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DECOMPOSITIONS OF DIRECTED SETS WITH ZERO

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ABSTRACT. A correspondence is shown between decompositions of directed ordered sets with zero and suitable neutral and complemented pairs of ideals satisfying certain conditions (P) and (P').

The aim of this paper is to show a relationship between direct decompositions of ordered sets and suitable ideals similarly as it was done for lattices ([1; Chapter 3, §4, Theorem 1]).

Let $(S,\leq)$ be an ordered set. If there is no danger of misunderstanding, we will denote it shortly by S. The inverse relation of $\leq$ is denoted by $\geq$.

For a subset $X\subseteq S$ of an ordered set S, we define an *upper* (*lower*) *cone* of X in S:

$$U_S(X):=\{x\in S:\ \ \forall a\in X:\ x\geq a\},$$
$$(L_S(X):=\{x\in S:\ \ \forall a\in X:\ x\leq a\}).$$

Remark. Subscripts will be omitted if there is no danger of misunderstanding.

We shall write briefly $L_SU_S(X)$ or $U_SL_S(X)$ instead of $L_S(U_S(X))$ or $U_S(L_S(X))$, respectively. If $A,B\subseteq S$, we denote by $L_S(A,B)$ and $U_S(A,B)$ the sets $L_S(A\cup B)$ and $U_S(A\cup B)$, respectively.

A subset $I\subseteq S$ is called an *ideal* of S if $L_SU_S(\{a,b\})\subseteq I$ whenever $a,b\in I$ (the case $I=\emptyset$ is not excluded).

Recall that an ordered set S is *directed* if $U_S(\{a,b\})\neq\emptyset$ for each $a,b\in S$. A lattice of all ideals in S will be denoted by $\mathrm{Id}(S)$. Let us note that the set $L(x)$ is an ideal for each element $x\in S$ and meet in $\mathrm{Id}(S)$ coincides with set-theoretic intersection. If S has the least element, it will be denoted by 0_S, and then the

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set of all non-void ideals forms a complete lattice $\mathrm{Id}_0(S)$, which is clearly a sublattice of $\mathrm{Id}(S)$.

For basic properties of ideals in ordered sets see [5].

Let L be a lattice. An element $a \in L$ is *neutral* if one of the following (equivalent) conditions is valid: (see [1])

(i) For each $x, y \in L$, the sublattice of L generated by $\{a, x, y\}$ is distributive.

(ii) There exists an embedding ϕ of L into the direct product $M \times N$ of lattices M, N such that M has a zero element 0, N has a unit element 1, and $\phi(a) = (0, 1)$.

Let $K, L \subseteq S$ and let $C_1(K,L) = \bigcup\{LU(a,b);\ a, b \in K \cup L\}$. Inductively, let $C_{n+1}(K,L) = \bigcup\{L_S U_S(a,b);\ a, b \in C_n(K,L)\}$ for each $n \in \mathbb{N}$.

LEMMA 1. *Let S be an ordered set, $K, L \in \mathrm{Id}_0(S)$. Then*

$$K \vee L = \bigcup\{C_n(K,L);\ n \in \mathbb{N}\}.$$

P r o o f. Evidently, $K \vee L \supseteq \bigcup\{C_n(K,L);\ n \in \mathbb{N}\} \supseteq K \cup L$. It suffices to show that the set $\bigcup\{C_n(K,L);\ n \in \mathbb{N}\}$ is an ideal.

Let $a \in C_m(K,L)$, $b \in C_n(K,L)$ for some $m, n \in \mathbb{N}$. Without loss of generality, we suppose $m \geq n$. Since the sets $C_n(K,L)$ form a chain, we have $a, b \in C_m(K,L)$. By the definition of $C_m(K,L)$, it is obvious that $LU(a,b) \subseteq C_{m+1}(K,L)$, hence $\bigcup\{C_n(K,L);\ n \in \mathbb{N}\}$ is an ideal. □

LEMMA 2. *Let A, B be ordered sets. Suppose $S = A \times B$ is a directed set with the zero element 0_S. Then also A has the zero element 0_A, and B has the zero element 0_B, and the sets $I = \{\langle a, 0_B\rangle;\ a \in A\}$, $J = \{\langle 0_A, b\rangle;\ b \in B\}$ are ideals of S. Moreover, for every $K, L \in \mathrm{Id}_0(S)$ we have $(K \vee L) \cap I = (K \cap I) \vee (L \cap I)$ and $(K \vee L) \cap J = (K \cap J) \vee (L \cap J)$, where the symbol $\vee$ denotes join in $\mathrm{Id}_0(S)$.*

P r o o f. Let $\mathfrak{a} = \langle a, 0_B\rangle$ and $\mathfrak{b} = \langle b, 0_B\rangle \in I$. Since S is directed, we obtain

$$\begin{aligned} L_S U_S(\mathfrak{a}, \mathfrak{b}) &= L_S\big(\{\langle z, w\rangle;\ z \in U_A(a,b),\ w \in B\}\big) \\ &= \{\langle q, 0_B\rangle;\ q \in L_A U_A(a,b)\} \subseteq I, \end{aligned}$$

thus I is an ideal. It can be done similarly for J.

The inclusion $(K \vee L) \cap I \supseteq (K \cap I) \vee (L \cap I)$ is obvious. By Lemma 1, it suffices to show that $C_n(K,L) \cap I \subseteq (K \cap I) \vee (L \cap I)$ for each $n \in \mathbb{N}$.

(1) Let $n = 1$. Then $C_1(K,L) = \bigcup\{L_S U_S(x,y);\ x, y \in K \cup L\}$. Without loss of generality, let $x \in K$, $y \in L$ and $x = \langle x_1, x_2\rangle$, $y = \langle y_1, y_2\rangle$. Suppose that $\langle a, 0_B\rangle \in C_1(K,L) \cap I$ and $\langle a, 0_B\rangle \in L_S U_S(x,y)$.

Then $a \in L_A U_A(x_1, y_1)$. Obviously, $\langle x_1, 0_B \rangle \in K$, $\langle y_1, 0_B \rangle \in L$, hence $\langle a, 0_B \rangle \in (K \cap I) \vee (L \cap I)$.

(2) Suppose that $C_n(K, L) \cap I \subseteq (K \cap I) \vee (L \cap I)$. We shall prove that this holds for $n + 1$.

Let $\langle b, 0_B \rangle \in C_{n+1}(K, L)$. Then

$$\langle b, 0_B \rangle \in L_S U_S(\langle x_1, x_2 \rangle, \langle y_1, y_2 \rangle) \qquad \text{for some} \quad \langle x_1, x_2 \rangle, \langle y_1, y_2 \rangle \in C_n(K, L).$$

This implies $b \in L_A U_A(x_1, y_1)$, where $\langle x_1, 0_B \rangle, \langle y_1, 0_B \rangle \in C_n(K, L) \cap I$. By the induction hypothesis, $\langle x_1, 0_B \rangle, \langle y_1, 0_B \rangle \in (K \cap I) \vee (L \cap I)$, therefore $\langle b, 0_B \rangle \in (K \cap I) \vee (L \cap I)$. □

THEOREM 1. *Let A, B be ordered sets. Suppose $S = A \times B$ is a directed set with the zero element 0_S.*

Then the sets $I = \{\langle a, 0_B \rangle ;\ a \in A\}$ and $J = \{\langle 0_A, b \rangle ;\ b \in B\}$, where 0_A or 0_B are the zero elements of A or B, respectively, are neutral and complemented elements in $\mathrm{Id}_0(S)$ satisfying the following conditions (P) *and* (P'):

(P): $\forall i \in I,\ j \in J\ \ \exists x \in S:\ \ L_S U_S(i, j) = L_S(x)$,

(P'): $\forall x \in S:\ \ L_S(x) = \bigcup \{L_S U_S(i, j) ;\ \ i \in L_S(x) \cap I,\ \ j \in L_S(x) \cap J\}$.

R e m a r k . The condition (P) is equivalent with the existence of $\sup(i, j)$ in S for every $i \in I$, $j \in J$.

P r o o f . Obviously, $I \cap J = \{\langle 0_A, 0_B \rangle\}$ and $\langle a, 0_B \rangle, \langle 0_A, b \rangle \in I \vee J$. For every $\langle a, b \rangle \in S$ we have

$$\langle a, b \rangle \in L_S U_S(\langle a, 0_B \rangle, \langle 0_A, b \rangle) \subseteq I \vee J,$$

therefore $I \vee J = S$.

If $K \in \mathrm{Id}_0(S)$, then clearly $K \supseteq (K \cap I) \vee (K \cap J)$. Suppose that $\langle a, b \rangle \in K$. Then $\langle a, 0_B \rangle \in K \cap I$ and $\langle 0_A, b \rangle \in K \cap J$, which implies $\langle a, 0_B \rangle, \langle 0_A, b \rangle \in (K \cap I) \vee (K \cap J)$, thus $\langle a, b \rangle \in (K \cap I) \vee (K \cap J)$ and $K = (K \cap I) \vee (K \cap J)$.

Let us prove that $K \cap I$ is an ideal in I:
if $\langle x, 0_B \rangle, \langle y, 0_B \rangle \in K \cap I$, then $L_I U_I(\langle x, 0_B \rangle, \langle y, 0_B \rangle) = L_I\{\langle w, 0_B \rangle ;\ w \in U_A(x, y)\} = \{\langle q, 0_B \rangle ;\ q \in L_A U_A(x, y)\} \subseteq K \cap I$.

Analogously, $K \cap J$ is an ideal in J.

Now we are going to prove that there exists an embedding ϕ of $\mathrm{Id}_0(S)$ to $\mathrm{Id}_0(I) \times \mathrm{Id}_0(J)$ such that $\phi(I) = \langle I, \{\langle 0_A, 0_B \rangle\}\rangle$ and $\phi(J) = \langle \{\langle 0_A, 0_B \rangle\}, J\rangle$.

Let us define the mapping ϕ: $\mathrm{Id}_0(S) \to \mathrm{Id}_0(I) \times \mathrm{Id}_0(J)$ by the rule:

$$\phi(K) = \langle K \cap I,\ K \cap J \rangle.$$

If $\phi(K) = \phi(L)$, then $K \cap I = L \cap I$, $K \cap J = L \cap J$, and therefore $K = (K \cap I) \vee (K \cap J) = (L \cap I) \vee (L \vee J) = L$, hence ϕ is injective. We shall prove that ϕ is a lattice homomorphism. Evidently,

$$\phi(K \cap L) = \phi(K) \cap \phi(L).$$

Denote by $\vee^*$ the join in $\mathrm{Id}_0(I)$.

By Lemma 2, $(K \vee L) \cap I = (K \cap I) \vee (L \cap I)$.

Let $M \in \mathrm{Id}_0(I)$ such that $M \supseteq (K \cap I) \cup (L \cap I)$. Let us prove, that $M \in \mathrm{Id}_0(S)$:
let $\langle a, 0_B \rangle, \langle b, 0_B \rangle \in M$; then

$$L_I U_I\big(\langle a, 0_B \rangle, \langle b, 0_B \rangle\big) = L_S U_S\big(\langle a, 0_B \rangle, \langle b, 0_B \rangle\big) \subseteq M\,,$$

thus M is an ideal in S.

Therefore $M \supseteq (K \cap I) \vee (L \cap I)$. But $(K \cap I) \vee (L \cap I)$ is an ideal in I which contains $(K \cap I) \cup (L \cap I)$, thus

$$(K \cap I) \vee (L \cap I) = (K \cap I) \vee^* (L \cap I) \quad \text{and} \quad (K \vee L) \cap I = (K \cap I) \vee^* (L \cap I)\,,$$

thus ϕ is a lattice homomorphism.

Further,

$$\phi(I) = \big\langle I, \{\langle 0_A, 0_B \rangle\}\big\rangle\,, \qquad \phi(J) = \big\langle \{\langle 0_A, 0_B \rangle\}, J\big\rangle\,,$$

i.e. I, J are neutral elements in $\mathrm{Id}_0(S)$.

Let $\langle a, 0_B \rangle \in I$ and $\langle 0_A, b \rangle \in J$. Then $L_S U_S\big(\langle a, 0_B \rangle, \langle 0_A, b \rangle\big) = L_S\big(\{\langle u, v \rangle;\ u \in U_A(a),\ v \in U_B(b)\}\big) = L_S\big(\langle a, b \rangle\big)$, thus the ideals I, J satisfy the condition (P).

Finally, we shall prove that I, J satisfy the condition (P'). It suffices to show that C is an ideal.

Let $a \in L_S U_S(i, j)$ and $b \in L_S U_S(i^*, j^*)$, where $i, i^* \in L_S(x) \cap I$, and $j, j^* \in L_S(x) \cap J$.

Then $i, i^*, j, j^* \le x$. Suppose $x = \langle m, n \rangle$, $i = \langle k, 0_B \rangle$, $i^* = \langle k^*, 0_B \rangle$, j $\langle 0_A, l \rangle$ and $j^* = \langle 0_A, l^* \rangle$. If $\alpha = \langle m, 0_B \rangle$ and $\beta = \langle 0_A, n \rangle$, then $i, i^* \le \alpha$ and $j, j^* \le \beta$.

Let $C = \bigcup\{L_S U_S(i, j);\ i \in L_S(x) \cap I,\ j \in L_S(x) \cap J\}$.

We obtain $U_S(i, i^*) \supseteq U_S(\alpha)$, and $U_S(j, j^*) \supseteq U_S(\beta)$. Hence $L_S U_S(a, b)$ $L_S U_S(i, i^*, j, j^*) \subseteq L_S U_S(\alpha, \beta) \subseteq C$. Thus C is an ideal, $C \supseteq \big(L_S(x) \cap I\big)$ $\big(L_S(x) \cap J\big)$, and by Lemma 2, we have $C = \big(L_S(x) \cap I\big) \vee \big(L_S(x) \cap J\big) = L_S(x)$.

We can state also the opposite statement:

THEOREM 2. *Let S be a directed ordered set with the zero 0_S and I, J be neutral and complemented ideals in $\mathrm{Id}_0(S)$. If these ideals satisfy the conditions (P) and (P'), then $S = I \times J$.*

Proof. For $x \in S$ we have: $x \in L_S(x) = L_S(x) \cap (I \vee J) = \big(L_S(x) \cap I\big) \vee \big(L_S(x) \cap J\big)$ by the neutrality of I, J. Thus by (P'), $x \in L_S U_S(i, j)$ for some $i \in L_S(x) \cap I$, $j \in L_S(x) \cap J$. This gives

$$L_S(x) \subseteq L_S U_S(i, j)\,, \quad U_S(i), U_S(j) \supseteq U_S(x)\,, \quad L_S U_S(i, j) \subseteq L_S U_S(x) = L_S(x)\,,$$

and consequently

$$L_S(x) = L_S U_S(i,j)\,. \tag{$*$}$$

Now we shall show that the foregoing expression is unique:

$$L_S(x) \cap I = L_S U_S(i,j) \cap I = \big(L_S(i) \vee L_S(j)\big) \cap I = L_S(i) \vee \big(L_S(j) \cap I\big) = L_S(i)\,.$$

Analogously,

$$L_S(x) \cap J = L_S(j)\,. \tag{$**$}$$

Let $\phi\colon S \to I \times J$ be a mapping defined by:

$$\phi(x) = \langle i, j\rangle\,,$$

where $i \in I$, $j \in J$ are such that $L_S(x) = L_S U_S(i,j)$.

Let $\langle i,j\rangle \in I \times J$. Since the ideals I, J satisfy the condition (P), there exists an element $x \in S$ with $L_S(x) = L_S U_S(i,j)$, and therefore ϕ is surjective. From the uniqueness condition ($**$) and ($*$) it is clear that ϕ is also injective.

Let $x, y \in S$, $x \le y$. Then $L_S(x) \subseteq L_S(y)$, and for $L_S(x) = L_S U_S(i,j)$, $L_S(y) = L_S U_S(i',j')$ we obtain: $L_S(i) = L_S(x) \cap I \subseteq L_S(y) \cap I = L_S(i')$, i.e. $i \le i'$. Analogously, $j \le j'$.

Conversely, if $i \le i'$, $j \le j'$, then $L_S U_S(i,j) \subseteq L_S U_S(i',j')$, thus $L_S(x) \subseteq L_S(y)$ and $x \le y$. In summary, we have $S = I \times J$. □

COROLLARY. *There exists a one-to-one correspondence between decompositions of directed ordered sets with zero and pairs of neutral complemented ideals satisfying the conditions* (P) *and* (P').

Remark. The following examples show that the foregoing conditions (P) and (P') are independent.

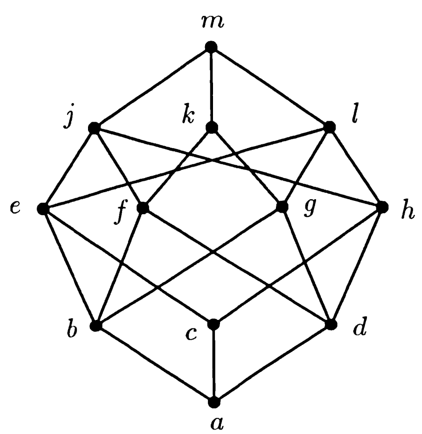

Figure 1.

E x a m p l e 1. Let S be an ordered set whose diagram is visualized in Fig. 1. Then $I = \{a, b, d, f, g, k\}$, $J = \{a, c\}$ are ideals of $\mathrm{Id}_0(S)$, $I \cap J = L(a) = \{0_S\}$ and $I \vee J = S$. The ideals I, J satisfy the condition (P):

$$LU(b,c) = L(e)\,, \quad LU(f,c) = L(j)\,, \quad LU(d,c) = L(h)\,, \quad LU(g,c) = L(l)\,, \quad LU(k,c) = L(m)\,;$$

I, J satisfy the condition (P') and are neutral elements of $\mathrm{Id}_0(S)$. Thus $S = I \times J$.

E x a m p l e 2. Let S be an ordered set visualized in Fig. 2. Let $I = \{a, c, 0\}$ and $J = \{b, d, 0\}$. Then $I \cap J = \{0\}$ and $I \vee J = S$. Since the lattice $\mathrm{Id}_0(S)$ is distributive (see Fig. 3), every element of $\mathrm{Id}_0(S)$ is neutral. However, the condition (P) for I and J is not satisfied: $c \in I$, $d \in J$, but $c \vee d$ does not exist. On the contrary, the condition (P') is valid: (subscripts are omitted), for

$$\begin{aligned}
x = 0:&\quad L(x) \cap I = \{0\}\,,\ L(x) \cap J = \{0\}\,,\ L(0) = LU(0)\,;\\
x = c:&\quad L(x) \cap I = \{c, 0\}\,,\ L(x) \cap J = \{0\}\,,\ L(c) = LU(c,0) \cup LU(0)\,;\\
x = a:&\quad L(x) \cap I = \{a, c, 0\}\,,\ L(x) \cap J = \{0\}\,,\\
&\quad L(a) = LU(a,0) \cup LU(c,0) \cup LU(0,0)\,;\\
x = e:&\quad L(x) \cap I = \{a, c, 0\}\,,\ L(x) \cap J = \{d, 0\}\,,\\
&\quad L(e) = LU(a,d) \cup LU(a,0) \cup LU(c,d) \cup LU(c,0) \cup LU(d,0) \cup LU(0,0)\,;\\
x = 1:&\quad L(x) \cap I = I\,,\ L(x) \cap J = J\,,\ L(1) = LU(a,b) = S\,.
\end{aligned}$$

It does not hold $S = I \times J$, because $\mathrm{card}(S) = 8$ and $\mathrm{card}(I \times J) = 9$.

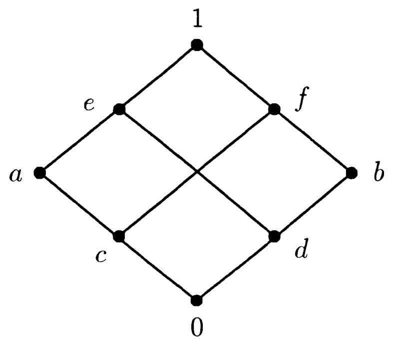

Figure 2.

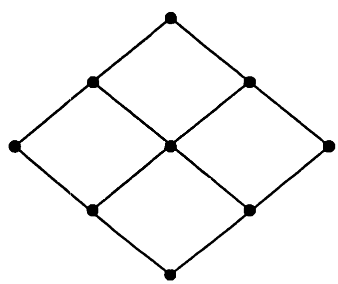

Figure 3.

E x a m p l e 3. Let S be an ordered set visualized in Fig. 4. It is an ordered set in Fig. 1 with one more element r, where r is a join of elements e, h. Thus the lattice $\mathrm{Id}_0(S)$ is the same as that for an ordered set in Fig. 1. Hence, $L(k)$,

$L(c)$ are complemented neutral ideals in $\mathrm{Id}_0(S)$. Let us show that the condition (P) is valid:

$$c \vee b = e\,, \qquad c \vee d = h\,, \qquad c \vee f = j\,, \qquad c \vee g = l\,, \qquad c \vee k = m\,,$$

and by symmetry, there exists a join of the remaining elements of S. However, the condition (P') is not valid: for an element r one has:

$$L_S(r) \cap L_S(k) = \{b, d, a\}\,, \quad L_S(r) \cap J = \{c, a\} \qquad \text{and}$$

$$\bigcup\{LU(i,j) :\ i \in L(k,r)\,,\ j \in L(c,r)\}$$
$$= L_S U_S(b,c) \cup L_S U_S(b,a) \cup L_S U_S(d,c) \cup L_S U_S(d,a) \cup L_S U_S(a,c)$$
$$= L_S(e) \cup L_S(h) \cup L_S(d) \cup L_S(b) \cup L_S(c) = L_S(e) \cup L_S(h) \neq L_S(r)\,.$$

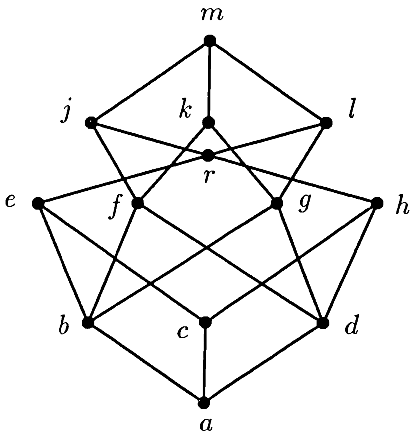

Figure 4.

From Examples 2, 3 we see that the conditions (P), (P') are independent.

Decompositions of ordered sets were studied also by M. Kolibiar, see [4], [5].

We can compare the above obtained results with those of [3].

DEFINITION. (see [3]) Let S be a directed ordered set. An equivalence θ on S will be called a *congruence relation* on S if the following conditions are satisfied:

(i) For each $a \in S$, $[a]\theta$ $\big(= \{x \in S\,;\ (x,a) \in \theta\}\big)$ is a convex subset of S.

(ii) If $a, b, c \in S$, $a \le c$, $b \le c$, and $(a,b) \in \theta$, then there is $d \in S$ such that $a \le d \le c$, $b \le d$ and $(a,d) \in \theta$.

(iii) If $a, b, u, v \in S$, $u \le a \le v$, $u \le b \le v$ and $(u,a) \in \theta$ $((a,v) \in \theta)$, then there is $t \in S$ such that $b \le t \le v$, $a \le t$, $(u \le t \le b,\ t \le a)$ and $(b,t) \in \theta$.

It is proven in [4] that if S satisfies the restricted ascending chain condition, there is a one-to-one correspondence between the direct product decompositions of S into two components, and congruences θ_1, θ_2 of S satisfying

(1) $\theta_1 \cap \theta_2 = \mathrm{id}_S$,
(2) given x_1 and x_2 of S, there exists an element $x \in S$ such that $(x, x_1) \in \theta_1$ and $(x, x_2) \in \theta_2$.

THEOREM 3. *Let S be a directed ordered set with the zero 0_S, and I, J be neutral and complemented ideals in $\mathrm{Id}_0(S)$ satisfying the conditions (P), (P'). Then there exist congruences θ_I, θ_J of S satisfying the conditions (1), (2) such that $S/_{\theta_I} \cong I$ and $S/_{\theta_J} \cong J$.*

P r o o f. It has been proven in Theorem 2 that for each $x, y \in S$ there exist unique $i_x, i_y \in I$, $j_x, j_y \in J$ such that $L(x) = LU(i_x, j_x)$, $L(y) = LU(i_y, j_y)$ and

$$x \leq y \iff i_x \leq i_y, \quad j_x \leq j_y.$$

Let us define the relation θ_I on S by the rule:

$$(x, y) \in \theta_I \iff i_x = i_y.$$

It is evident that θ_I is an equivalence. We shall prove that θ_I is a congruence:

(i) If $x \leq y \leq z$ and $(x, z) \in \theta_I$, then $i_x \leq i_y \leq i_z$, but $i_x = i_z$, hence $i_x = i_y = i_z$.
(ii) It suffices to put $L(d) = LU(i_a, j_c)$. Then $a \leq d \leq c$, $b \leq d$ and $(a, d) \in \theta_I$.
(iii) We put $L(t) = LU(i_b, j_v)$. Then $b \leq t \leq v$, $a \leq t$ and $(b, t) \in \theta_I$.

Analogously, we obtain a congruence θ_J:

$$(x, y) \in \theta_J \iff j_x = j_y.$$

If $(x, y) \in \theta_I \cap \theta_J$, then $i_x = i_y$, $j_x = j_y$, hence $x = y$ and

$$\theta_I \cap \theta_J = \mathrm{id}_S.$$

If $x_1, x_2 \in S$, then for $L(x) = LU(i_{x_1}, j_{x_2})$ we have

$$(x, x_1) \in \theta_I, \qquad (x, x_2) \in \theta_J.$$

Finally, θ_I, θ_J are decomposition congruences of S.

The proof of the remaining part of the theorem is clear. □

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