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ASYMPTOTIC BEHAVIOUR OF ALL SOLUTIONS OF DIFFERENTIAL SYSTEMS WITH DEVIATING ARGUMENTS

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This paper deals with systems of differential equations with deviating arguments of the form

$$\text{(A)}\qquad \begin{aligned} &(p_i(t)x_i'(t))' = a_i(t)x_{i+1}(t), \quad i = 1, \dots, n-2, \ n \geqslant 2 \\ &(p_{n-1}(t)x_{n-1}'(t))' = a_{n-1}(t)g(x_n(\tau_n(t))) \\ &(p_n(t)x_n'(t))' = f(t, x_1(\tau_1(t))), \quad t \geqslant a, \end{aligned}$$

where the following conditions are always assumed:

(a) $0 < p_i(t) \in C[a; \infty)$, $\displaystyle\int^{\infty} \frac{ds}{p_i(s)} < \infty$, $\quad i = 1, \dots, n$;

(b) $0 \leqslant a_i(t) \in C[a; \infty)$, $\displaystyle\int^{\infty} a_i(s)\,ds < \infty$, $\quad i = 1, \dots, n-1$ and $a_i(t)$ are not identically zero on any subinterval of $[a; \infty)$;

(c) $g(u) \in C(-\infty; \infty)$, $|g(u)| \leqslant K|u|^{\beta}$ for $0 < \beta \leqslant 1$, $0 < K$-const,

(d) $f(t, v) \in C([a; \infty) \times (-\infty; \infty))$ and $|f(t, v)| \leqslant \omega(t, |v|)$ for $(t, v) \in [a; \infty) \times (-\infty; \infty)$, where $\omega(t, z) \in C([a; \infty) \times [0; \infty))$ and $\omega(t, z)$, $\omega(t, z)/z$ are nondecreasing in z;

(e) $\tau_i(t) \in C[a; \infty)$, $\lim\limits_{t \to \infty} \tau_i(t) = \infty$, $i = 1, n$ and $\tau_n(t) \leqslant t$ for $t \geqslant a$.

The term "solution $x(t) = (x_1(t), \dots, x_n(t))$ of (A)" will be understood in the sequel to refer to a solution of (A) which exists on an interval $[T_x; \infty) \subset [a; \infty)$ and satisfies the condition

$$\sup\left\{\sum_{i=1}^{n} |x_i(t)| : t \geqslant T\right\} > 0 \quad \text{for every} \quad T \geqslant T_x.$$

Such a solution is said to be oscillatory, resp. weakly oscillatory, if each component (resp. at least one component) has arbitrarily large zeros.

Some asymptotic behaviours of solutions of two-dimensional differential systems with deviating argument are studied in the papers by Kitamura and Kusano [1—3]. Our purpose in this work is to give an analogue of the theorem of [1], which enables us to classify all the solutions of (A) according to the asymptotic behaviour.

Let $i_k = \{1, 2, \dots, 2n-1\}$, $1 \leqslant k \leqslant 2n-1$ and $t, s \in [a; \infty)$. We define

$$J_0(t,s) = I_0(t,s) = 1$$

$$I_k(t,s; y_{i_k}, \dots, y_{i_1}) = \int_s^t y_{i_k}(x) I_{k-1}(x, s; y_{i_{k-1}}, \dots, y_{i_1})\,dx,$$

$$J_k(t, s; y_{i_k}, \dots, y_{i_1}) = \int_s^t y_{i_1}(x) J_{k-1}(t, x; y_{i_k}, \dots, y_{i_2})\,dx.$$

We introduce the notation for $t \geqslant a$:

$$\pi_0^i(t) = \alpha_0^i(t) = \varrho_0(t) = 1$$

$$\pi_{2j+1}^i(t) = J_{2j+1}\left(\infty, t; \frac{1}{p_{i+j}}, a_{i+j-1}, \dots, a_i, \frac{1}{p_i}\right)$$

$$\pi_{2j+2}^i(t) = J_{2j+2}\left(\infty, t; a_{i+j}, \frac{1}{p_{i+j}}, \dots, a_i, \frac{1}{p_i}\right), \ 0 \leqslant j \leqslant n-i-1,\ 1 \leqslant i \leqslant n-1;$$

$$\pi_{2n-2i+1}^i(t) = J_{2n-2i}\left(\infty, t; a_{n-1}(\pi_1^n(\tau_n))^\beta, \frac{1}{p_{n-1}}, \dots, a_i, \frac{1}{p_i}\right), \ 1 \leqslant i \leqslant n-1,$$

$$\pi_1^n(t) = J_1\left(\infty, t; \frac{1}{p_n}\right),$$

$$\alpha_{2j+1}^i(t) = J_{2j+1}\left(\infty, t; a_{i-j}, \frac{1}{p_{i-j+1}}, \dots, \frac{1}{p_i}, a_i\right), \ i \leqslant i \leqslant n-1,\ 0 \leqslant j \leqslant i-1;$$

$$\alpha_{2j+2}^i(t) = J_{2j+2}\left(\infty, t; \frac{1}{p_{i-j}}, a_{i-j}, \dots, \frac{1}{p_i}, a_i\right), \ 1 \leqslant i \leqslant n-2,\ 0 \leqslant j \leqslant i-1,$$

$$\varrho_{2j+1}(t) = J_{2j+1}\left(\infty, t; \frac{1}{p_{n-j}}, a_{n-j}, \dots, a_{n-1}, \frac{1}{p_n}\right), \ 0 \leqslant j \leqslant n-1,$$

$$\varrho_{2j+2}(t) = J_{2j+2}\left(\infty, t; a_{n-j-1}, \frac{1}{p_{n-j}}, \dots, a_{n-1}, \frac{1}{p_n}\right), \ 0 \leqslant j \leqslant n-2.$$

Lemma 1. *Let* (a), (b) *be valid. Then*

1) $\lim\limits_{t\to\infty} \pi_k^i(t) = 0$, $1 \leqslant i \leqslant n$, $1 \leqslant k \leqslant 2n-2i+1$,

$\lim\limits_{t\to\infty} \alpha_k^i(t) = 0$, $1 \leqslant i \leqslant n-1$, $1 \leqslant k \leqslant 2i-1$;

2) $\lim_{t\to\infty} \frac{\pi_k^i(t)}{\pi_j^i(t)} = 0,\ k > j,\quad k,j = 1, \ldots, 2n-2i+1,\ 1 \leqslant i \leqslant n.$

Proof. From (a), (b) we see that $\lim_{t\to\infty} \pi_1^i(t) = \lim_{t\to\infty} \alpha_1^i(t) = 0,\ 1 \leqslant i \leqslant n,$ $1 \leqslant j \leqslant n-1$. Let for some $k \in \{1, \ldots, 2(n-i)\}$, $1 \leqslant i \leqslant n-1$ $\lim_{t\to\infty} \pi_k^i(t) = 0$ hold. If k is even, i.e. $k = 2m$, then

$$\pi_{k+1}^i(t) = J_{2m+1}\left(\infty, t; \frac{1}{p_{i+m}}, a_{i+m-1}, \ldots, a_i, \frac{1}{p_i}\right) \leqslant \pi_1^{i+m}(t)\,\pi_{2m}^i(t) \to 0 \text{ for } t \to \infty.$$

If k is odd, i.e. $k = 2m+1$, then

$$\pi_{k+1}^i(t) = J_{2m+2}\left(\infty, t; a_{i+m}, \frac{1}{p_{i+m}}, \ldots, a_i, \frac{1}{p_i}\right) \leqslant \alpha_1^{i+m}(t)\,\pi_{2m+1}^i(t) \to 0 \text{ for } t \to \infty.$$

Analogously it can be shown that $\alpha_k^i(t)$ has the property 1). Using the l'Hospital theorem and the properties 1) we prove the case 2).

Corollary 1. *For $t_0 \geqslant a$ there exist constants A_j, B_k such that*

$$\varrho_j(t) \leqslant A_j \varrho_{j-1}(t),\ \pi_k^i(t) \leqslant B_k \pi_{k-1}^i(t),\ 1 \leqslant i \leqslant n,\ 1 \leqslant j \leqslant 2n-2,\ 1 \leqslant k \leqslant 2n-2i+1.$$

Lemma 2. *Let $x(t) = (x_1(t), \ldots, x_n(t))$ be a solution of* (A) *on the interval* $[a; \infty)$. *Then the following relations hold for $t \geqslant T \geqslant a$:*

$$\begin{aligned}|x_i(t)| \leqslant{} & \sum_{j=0}^{k-i-1} |x_{i+j}(T)|\, I_{2j}\left(t, T; \frac{1}{p_i}, a_i, \frac{1}{p_{i+1}}, a_{i+1}, \ldots, a_{i+j-1}\right) + \\ & + \sum_{j=0}^{k-i-1} |p_{i+j}(T)\, x'_{i+j}(T)|\, I_{2j+1}\left(t, T; \frac{1}{p_i}, a_i, \ldots, a_{i+j-1}, \frac{1}{p_{i+j}}\right) + \\ & + I_{2(k-i)}\left(t, T; \frac{1}{p_i}, a_i, \frac{1}{p_{i+1}}, \ldots, \frac{1}{p_{k-1}}, a_{k-1}|x_k|\right),\end{aligned} \tag{1}$$

$1 \leqslant i \leqslant n-2,\ i+1 \leqslant k \leqslant n-1;$

$$\begin{aligned}|x_i(t)| \leqslant{} & \sum_{j=0}^{n-i-1} |x_{i+j}(T)|\, I_{2j}\left(t, T; \frac{1}{p_i}, a_i, \frac{1}{p_{i+1}}, \ldots, a_{i+j-1}\right) + \\ & + \sum_{j=0}^{n-i-1} |p_{i+j}(T)\, x'_{i+j}(T)|\, I_{2j+1}\left(t, T; \frac{1}{p_i}, a_i, \ldots, \frac{1}{p_{i+j}}\right) + \\ & + M^{\beta} K I_{2n-2i}\left(t, T; \frac{1}{p_i}, a_i, \ldots, \frac{1}{p_{n-1}}, a_{n-1}\right) +\end{aligned} \tag{2}$$

$$+ NK\int_T^t \omega(s, |x_1(\tau_1(s))|)\, J_{2n-2i+1}\left(t, s; \frac{1}{p_i}, a_i, \ldots, a_{n-1}, \frac{1}{p_n}\right) ds,$$

$$1 \leqslant i \leqslant n-1,\; M = |x_n(T)| + |p_n(T)\, x_n'(T)|\, \pi_1^n(T),\; N = \beta M^{\beta-1}.$$

Proof. Let us integrate the first $(n-2)$ equations of (A) from $T \geqslant a$ to $t \geqslant T$ we get

$$|x_i(t)| \leqslant |x_i(T)| + |p_i(T)\, x_i'(T)|\, I_1\left(t, T; \frac{1}{p_i}\right) + \int_T^t \frac{1}{p_i(u)} \int_T^u a_i(s)\, |x_{i+1}(s)|\, ds\, du, \quad 1 \leqslant i \leqslant n-2, \tag{3}$$

which are the inequalities (1) (with $1 \leqslant i \leqslant n-2, k = i+1$). By substituting for $|x_{i+1}(t)|$ in (3) successively we have the inequalities (1).

From the $(n-1)$st and nth equations of (A) we have for $t \geqslant T \geqslant a$

$$|x_{n-1}(t) \leqslant |x_{n-1}(T)| + |p_{n-1}(T)\, x_{n-1}'(T)|\, I_1\left(t, T; \frac{1}{p_{n-1}}\right) + K\int_T^t \frac{1}{p_{n-1}(u)} \int_T^u a_{n-1}(s)\, |x_n(\tau_n(s))|^\beta\, ds\, du, \tag{4}$$

$$|x_n(t) \leqslant |x_n(T)| + |p_n(T)\, x_n'(T)|\, \pi_1^n(T) + \int_T^t \frac{1}{p_n(s)} \int_T^s \omega(u, |x_1(\tau_1(u))|)\, du\, ds \tag{5}$$

and using the Taylor theorem

$$|x_n(\tau_n(t))|^\beta \leqslant M^\beta + N\int_T^t \frac{1}{p_n(s)} \int_T^s \omega(u, |x_1(\tau_1(u))|)\, du\, ds,$$

for $t \geqslant T_1 \geqslant T$ such that $\tau_n(t) \geqslant T$ for $t \geqslant T_1$ and from (4)

$$|x_{n-1}(t)| \leqslant |x_{n-1}(T_1)| + |p_{n-1}(T_1)\, x_{n-1}'(T_1)|\, I_1\left(t, T_1; \frac{1}{p_{n-1}}\right) +$$

$$+ KM^\beta I_2\left(t, T_1; \frac{1}{p_{n-1}}, a_{n-1}\right) + KNI_4\left(t, T_1; \frac{1}{p_{n-1}}, a_{n-1}, \frac{1}{p_n}, \omega(\cdot, |x_1(\tau_1(\cdot))|\right).$$

When we substitute the last inequality for $|x_{n-1}(t)|$ in (1) (with $k = n-1$, $T = T_1$), at the end we get (2).

The main result of this paper is the following theorem, which describes possible behaviours of all solutions of (A).

Theorem 1. *Let* (a)—(e) *be satisfied and*

$$\int^{\infty} \omega(t, c\pi^1_{k-1}(\tau_1(t)))\, \varrho_{2n-k-1}(t)\, \mathrm{d}t < \infty \quad \text{for } k = 1, \ldots, 2n-1 \tag{6}$$

and for all $c > 0$. If $(x_1(t), \ldots, x_n(t))$ is a solution of (A), then exactly one of the following cases holds:

(I) $\limsup_{t\to\infty} |x_i(t)| = \infty, \quad i = 1, \ldots, n;$

(II) there exist an integer k, $0 \leqslant k \leqslant 2n-3$ and a nonzero number b_k such that

$$\lim_{t\to\infty} \frac{x_i(t)}{\pi^i_{k-2i+2}(t)} = b_k, \quad \lim_{t\to\infty} x_j(t)\, \alpha^{j-1}_{2j-k-2}(t) = 0, \quad 1 \leqslant i \leqslant \left[\frac{k}{2}\right] + 1 < j \leqslant n;$$

(III) there exists a nonzero number b_{2n-2} such that

$$\lim_{t\to\infty} x_n(t) = b_{2n-2}, \quad \lim_{t\to\infty} \frac{x_i(t)}{\pi^i_{2n-2i}(t)} = g(b_{2n-2}), \quad 1 \leqslant i \leqslant n-1;$$

(IV) there exists a nonzero number b_{2n-1} such that

$$\lim_{t\to\infty} \frac{x_n(t)}{\pi^n_1(t)} = b_{2n-1}, \quad \limsup_{t\to\infty} \frac{|x_i(t)|}{\pi^i_{2n-2i+1}(t)} < \infty, \quad 1 \leqslant i \leqslant n-1$$

(V) $\lim_{t\to\infty} \dfrac{x_i(t)}{\pi^i_{2n-2i+1}(t)} = 0, \quad 1 \leqslant i \leqslant n.$

Proof. Let $(x_1(t), \ldots, x_n(t))$ be a solution of (A) defined on $[t_0; \infty)$ and let $t_1 \geqslant t_0$ such that $\min(\tau_1(t), \tau_n(t)) \geqslant t_0$ for $t \geqslant t_1$. We shall show, if $\limsup_{t\to\infty} |x_1(t)| = \infty$, then $\limsup_{t\to\infty} |x_i(t)| = \infty$ for $i = 2, \ldots, n$. Suppose the contrary. Then there exists an integer j, $2 \leqslant j \leqslant n$ such that $\limsup_{t\to\infty} |x_j(t)| < \infty$, which means there exist a $T \geqslant t_1$ and a positive constatnt L such that $|x_j(t)| \leqslant L$ for $t \geqslant T$ (if $j = n$, let a T be such that $|x_n(\tau_n(t))| \leqslant L$ for $t \geqslant T$ too). From (1) with $i = 1$, $k = j$ (resp. (2) with $j = n$) by Lemma 1 we get that $\limsup_{t\to\infty} |x_1(t)| < \infty$, which contradicts the assumption.

Now, let $\limsup_{t\to\infty} |x_1(t)| < \infty$, then there exist a positive constant c and a $T \geqslant t_1$ such that $|x_1(t)| \leqslant c$, $|x_1(\tau_1(t))| \leqslant c$ for $t \geqslant T$. First we assume that (6) for $k = 1$ holds. Using Lemma 1, (2) (for $i = 2$) we can easily show that

$$\left| \int_T^{\infty} \frac{1}{p_1(u)} \int_T^u a_1(s)\, x_2(s)\, \mathrm{d}s\, \mathrm{d}u \right| < \infty$$

and from the first equation of (A) we get for $t \geqslant T$

$$x_1(t) = b_0 - p_1(T)\, x_1'(T)\, \pi_1^1(t) - \int_t^\infty \frac{1}{p_1(u)} \int_T^u a_1(s)\, x_2(s)\, ds\, du, \tag{7}$$

where

$$b_0 = x_1(T) + p_1(T)\, x_1'(T)\, \pi_1^1(T) + \int_T^\infty \frac{1}{p_1(u)} \int_T^u a_1(s)\, x_2(s)\, ds\, du.$$

So, we have $\lim_{t \to \infty} x_1(t) = b_0$ and by (2) and corollary 1

$$\alpha_{2i-2}^{i-1}(t)\, |x_i(t)| \leqslant \alpha_{2i-2}^{i-1}(t) \Bigg[\sum_{j=0}^{n-i-1} |x_{i+j}(T)|\, I_{2j}\left(\infty, T; \frac{1}{p_i}, a_i, \ldots, a_{i+j-1}\right) +$$

$$+ \sum_{j=0}^{n-i-1} |p_{i+j}(T)\, x_{i+j}'(T)|\, I_{2j+1}\left(\infty, T; \frac{1}{p_i}, a_i, \ldots, \frac{1}{p_{i+j}}\right) + M^\beta K \alpha_{2n-2i}^{n-1}(T) +$$

$$+ NK \int_T^{T_1} \omega(s, c)\, \varrho_{2n-2i+1}(s)\, ds \Bigg] + NKA_{2n-1} \int_{T_1}^t \omega(s, c)\, \varrho_{2n-2}(s)\, ds$$

$t \geqslant T_1 \geqslant T$, $i = 2, \ldots, n-1$. The right-hand side of this inequality can be made arbitrarily small by taking T_1 sufficiently large and then letting t without bound. Thus $\lim_{t \to \infty} \alpha_{2i-2}^{i-1}(t)\, x_i(t) = 0$, $i = 2, \ldots, n-1$. Analogously we get from (5)

$$\alpha_{2n-2}^{n-1}(t)\, |x_n(t)| \leqslant \alpha_{2n-2}^{n-1}(t) \left[M + \int_T^{T_1} \frac{1}{p_n(s)} \int_T^s \omega(u, c)\, du\, ds \right] +$$

$$+ A_{2n-1} \int_{T_1}^t \omega(u, c)\, \varrho_{2n-2}(u)\, du, \quad t \geqslant T_1$$

which implies $\lim_{t \to \infty} \alpha_{2n-2}^{n-1}(t)\, x_n(t) = 0$. Hence Case (II) for $k = 0$, $b_0 \neq 0$ occurs.

Now, let $b_0 = 0$. We shall show that if for any integer k, $0 \leqslant k \leqslant 2n-4$, $b_k = 0$ and (6) (with $k+1$) hold, then the components of the solution of (A) are subject to Case (II) with $k+1$.

Let us consider the following two cases:

a) Let k be even, i.e. $k = 2m$. The components of the solution of (A) have these properties for $t \geqslant T$

$$x_i(t) = \int_t^\infty \frac{1}{p_i(s)} \int_s^\infty a_i(u)\, x_{i+1}(u)\, du\, du, \quad i = 1, 2, \ldots, m, \tag{8}$$

$$x_{m+1}(t) = -p_{m+1}(T)\,x'_{m+1}(T)\,\pi_1^{m+1}(t) - \int_t^\infty \frac{1}{p_{m+1}(s)} \int_T^s a_{m+1}(u)\,x_{m+2}(u)\,\mathrm{d}u\,\mathrm{d}s \tag{9}$$

$$x_j(t) = x_j(T) + p_j(T)\,x'_j(T) \int_T^t \frac{\mathrm{d}s}{p_j(s)} + \int_T^t \frac{1}{p_j(s)} \int_T^s a_j(u)\,x_{j+1}(u)\,\mathrm{d}u\,\mathrm{d}s \tag{10}$$

$$j = m+2, \ldots, n-2,$$

$$x_{n-1}(t) = x_{n-1}(T) + p_{n-1}(T)\,x'_{n-1}(T) \int_T^t \frac{\mathrm{d}s}{p_{n-1}(s)} + \int_T^t \frac{1}{p_{n-1}(s)} \int_T^s a_{n-1}(u)\,g(x_n(\tau_n(u)))\,\mathrm{d}u\,\mathrm{d}s \tag{11}$$

$$x_n(t) = x_n(T) + p_n(T)\,x'_n(T) \int_T^t \frac{1}{p_n(s)} + \int_T^t \frac{1}{p_n(s)} \int_T^s f(u, x_1(\tau_1(u)))\,\mathrm{d}u\,\mathrm{d}s. \tag{12}$$

(8), (9) give using (2) (with $i = m+2$)

$$|x_1(t)| \leqslant \pi_{2m+1}^1(t) \left[|p_{m+1}(T)\,x'_{m+1}(T)| + \int_T^\infty a_{m+1}(s)\,|x_{m+2}(s)|\,\mathrm{d}s \right] \leqslant$$

$$\leqslant \pi_{2m+1}^1(t) \Bigg[\sum_{i=0}^{n-m-2} |p_{m+i+1}(T)\,x'_{m+i+1}(T)|\, I_{2i}\left(\infty, T; a_{m+1}, \frac{1}{p_{m+2}}, \ldots, \frac{1}{p_{m+i+1}}\right) +$$

$$+ \sum_{i=0}^{n-m-3} |x_{m+i+2}(T)|\, \alpha_{2i+1}^{m+i+1}(T) + KM^\beta \alpha_{2n-2m-3}^{n-1}(T) +$$

$$+ KN \int_T^\infty \omega(s, |x_1(\tau_1(s))|\, \varrho_{2n-2m-2}(s)\,\mathrm{d}s \Bigg], \quad t \geqslant T.$$

By Lemma 1 there exists a positive constant A such that

$$\frac{|x_1(t)|}{\pi_{2m+1}^1(t)} \leqslant A + KN \int_T^\infty \omega(s, |x_1(\tau_1(s))|)\, \varrho_{2n-2m-2}(s)\,\mathrm{d}s, \quad t \geqslant T. \tag{13}$$

We shall show that $x_1(t) = O(\pi_{2m+1}^1(t))$ as $t \to \infty$. Since $x_1(t) = o(\pi_{2m}^1(t))$ as $t \to \infty$ and $\pi_{2m+1}^1(t) = o(\pi_{2m}^1(t))$ as $t \to \infty$ by Lemma 1 we can choose a $T_1 \geqslant T$ such that $T_0 = \inf_{s \geqslant T_1} (\min(\tau_1(s), s)) \geqslant T$, $|x_1(\tau_1(t))| \leqslant \pi_{2m}^1(\tau_1(t))$, $\pi_{2m}^1(\tau_1(t)) \geqslant \pi_{2m+1}^1(\tau_1(t))$ for $t \geqslant T_0$ and

$$\int_{T_1}^\infty \omega(s, \pi_{2m}^1(\tau_1(s)))\, \varrho_{2n-2m-2}(s)\,\mathrm{d}s \leqslant \frac{1}{4KN}.$$

Let us define

$$u_{2m}(t) = \sup_{s \geqslant t} \frac{|x_1(s)|}{\pi^1_{2m}(s)}, \quad t \geqslant T_0. \tag{14}$$

When we use (14) and the monotony of function $\omega(t, z)/z$, we obtain from (13)

$$\begin{aligned} \frac{\pi^1_{2m}(t)}{\pi^1_{2m+1}(t)} u_{2m}(t) \leqslant A + KN \int_T^{T_1} \omega(s, |x_1(\tau_1(s))| \varrho_{2n-2m-2}(s) \, ds + \\ + KN \int_{T_1}^{\infty} \omega(s, \pi^1_{2m}(\tau_1(s))) \, u_{2m}(\tau_1(s)) \, \varrho_{2n-2m-2}(s) \, ds. \end{aligned} \tag{15}$$

For each $t \geqslant T_1$ let I_t, J_t denote the sets

$$t_t = \{s \in [T_1; \infty), \ \tau_1(s) \leqslant t\}, \qquad J_t = \{s \in [T_1; \infty), \ \tau_1(s) > t\}.$$

Since $u_{2m}(\tau_1(s)) \leqslant u_{2m}(t)$ for $s \in J_t$;

$$\frac{\pi^1_{2m}(\tau_1(s)) \, u_{2m}(\tau_1(s))}{\pi^1_{2m+1}(\tau_1(s))} \leqslant \sup_{T_0 \leqslant v \leqslant t} \frac{\pi^1_{2m}(v) \, u_{2m}(v)}{\pi^1_{2m+1}(v)} \quad \text{for} \quad s \in I_t$$

the right-hand side of (15) is bounded from above by

$$A^* + KN \sup_{T_0 \leqslant v \leqslant t} \frac{\pi^1_{2m}(v) \, u_{2m}(v)}{\pi^1_{2m+1}(v)} \int_{I_t \cap [T_1; \infty)} \omega(s, \pi^1_{2m}(\tau_1(s))) \frac{\pi^1_{2m+1}(\tau_1(s))}{\pi^1_{2m}(\tau_1(s))} \varrho_{2n-2m-2}(s) \, ds +$$

$$+ KNu_{2m}(t) \int_{J_t \cap [T_1; \infty)} \omega(s, \ \pi^1_{2m}(\tau_1(s))) \, \varrho_{2n-2m-2}(s) \, ds \leqslant$$

$$\leqslant A^* + KN \sup_{T_0 \leqslant v \leqslant t} \frac{\pi^1_{2m}(v) \, u_{2m}(v)}{\pi^1_{2m+1}(v)} \int_{T_1}^{\infty} \omega(s, \pi^1_{2m}(\tau_1(s))) \, \varrho_{2n-2m-2}(s) \, ds +$$

$$+ KN \frac{\pi^1_{2m}(t) \, u_{2m}(t)}{\pi^1_{2m+1}(t)} \int_{T_1}^{\infty} \omega(s, \ \pi^1_{2m}(\tau_1(s))) \, \varrho_{2n-2m-2}(s) \, ds \leqslant$$

$$\leqslant A^* + \frac{1}{4} \sup_{T_0 \leqslant v \leqslant t} \frac{\pi^1_{2m}(v) \, u_{2m}(v)}{\pi^1_{2m+1}(v)} + \frac{1}{4} \frac{\pi^1_{2m}(t) \, u_{2m}(t)}{\pi^1_{2m+1}(t)}, \qquad t \geqslant T_1,$$

where $A^* = A + KN \int_T^{T_1} \omega(s, |x_1(\tau_1(s))|) \, \varrho_{2n-2m-2}(s) \, ds$. From this last inequality we get

$$\frac{3}{4} \frac{\pi^1_{2m}(t) \, u_{2m}(t)}{\pi^1_{2m+1}(t)} \leqslant \bar{A} + \frac{1}{4} \sup_{T_1 \leqslant v \leqslant t} \frac{\pi^1_{2m}(v) \, u_{2m}(v)}{\pi^1_{2m+1}(v)}, \ t \geqslant T_1, \ \bar{A} > 0, \text{ const.}$$

and hence

$$\frac{|x_1(t)|}{\pi^1_{2m+1}(t)} \leqslant \sup_{T_1 \leqslant v \leqslant t} \frac{\pi^1_{2m}(v)\, u_{2m}(v)}{\pi^1_{2m+1}(v)} \leqslant 2\bar{A}, \quad t \geqslant T_1.$$

So $x_1(t) = O(\pi^1_{2m+1}(t))$ as $t \to \infty$ and (6) (with $k = 2m + 1$) yields that the function $\omega(t, \pi^1_{2m+1}(\tau_1(t)))\, \varrho_{2n-2m-2}(t)$ is integrable at ∞. Taking into account (2) (for $i = m + 2$) we get by Lemma 1 that $a_{m+1}(t)\, x_{m+2}(t) \in L_1[T; \infty)$ and the $(m + 1)$st equation of (A) implies

$$p_{m+1}(t)\, x'_{m+1}(t) = -b_{2m+1} - \int_t^\infty a_{m+1}(s)\, x_{m+2}(s)\, ds, \quad t \geqslant T$$

where

$$-b_{2m+1} = p_{m+1}(T)\, x'_{m+1}(T) + \int_T^\infty a_{m+1}(s)\, x_{m+2}(s)\, ds.$$

We transform (9)

$$x_{m+1}(t) = -p_{m+1}(T)\, x'_{m+1}(T)\, \pi_1^{m+1}(t) -$$

$$- \int_t^\infty \frac{1}{p_{m+1}(s)} \left[\int_T^\infty a_{m+1}(u)\, x_{m+2}(u)\, du - \int_s^\infty a_{m+1}(u)\, x_{m+2}(u)\, du \right] ds = \quad (16)$$

$$= b_{2m+1}\pi_1^{m+1}(t) + \int_t^\infty \frac{1}{p_{m+1}(s)} \int_s^\infty a_{m+1}(u)\, x_{m+2}(u)\, du\, ds, \quad t \geqslant T.$$

So we have $\lim\limits_{t \to \infty} \dfrac{x_{m+1}(t)}{\pi_1^{m+1}(t)} = b_{2m+1}$. Taking this into consideration we have from (8) $\lim\limits_{t \to \infty} \dfrac{x_i(t)}{\pi^i_{2m-2i+3}(t)} = b_{2m+1}$, $i = 1, \ldots, m$. As in the previous case we can easily show using (2) (for $i = m + 2, \ldots, n - 1$) and (5) that $\lim\limits_{t \to \infty} x_i(t)\, \alpha^{i-1}_{2i-2m-3}(t) = 0$, $i = m + 2, \ldots, n$. Hence case (II) can occur for $k + 1$.

b) Now, let k be odd, i.e. $k = 2m + 1$ and $b_k = 0$. The components of the solution of (A) satisfy (8), (10), (11), (12) and from (16) we have for the $(m + 1)$st component

$$x_{m+1}(t) = \int_t^\infty \frac{1}{p_{m+1}(s)} \int_s^\infty a_{m+1}(u)\, x_{m+2}(u)\, du\, ds, \quad t \geqslant T. \quad (17)$$

By transforming (8), (10) (for $j = m + 2$), (17) and (2) (for $i = m + 3$) we get

$$|x_1(t)| \leqslant \pi^1_{2m+2}(t) \left[\sum_{i=0}^{n-m-3} |x_{m+i+2}(T)\, \alpha_{2i}^{m+i+1}(T) + \right.$$

$$+\sum_{i=0}^{n-m-3} |p_{m+i+2}(T)\, x'_{m+i+2}(T)|\, I_{2i+1}\left(\infty,\, T;\, \frac{1}{p_{m+2}},\, a_{m+2},\, \ldots,\, \frac{1}{p_{m+2+i}}\right) +$$

$$+ KM^{\beta} \alpha_{2n-2m-4}^{n-1}(T) + KN \int_T^{\infty} \omega(s,\, |x_1(\tau_1(s))|)\, \varrho_{2n-2m-3}(s)\, \mathrm{d}s \Bigg]$$

from which by Lemma 1

$$\frac{|x_1(t)|}{\pi^1_{2m+2}(t)} \leqslant \bar{A}_2 + KN \int_T^{\infty} \omega(s,\, |x_1(\tau_1(s))|)\, \varrho_{2n-2m-3}(s)\, \mathrm{d}s, \tag{18}$$

$t \geqslant T$, $\bar{A}_2$ is a suitable positive constant. We shall show that $x_1(t) = O(\pi^1_{2m+2}(t))$ as $t \to \infty$ if (6) (with $k = 2m + 2$) holds. We know that $x_1(t) = o(\pi^1_{2m+1}(t))$, $\pi^1_{2m+2}(t) = o(\pi^1_{2m+1}(t))$ as $t \to \infty$. Defining

$$u_{2m+1}(t) = \sup_{s \geqslant t} \frac{|x_1(s)|}{\pi^1_{2m+1}(s)}, \quad t \geqslant T_0$$

and applying the same type of arguments that we used to prove the case a) we conclude from the last inequality (18) that $x_1(t) = O(\pi^1_{2m+2}(t))$ as $t \to \infty$. Owing to (6), the nondecreasing of the function $\omega(t, z)$, Lemma 1 and (2) (with $i = m + 3$) the integrals

$$\int_T^{\infty} \omega(t,\, \pi^1_{2m+2}(\tau_1(t)))\, \varrho_{2n-2m-3}(t)\, \mathrm{d}t, \qquad \int_T^{\infty} \frac{1}{p_{m+2}(s)} \int_T^{s} a_{m+2}(u)\, x_{m+3}(u)\, \mathrm{d}u\, \mathrm{d}s$$

are convergent. Therefore we get from the $(m + 2)$nd equation of (A)

$$x_{m+2}(t) = b_{2m+2} - p_{m+2}(T)\, x'_{m+2}(T)\, \pi_1^{m+2}(t) -$$

$$- \int_t^{\infty} \frac{1}{p_{m+2}(s)} \int_T^{s} a_{m+2}(u)\, x_{m+3}(u)\, \mathrm{d}u\, \mathrm{d}s, \quad t \geqslant T$$

where

$$b_{2m+2} = x_{m+2}(T) + p_{m+2}(T)\, x'_{m+2}(T)\, \pi_1^{m+2}(T) +$$

$$+ \int_T^{\infty} \frac{1}{p_{m+2}(s)} \int_T^{s} a_{m+2}(u)\, x_{m+3}(u)\, \mathrm{d}u\, \mathrm{d}s,$$

and hence $\lim_{t \to \infty} x_{m+2}(t) = b_{2m+2}$ and (8), (17) yield

$$\lim_{t \to \infty} \frac{x_i(t)}{\pi^i_{2m-2i+4}(t)} = b_{2m+2}, \quad i = 1,\, \ldots,\, m+1.$$

As described above we can show using (2) (for $i = m + 3, \ldots, n - 1$) and (5) that

$\lim_{t\to\infty} x_i(t)\,\alpha_{2i-2m-4}^{i-1}(t) = 0$, $i = m + 3, \ldots, n$. So the solution of (A) is subject to Case (II) for $k = 2m + 2$, $0 \leqslant k < 2n - 2$.

Now we shall show that Case (III) can occur. Let $b_{2n-3} = 0$ and the assumption (6) hold. The components of a solution of (A) satisfying (8) (for $i = 1, \ldots, n-2$), (12) and

$$x_{n-1}(t) = \int_t^\infty \frac{1}{p_{n-1}(s)} \int_s^\infty a_{n-1}(u)\, g(x_n(\tau_n(u)))\, du\, ds, \quad t \geqslant T. \tag{19}$$

Using these properties we get

$$\frac{|x_1(t)|}{\pi_{2n-1}^1(t)} \leqslant KM^\beta + KN \int_T^\infty \omega(s, |x_1(\tau_1(s))|)\, \varrho_1(s)\, ds, \quad t \geqslant T.$$

Repeating the procedure used in the previous cases, defining the function

$$u_{2n-3}(t) = \sup_{s \geqslant t} \frac{|x_1(s)|}{\pi_{2n-3}^1(s)}, \quad t \geqslant T_0$$

we obtain that $x_1(t) = 0(\pi_{2n-2}^1(t))$ as $t \to \infty$. Therefore

$$\left| \int_T^\infty \frac{1}{p_n(s)} \int_T^s f(u, x_1(\tau_1(u)))\, du\, ds \right| \leqslant \int_T^\infty \omega(s, c\pi_{2n-3}^1(\tau_1(s)))\, \varrho_1(s)\, ds < \infty$$

c is a suitable positive constant. From the nth equation of (A) we have

$$x_n(t) = b_{2n-2} - p_n(T)\, x_n'(T)\, \pi_1^n(t) - \int_t^\infty \frac{1}{p_n(s)} \int_T^s f(u, x_1(\tau_1(u)))\, du\, ds, \tag{20}$$

$t \geqslant T$, where

$$b_{2n-2} = x_n(T) + p_n(T)\, x_n'(T)\, \pi_1^n(T) + \int_T^\infty \frac{1}{p_n(s)} \int_T^s f(u, x_1(\tau_1(u)))\, du\, ds$$

and therefore $\lim_{t\to\infty} x_n(t) = b_{2n-2}$. Owing to a continuity of the functions g and τ_n $\lim_{t\to\infty} g(x_n(\tau_n(t))) = g(b_{2n-2})$ and (19) yields $\lim_{t\to\infty} \dfrac{x_{n-1}(t)}{\pi_2^{n-1}(t)} = g(b_{2n-2})$. Further from (8) we have $\lim_{t\to\infty} \dfrac{x_i(t)}{\pi_{2n-2i}^i(t)} = g(b_{2n-2})$, $i = 1, \ldots, n-1$, which is Case (III) for $b_{2n-2} \neq 0$.

If $b_{2n-2} = 0$ and (6) (with $k = 2n - 1$) hold, we shall show that case (IV) can occur. From (20) with $b_{2n-2} = 0$ we have

$$|x_n(t)| \leqslant \pi_1^n(t)\left[|p_n(T)\,x_n'(T)| + \int_T^\infty \omega(u, |x_1(\tau_1(u))|)\,\mathrm{d}u\right], \quad t \geqslant T$$

and hence

$$\begin{aligned}|x_n(\tau_n(t))|^\beta \leqslant (\pi_1^n(\tau_n(t))^\beta\Big[&|p_n(T_1)\,x_n'(T_1)|^\beta + \\ &+ \beta|p_n(T_1)\,x_n'(T_1)|^{\beta-1}\int_{T_1}^\infty \omega(u, |x_1(\tau_1(u))|)\,\mathrm{d}u\Big]\end{aligned} \tag{21}$$

for $t \geqslant T_1 \geqslant T$ such that $\tau_n(t) \geqslant T$ for $t \geqslant T_1$. Combining (8), for $i = 1, \ldots, n-2$ (19), (21) we obtain

$$\frac{|x_1(t)|}{\pi_{2n-1}^1(t)} \leqslant |p_n(T_1)\,x_n'(T_1)|^\beta + \beta|p_n(T_1)\,x_n'(T_1)|^{\beta-1}\int_{T_1}^\infty \omega(u, |x_1(\tau_1(u))|)\,\mathrm{d}u, \quad t \geqslant T_1.$$

By taking into account (6) (with $k = 2n-1$), defining a function

$$u_{2n-1}(t) = \sup_{s \geqslant t} \frac{|x_1(s)|}{\pi_{2n-2}^1(s)}, \quad t \geqslant T_0$$

and using the method described earlier we show that $x_1(t) = 0(\pi_{2n-1}^1(t))$ as $t \to \infty$ and therefore $f(t, x_1(\tau_1(t))) \in L_1[T_1; \infty)$. The nth equation of (A) implies

$$p_n(t)\,x_n'(t) = -b_{2n-1} - \int_t^\infty f(s, x_1(\tau_1(s)))) \,\mathrm{d}s, \quad t \geqslant T_1,$$

where $-b_{2n-1} = p_n(T_1)\,x_n'(T_1) + \int_{T_1}^\infty f(s, x_1(\tau_1(s)))\,\mathrm{d}s$. From (20) with $b_{2n-2} = 0$, $T = T_1$ we get

$$\begin{aligned}x_n(t) &= -p_n(T_1)\,x_n'(T_1)\,\pi_1^n(t) - \\ &- \int_t^\infty \frac{1}{p_n(s)}\left[\int_{T_1}^\infty f(u, x_1(\tau_1(u)))\,\mathrm{d}u - \int_s^\infty f(u, x_1(\tau_1(u)))\,\mathrm{d}u\right]\mathrm{d}s = \\ &= b_{2n-1}\pi_1^n(t) + \int_t^\infty \frac{1}{p_n(s)}\int_s^\infty f(u, x_1(\tau_1(u)))\,\mathrm{d}u\,\mathrm{d}s, \quad t \geqslant T_1\end{aligned}$$

which yields $\lim_{t\to\infty} \frac{x_n(t)}{\pi_1^n(t)} = b_{2n-1}$. It means that $x_n(t) = 0(\pi_1^n(t))$ as $t \to \infty$ and from (19) we obtain $\frac{|x_{n-1}(t)|}{\pi_3^{n-1}(t)} \leqslant K \cdot c$, c is a suitable positive constant, $t \geqslant T_2$, T_2 is sufficiently large, i.e. $\limsup_{t\to\infty} \frac{|x_{n-1}(t)|}{\pi_3^{n-1}(t)} < \infty$. Analogously it can be easily shown by using (8) (for $i = 1, \ldots, n-2$) that $\limsup_{t\to\infty} \frac{|x_i(t)|}{\pi^i_{2n-2i+1}(t)} < \infty$ $i = 1, \ldots, n-2$. Case (V) of the theorem occurs for $b_{2n-1} = 0$. Thus the proof of Theorem 1 is complete.

Corollary. *Let all assumptions of* Theorem 1 *be fulfilled and let* $g(u) = |u|^\beta \operatorname{sgn} u$. *then Cases* (I), (II), (III), (V) *and*

$$\text{(IV}'\text{)} \qquad \lim_{t\to\infty} \frac{x_i(t)}{\pi^i_{2n-2i+1}(t)} = b_{2n-1} \neq 0, \quad i = 1, \ldots, n,$$

hold.

The following theorem describes some properties of all oscillatory solutions of (A) for which the following assumption is needed.

(f) Let there exist a continuous increasing function η on $[a; \infty)$ such that $\eta(t) \leqslant \tau_n(t)$, $\lim_{t\to\infty} \eta(t) = \infty$.

We use the notation $h(t) = \inf_{s \geqslant \eta(t)} \{\min(\tau_1(s), s)\}$. We say that the condition (G) is satisfied if there exists a sequence $\{t_n\}_{n=1}^\infty$ such that $t_n \to \infty$ as $n \to \infty$ and $h(t_n) = t_n$ for $n = 1, 2, \ldots$.

Theorem 2. *Assume that* $\beta = 1$ *and* (a)—(f), (G) *are satisfied. If* (6) *for* $k = 1, \ldots, 2n-1$ *hold, then every oscillatory solution* $(x_1(t), \ldots, x_n(t))$ *of* (A) *has the property of Case* (I).

Proof. Let $(x_1(t), \ldots, x_n(t))$ be an oscillatory solution of (A) on $[t_0; \infty)$. Choose a $T \geqslant t_0$ so that $\min(\tau_1(t), \eta(t)) \geqslant t_0$ as $t \geqslant T$. Since the solution is oscillatory by hypothesis, the Cases (II)—(IV) can never occur, so it must satisfy either Case (I) or Case (V). Suppose that Case (V) is true. Hence $x_1(t) = o(\pi^1_{2n-1}(t))$ as $t \to \infty$. We can choose a $T_1 \geqslant T$ such that $|x_1(\tau_1(t))| \leqslant c\pi^1_{2n-1}(\tau_1(t))$, $t \geqslant T_1$, c — is a suitable constant. Then $f(t, x_1(\tau_1(t))) \in L_1[T_1; \infty)$ by (6) (with $k = 2n-1$) and the nth equation of (A) implies

$$p_n(t)\, x_n'(t) = p_n(T_1)\, x_n'(T_1) + \int_{T_1}^\infty f(s, x_1(\tau_1(s)))\, \mathrm{d}s - \int_t^\infty f(s, x_1(\tau_1(s)))\, \mathrm{d}s. \quad (22)$$

Since $x_n'(t)$ is oscillatory by hypothesis, we must have

$$p_n(T_1)\, x_n'(T_1) + \int_{T_1}^\infty f(s, x_1(\tau_1(s)))\, \mathrm{d}s = 0.$$

Further

$$\left| \int_{T_1}^\infty \frac{1}{p_n(s)} \int_s^\infty f(u, x_1(\tau_1(u)))\, \mathrm{d}u\, \mathrm{d}s \right| \leqslant \pi_1^n(T_1) \int_{T_1}^\infty \omega(s, c\pi^1_{2n-1}(\tau_1(s)))\, \mathrm{d}s$$

and taking this into account we get by (22)

$$x_n(t) = x_n(T_1) - \int_{T_1}^{\infty} \frac{1}{p_n(s)} \int_s^{\infty} f(u, x_1(\tau_1(u)))\, du\, ds + \tag{23}$$
$$+ \int_t^{\infty} \frac{1}{p_n(s)} \int_s^{\infty} f(u, x_1(\tau_1(u)))\, du\, ds, \qquad t \geqslant T_1.$$

Since $x_n(t)$ is oscillatory we get again

$$x_n(T_1) - \int_{T_1}^{\infty} \frac{1}{p_n(s)} \int_s^{\infty} f(u, x_1(\tau_1(u)))\, du\, ds = 0.$$

Using this fact for transforming (23) we obtain

$$|x_n(t)| \leqslant \int_t^{\infty} \frac{ds}{p_n(s)} \int_t^{\infty} \omega(s, |x_1(\tau_1(s))|)\, ds, \quad t \geqslant T_1, \tag{24}$$

which implies by (6) (with $k = 2n - 1$) that $x_n(t) = o(\pi_1^n(t))$ as $t \to \infty$ and so the integrals

$$\int_{T_1}^{\infty} a_{n-1}(s) g(x_n(\tau_n(s)))\, ds, \quad \int_{T_1}^{\infty} \frac{1}{p_{n-1}(s)} \int_s^{\infty} a_{n-1}(u) g(x_n(\tau_n(u))\, du\, ds$$

are convergent. The functions $x'_{n-1}(t)$, $x_{n-1}(t)$ are oscillatory and analogously as above we get from the $(n-1)$st equation of (A)

$$|x_{n-1}(t)| \leqslant K \int_t^{\infty} \frac{1}{p_{n-1}(s)} \int_s^{\infty} a_{n-1}(u) |x_n(\tau_n(u))|\, du\, ds. \tag{25}$$

Similarly as above, since $x_i(t) = o(\pi^i_{2n-2i+1}(t))$ as $t \to \infty$, $i = 2, \ldots, n-1$ the integrals

$$\int^{\infty} a_{i-1}(s) x_i(s)\, ds, \quad \int^{\infty} \frac{1}{p_{i-1}(s)} \int_s^{\infty} a_{i-1}(u) x_i(u)\, du\, ds$$

are convergent. By hypothesis $x_{i-1}(t)$, $x'_{i-1}(t)$ are oscillatory. So, the $(i-1)$st equation of (A) implies

$$|x_{i-1}(t)| \leqslant \int_t^{\infty} \frac{1}{p_{i-1}(s)} \int_s^{\infty} a_{i-1}(u) |x_i(u)|\, du\, ds, \quad t \geqslant T_i \tag{26}$$

a T_i is sufficiently large, $i = 2, \ldots, n-1$. Combining (24), (25), (26) we have

$$|x_1(t)| \leqslant K\pi^1_{2n-1}(t) \int_{\eta(t)}^{\infty} \omega(s, |x_1(\tau_1(s))|)\, ds, \quad t \geqslant T_n \tag{27}$$

aT_n — sufficiently large. We choose T_1^*, T_2^* such that $T_n < T_1^* < T_2^*$, $T_0^* = h(T_2^*) > T_1^*$, $|x_1(t)| \leqslant \pi_{2n-1}^1(t)$ for $t \geqslant T_0^*$. Let us define

$$v(t) = \sup_{s \geqslant t} \frac{|x_1(s)|}{\pi_{2n-1}^1(s)}, \quad t \geqslant T_0^*.$$

Owing to the nondecreasing of the function $\omega(t, z)/z$ in z we get from (27)

$$v(t) \leqslant K \int_{\eta(t)}^{\infty} \omega(s, \pi_{2n-1}^1(\tau_1(s)) v(\tau_1(s))\, ds \leqslant Kv(h(t)) \int_{\eta(t)}^{\infty} \omega(s, \pi_{2n-1}^1(\tau_1(s))\, ds$$

and hence

$$\frac{v(t)}{v(h(t))} \leqslant K \int_{\eta(t)}^{\infty} \omega(s, \pi_{2n-1}^1(\tau_1(s))\, ds. \tag{28}$$

This is a contradiction because the right-hand side of (28) tends to zero as $t \to \infty$ while the left-hand side equal to 1 along a sequence diverging to infinity by (G). From that it follows that Case (I) is the only possibility.

Example. Consider the system

$$(t^2 x_1'(t))' = 30 t^{-\frac{13}{2}} (x_2(t^{\frac{2}{3}}))^{\frac{1}{2}}$$

$$(t^{\frac{11}{10}} x_2'(t))' = \frac{24}{10} t^3 (x_1(t^{\frac{1}{5}}))^2, \quad t > 0$$

One can easily check that condition (6) with $k = 2$ is not satisfied and the system has the solution $(x_1(t), x_2(t)) = (t^{-6}, t^{3/2})$ which has the following properties

$$\lim_{t \to \infty} x_1(t) = \lim_{t \to \infty} \frac{x_1(t)}{\pi_1^1(t)} = 0$$

$$\lim_{t \to \infty} \alpha_2^1(t) x_2(t) = \lim_{t \to \infty} \alpha_1^1(t) x_2(t) = 0$$

$$\lim_{t \to \infty} \frac{x_1(t)}{\pi_2^1(t)} = \infty, \qquad \lim_{t \to \infty} x_2(t) = \infty.$$

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АСИМПТОТИЧЕСКИЕ ПОВЕДЕНИЕ РЕШЕНИЙ СИСТЕМ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ С ОТКЛОНЯЮЩИМСЯ АРГУМЕНТОМ

Božena Mihaliková

Резюме

В статье приведены достаточные условия, которые позволяют определить асимптотическое поведение всех решений систем

$$(p_i(t)x_i'(t))' = a_i(t)x_{i+1}(t) \quad i = 1, \ldots, n, n \geqslant 2$$

$$(p_{n-1}(t)x_{n-1}'(t))' = a_{n-1}(t)g(x_n(\tau_n(t))$$

$$(p_n(t)x_n'(t))' = f(t, x_1(\tau_1(t))), \quad t \geqslant a.$$