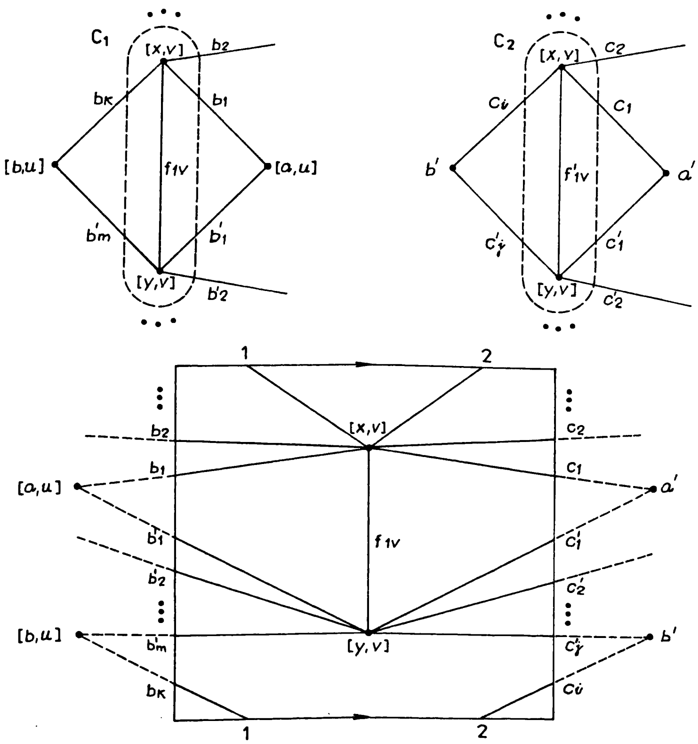

Fig. 2

3. Applications to new genus results

Here we give only a small number of the consequences of Theorem 1. The reader will be able to find many other applications. Recall that C_n denotes the circuit of n vertices, Q_s the graph of the s-dimensional cube, $K_{m,n}$ the complete bipartite graph.

Let $G_1, G_2, \ldots, G_n$ be graphs. Define graph H_n as follows:

$$H_1 = G_1$$
$$H_k = H_{k-1} \wedge G_k \quad \text{for} \quad k = 2, 3, \ldots, n.$$

Since the conjunction of two graphs is bipartite if and only if at least one of them is bipartite (see, e.g., Weichsel [7]), it follows that if at least one of the G_i is bipartite, so is the graph H_n; this fact will often be employed. If not assumed otherwise, m_i and n_i denotes the number of edges and vertices of the graph G_i, respectively.

From now on let $X(i, n)$ denote the product of the numbers $x_i, x_{i+1}, \ldots, x_n$ (analogously the product $b_j . b_{j+1} . \ldots . b_r$ will be denoted by $B(j, r)$, etc.)

Theorem 2. *Let the connected bipartite graph G_1 have an orientable or a nonorientable quadrilateral embedding, respectively. Let $G_2, \ldots, G_n$ be connected graphs, k of them bipartite. Then we have*

$$\gamma(H_n) = 2^{n-3} . M(1, n) - N(1, n)/2 + 2^k, \quad \textit{or}$$
$$\tilde{\gamma}(H_n) = 2^{n-2} . M(1, n) - N(1, n) + 2^k, \quad \textit{respectively.}$$

Proof. We apply Theorem 1 $n - 1$ times, to obtain a quadrilateral embedding of H_n. Since $v(H_n) = N(1, n)$, $e(H_n) = 2^{n-1} . M(1, n)$ and $c(H_n) = 2^k$, the statement follows from Remark 1.

Theorem 3. *The graph Q_s has an orientable* DQE *whenever $s \geqq 2$ and it has a nonorientable* DQE *whenever $s \geqq 6$.*

Proof. We use the quadrilateral embedding of Q_s given in [2] ($s \geqq 2$) or in [5] ($s \gtreqqless 6$) for the orientable or the nonorientable case, respectively. In both cases we can choose the diagonals in the same way. We denote the vertices of Q_s by a binary sequence $(a_1 a_2 \ldots a_s)$ of length s, where $a_i = 0$ or 1, in such a way that two vertices are adjacent whenever their sequences differ in exactly one place. Let a denote an arbitrary binary sequence of length $s - 3$. We can join the vertices $(0a00)$ and $(1a00)$, resp. $(0a01)$ and $(1a11)$, resp. $(0a11)$ and $(1a10)$, resp. $(0a10)$ and $(1a00)$, by a diagonal situated inside of quadrangle $(0a00)$ $(0a01)$ $(1a01)$ $(1a00)$, resp. $(0a01)$ $(0a11)$ $(1a11)$ $(1a01)$, resp. $(0a11)$ $(0a10)$ $(1a10)$ $(1a11)$, resp. $(0a10)$ $(0a00)$ $(1a00)$ $(1a10)$. These quadrangles are located on the cylinders added in the last step of construction of the given embeddings.

Corollary 1. *Let $G_i = Q_{s_i}$ for $i = 1, \ldots, n$, $s_1 \geqq 2$ in the orientable case, $s_1 \geqq 6$ in the nonorientable case and $s = s_1 + \ldots + s_n$. Then*

$$\gamma(H_n) = 2^{s-3}.(S(1,n) - 4) + 2^{n-1};$$
$$\tilde{\gamma}(H_n) = 2^{s-2}.(S(1,n) - 4) + 2^{n-1}.$$

Theorem 4. *The graph $K_{2r,2q}$ has an orientable* DQE *for each $r \geqq 1$, $q \geqq 1$ and it has a nonorientable* DQE *whenever $r \geqq 1$, $q \geqq 2$.*

Proof. Let $V_1 \cup V_2$ be a partition of vertices of $K_{2r,2q}$. Denote $1, 2, \ldots, 2r$, resp. $1', 2', \ldots, (2q)'$ the vertices from V_1, resp. V_2.

In the orientable case we take the quadrilateral embedding of $K_{2r,2q}$ given in [6]. Then we can join the vertices i and $i+1$ from V_1 by a diagonal in the quadrangle i, $1'$, $i+1$, $(2q)'$ for $i = 1, 3, 5, \ldots, 2r-1$ and the vertices j', $(j+1)'$ in the quadrangle $1, j', 2r, (j+1)'$ for $j = 1, 3, 5, \ldots, 2q-1$.

In the nonorientable case we will use the embedding given in [7]. We join the vertices i and $i+1$ by a diagonal inside the quadrangle $(2q)', i, 1', i+1$ for $i = 2, 4, \ldots, 2r-2$ and the vertices 1 and $2r$ in $(2q)', 2r, 1', 1$. The vertices $j', (j+1)'$ for $j = 3, 5, \ldots, 2q-1$ are joined inside $j', 1, (j+1)', 2$ and $1'$ with $2'$ in $1', 4, 2', 2$.

Corollary 2. *Let $G_1 = K_{2r_1, 2q_1}$, $G_i = K_{r_i, q_i}$ for $i = 2, \ldots, n$, $q_1 \geqq 2$ in the nonorientable case. Let $m_j = r_j . q_j$ and $n_j = r_j + q_j$ for $j = 1, 2, \ldots, n$. Then*

$$\gamma(H_n) = 2^{n-1}.(M(1,n) + 1) - N(1,n),$$
$$\tilde{\gamma}(H_n) = 2^{n-1}(2.M(1,n) + 1) - 2.N(1,n).$$

Theorem 5. *The conjunction of two circuits C_m and C_n has an orientable quadrilateral embedding. Moreover this embedding is a* DQE *if and only if at least one of the numbers m, n is even.*

Proof. Denote the vertices of the first circuit by $1, \ldots, m$ and by $1, \ldots, n$ those of the second in the cyclic ordering and define the local rotations as follows:

$$P_{ij} = ([i-1, j-1], [i-1, j+1], [i+1, j+1], [i+1, j-1]), \quad \text{for} \quad i = 1, 2, \ldots, m \quad \text{and} \quad j = 1, 2, \ldots, n.$$

Then the orbits (each corresponding to a quadrilateral face) determined by the rotation $P([x,y],[u,v]) = ([u,v], P_{uv}([x,y]))$ are

$$[i,j][i+1,j+1][i+2,j][i+1,j-1],$$

where the first index is taken modulo m, the second modulo n and we write m (resp. n) instead of 0. This choice $P = (P_{11}, P_{12}, \ldots, P_{mn})$ determines a quadrilateral embedding for each of the components of the graph $C_m \wedge C_n$ in the orientable manifold S.

Now let n be even. We can join vertices $[i,j]$ and $[i+2,j]$ by a diagonal for $i = 4k+1$ and $i = 4k+2$, $k = 0, 1, 2, \ldots$ and $j = 1, 2, \ldots, m$ in the quadrangle $[i,j][i+1,j+1][i+2,j][i+1,j-1]$.

If both m, n are odd, then $v(C_m \wedge C_n)$ is odd, therefore $C_m \wedge C_n$ cannot have DQE.

Corollary 3. *For the genus of the conjunction of two circuits C_m and C_n we have*

$$\begin{aligned} \gamma(C_m \wedge C_n) &= 2 \quad \text{if both } m \text{ and } n \text{ are even;} \\ &= 1 \quad \text{otherwise.} \end{aligned}$$

Proof. The graph $C_m \wedge C_n$ is connected if and only if m or n is odd (see, e.g., Weichsel [9]). On the other hand it is easy to see that $C_m \wedge C_n$ has two isomorphic components for both m and n even. From the nonplanarity of $C_m \wedge C_n$ (Farsan, Waller [3]) and the additivity of the genus parameter over the components ([1]) it follows that

$$\begin{aligned} &\gamma(C_m \wedge C_n) \geqq 2 \quad \text{if both } m \text{ and } n \text{ are even, and} \\ &\gamma(C_m \wedge C_n) \geqq 1 \quad \text{otherwise.} \end{aligned}$$

Now it is sufficient to show that each of the components of $C_m \wedge C_n$ can be embedded in S_1 (the torus). We use a quadrilateral embedding of $C_m \wedge C_n$ given in Theorem 5. For the number of regions $r(K)$ of any component K of $C_m \wedge C_n$ we have $4 \,.\, r(K) = 2 \,.\, e(K)$, because each of the regions is a quadrangle and each of the edges belongs to two regions. Moreover, $e(K) = 2 \,.\, v(K)$ and therefore we have $\gamma(S) = 1$, because

$$2 \,.\, (1 - \gamma(S)) = v(K) - e(K) + r(K).$$

Theorem 6. *The graph $C_4 \wedge C_m$ has a nonorientable* DQE.

Proof. The denotation of the vertices, the definition of the local rotations and a choice of diagonals are the same as in the proof of Theorem 5. The labelling s is equal to 1 for every edge of the quadrangle $[1, 1], [2, m], [3, 1], [4, m]$ and $s(e) = 0$ otherwise. Since in the cycle $[1, 1], [2, 1], [1, 3], [2, 4], \ldots, [2, m]$ (if m is odd we have $\ldots, [1, m], [2, 1], [1, 2], \ldots$ inside) only the edge $([1, 1][2, u])$ has the labelling 1, the embedding scheme (P, s) defines the nonorientable embedding.

Corollary 4. *Let $G_1 = C_{2m_1}$, $G_i = C_{m_i}$, for $i = 2, \ldots, n$ and $m_1 \geqq 2$ in the orientable case and $m_1 = 2$ in the nonorientable case. Let $k = |\{m_i;\ 2 \leqq i \leqq n,\ m_i$ is even$\}|$. Then*

$$\begin{aligned} \gamma(H_n) &= M(1, n) \,.\, (2^{n-2} - 1) + 2^k, \\ \tilde{\gamma}(H_n) &= 2 \,.\, M(1, n) \,.\, (2^{n-2} - 1) + 2^k. \end{aligned}$$

Proof. By proofs of Theorems 5 and 6 the graph H_2 has DQE in both cases. Since $e(H_n) = 2^{n-1} \,.\, 2 \,.\, M(1, n)$, $v(H_n) = 2 \,.\, M(1, n)$ and $c(H_n) = 2^k$, the statement is proved.

For the special cases of the powers of the conjunction of the graphs Q_s, $K_{2r,2q}$ and C_{2m} we have

Corollary 5. *Let $s \geqq 2$, $r \geqq 1$, $q \geqq 1$, $m \geqq 2$ in the orientable case and $s \geqq 6$, $r \geqq 1$, $q \geqq 2$, $m = 2$ is the nonorientable case. Let*

a) $G_i = Q_s$
b) $G_i = K_{2r,2q}$
c) $G_i = C_{2m}$

for $i = 1, \dots, n$. Then

a) $\gamma(H_n) = 2^{s-1}.(2^{-2}.s^n - 1) + 2^{n-1}$;
$\tilde{\gamma}(H_n) = 2^{n.s}.(2^{-2}.s^n - 1) + 2^{n-1}$;
b) $\gamma(H_n) = 2^{n-1}.(4^{n-1}.(r.q)^n - (r+q)^n + 1)$;
$\tilde{\gamma}(H_n) = 2^n.(4^{n-1}.(r.q)^n - (r+q)^n + 2^{-1})$;
c) $\gamma(H_n) = 2^{n-1}.(m^n.(2^{n-2} - 1) + 1)$;
$\tilde{\gamma}(H_n) = 2^{3n-2} - 2^{2n} + 2^{n-1}$.

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ЧЕТЫРЕХСТРОННИЕ ВЛОЖЕНИЯ СОЕДИНЕНИЯ ГРАФОВ

Vladimír Železník

Резюме

В статье занимается четырехстронними вложениями двудольных графов, порожденных соединением графов, в ориентируемую или неориентируемую поверхность. Главным результатом данной работы является следующая теорема:

Если двудольный граф имеет диагонализируемое четырехстороннее вложение (ориентируемое или неориентируемое), то и соединение зтого графа с любым графом имееет такое вложение.

Как следствие получен ряд формул для ориентируемого и неориентируемого рода графа, полученного соединением n графов.