Ján Futák

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ON THE ASYMPTOTIC BEHAVIOUR OF THE SOLUTIONS OF NONLINEAR DELAY DIFFERENTIAL SYSTEM

JÁN FUTÁK

I.

Let R^n denote the n — dimensional vector space. Let $P(t)$, $Q(t)$, $R(t)$, be $n \times n$ regular matrices such that $P(t), Q(t) \in C(J \equiv [t_0, \infty), R^n)$, and $R(t) \in C((-\infty, \infty), R^n)$.

We consider a nonlinear delay differential system of the form

$$(1) \qquad z'(t) = P(t) f(t, Q(t)z(t), R[h(t)]z[h(t)]),$$

where z, f are n — dimensional vectors.

Throughout the paper we assume that $f(t, u, v) \in C(D \equiv J \times R^n \times R^n, R^n)$ and $h(t) \in C(J, (-\infty, \infty))$, $h(t) \leq t$. The symbol $\|\cdot\|$ denotes come convenient norm of vector or matrix.

The fundamental initial problem is formulated as follows: Let Φ be the set of all continuous vector valued function $\varphi(t)$ defined on the initial set E_{t_0}, where

$$E_{t_0} = [\inf_{t \in J} h(t), t_0], \quad \text{if } \inf_{t \in J} h(t) > -\infty, \quad \text{and } E_{t_0} = (-\infty, t_0]$$

otherwise. It is to find a solution $z(t)$ (a vector valued function) of the system (1) on the interval J satisfying the initial conditions

$$(2) \qquad z(t_0) = \varphi(t_0), \quad z[h(t)] \equiv \varphi[h(t)], \quad h(t) < t_0, \quad \varphi \in \Phi.$$

The asymptotic behaviour of solution of systems of ordinary nonlinear differential equations is treated in many papers by various authors, as Brauer and Wong [1], Trench [10], Hallam and Heidel [5], Hosam El—Din [6], Švec [8] and others. The asymptotic behaviour of functional differential equations was studied by Švec [9], Kato [7], and others. The existence and boundedness of the solutions of nonlinear delay differential system is studied in [4].

In this paper we will investigate asymptotic behaviour of the solutions of (1). Further, we will study asymptotic properties of solutions of perturbed nonlinear delay differential system with respect to the behaviour of solutions of the unperturbed linear system of ordinary differential equations.

Denote by Z the set of all solutions $z(t)$ of all initial problems (1), (2).

We will say that a solution $z \in Z$ has the property (V), if

$$\text{(V)} \qquad z(t) \to c \quad \text{as} \quad t \to \infty,$$

holds, where $c \in R^n$ is a constant vector.

Theorem 1. *Suppose that*

(i) $\omega_1(r), \omega_2(r) \in C([0, \infty), (0, \infty))$ *are nondecreasing and bounded functions,*

(ii) $\psi_1(t), \psi_2(t) \in C(J, [0, \infty))$ *and*

$$\text{(3)} \qquad \int_{t_0} \psi_1(t)\,\mathrm{d}t < \infty, \quad \int_{t_0}^{\infty} \psi_2(t)\,\mathrm{d}t < \infty,$$

$$\text{(iii)} \qquad \|P(t)[f(t, Q(t)u, R[h(t)]v) - f(t, Q(t)\bar{u}, R[h(t)]\bar{v})]\| \leq \\ \leq \psi_1(t)\omega_1(\|u - \bar{u}\|) + \psi_2(t)\omega_2(\|v - \bar{v}\|), \quad \text{on } D,$$

$$\text{(iv)} \qquad \int_{t_0}^{\infty} \|P(t)f(t, 0, 0)\|\,\mathrm{d}t < K < \infty.$$

Then every solution $z \in Z$ is defined on J and for this solution there exists a constant vector $c \in R^n$ such that the property (V) *holds. Conversely, for every constant vector $c \in R^n$ there exists a solution $z \in Z$ on J such that* (V) *holds.*

Proof. *a*) Every solution $z \in Z$ can be written in the form

$$\text{(4)} \qquad z(t) = z(t_0) + \int_{t_0}^{t} P(s)f(s, Q(s)z(s), R[h(s)]z[h(s)])\,\mathrm{d}s, \quad t \in J.$$

From the assumptions of theorem and from (4) we obtain:

$$\|z(t)\| \leq \|\varphi(t_0)\| + \int_{t_0}^{t} \|P(s)f(s, Q(s)z(s), R[h(s)]z[h(s)])\|\,\mathrm{d}s \leq$$

$$\leq \|\varphi(t_0)\| + \int_{t_0}^{t} \|P(s)[f(s, Q(s)z(s), R[h(s)]z[h(s)]) - f(s, 0, 0) +$$

$$+ f(s, 0, 0)]\|\,\mathrm{d}s \leq \|\varphi(t_0)\| + \int_{t_0}^{t} [\psi_1(s)\omega_1(\|z(s)\|) + \psi_2(s)\omega_2(\|z[h(s)]\|)]\,\mathrm{d}s +$$

$$+ \int_{t_0}^{t} \|P(s)f(s, 0, 0\|\,\mathrm{d}s.$$

Using (3) and (iv), from the last inequality it follows that $z(t)$ is bounded on J.

This guarantees the existence of $z\in Z$ on J and

$$\int_{t_0}^{\infty} P(t)f(t, Q((t)z(t), R[h(t)]z[h(t)])\,\mathrm{d}t<\infty.$$

Therefore from (4) it follows that $z(t)\to c$ as $t\to\infty$. Hence (V) holds.

b) Let $c\in R^n$ be a constant vector. Let G be the set of all vector valued functions $g(t)$ continuous on $E_{t_0}\cup J$ and bounded in the norm $\|\cdot\|$.

Define, on the set G, a norm by

$$\|g(t)\|_* = \sup_{t\in E_{t_0}\cup J}\|g(t)\|.$$

The set G endowed with the norm $\|\cdot\|_*$ is a Banach space.

Denote

$$F=\{g(t);\ \|g(t)\|_*\leqslant\lambda,\ t\in E_{t_0}\cup J\}\subset G,$$

where

$$\lambda\geqslant\|c\|+K_1+K_2+K \text{ and } K_{11}>\omega_1(a)\int_{t_0}^{\infty}\psi_1(t)\,\mathrm{d}t,\ K_2>\omega_2(a)\int_{t_0}^{\infty}\psi_2(t)\,\mathrm{d}t,$$

for every real number $a\geqslant 0$.

Define an operator T on F by the equations

$$(5)\qquad (Tg)(t)=c-\int_{t_0}^{\infty}P(s)f(s, Q(s)g(s), R[h(s)]g[h(s)])\,\mathrm{d}s,\quad t\in E_{t_0},$$

$$(Tg)(t)=c-\int_{t}^{\infty}P(s)f(s, Q(s)g(s), R[h(s)]g[h(s)])\,\mathrm{d}s,\quad t\in J,$$

where $g[h(t)]\equiv\varphi[h(t)]$, $h(t)<t_0$.

It is evident that F is a convex closed set.

We show that $TF\subset F$.

If $t\in E_{t_0}$, then

$$\|(Tg)(t)\|\leqslant\|c\|+\int_{t_0}^{\infty}\|P(s)f(s, Q(s)g(s), R[h(s)]g[h(s)])\|\,\mathrm{d}s\leqslant$$

$$\leqslant\|c\|+\int_{t_0}^{\infty}[\psi_1(s)\omega_1(\|g(s)\|+\psi_2(s)\omega_2(\|g[h(s)]\|)\,\mathrm{d}s+\int_{t_0}^{\infty}\|P(s)f(s,0,0)\|\,\mathrm{d}s\leqslant$$

$$\leqslant\|c\|+\omega_1(\lambda)\int_{t_0}^{\infty}\psi_1(s)\,\mathrm{d}s+\omega_2(\lambda)\int_{t_0}^{\infty}\psi_2(s)\,\mathrm{d}s+K\leqslant\|c\|+K_1+K_2+K\leqslant\lambda.$$

Since

$$\|c\|+\int_{t}^{\infty}\|P(s)f(s, Q(s)g(s), R[h(s)]g[h(s)])\|\,\mathrm{d}s\leqslant\|c\|+$$

$$+\int_{t_0}^{\infty} \|P(s)f(s, Q(s)g(s), R[h(s)]g[h(s)])\| \, \mathrm{d}s, \quad \text{for } t \in J$$

the same reasoning as above gives $\|(Tg)(t)\| \leq \lambda$. Therefore for each $t \in E_{t_0} \cup J$ we have $\|(Tg)(t)\|_* \leq \lambda$.

Further, we show that T is continuous on F. Let $\{g_n\}_{n=1}^{\infty}$ be a sequence in F such that $\|g_n - g\|_* \to 0$ as $n \to \infty$. Since F is a closed set, it follows that $g \in F$ and for $t \in J$ we have

$$\|(Tg_n)(t) - (Tg)(t)\| = \Big\| \int_t^{\infty} P(s)f(s, Q(s)g(s), R[h(s)]g[h(s)]) \, \mathrm{d}s -$$

$$- \int_t^{\infty} P(s)f(s, Q(s)g_n(s), R[h(s)]g_n[h(s)]) \, \mathrm{d}s \Big\| \leq$$

$$\leq \int_{t_0}^{\infty} \|P(s)[f(s, Q(s)g(s), R[h(s)]g[h(s)])$$

$$-f(s, Q(s)g_n(s), R[h(s)]g_n[h(s)])]\| \, \mathrm{d}s \overset{\mathrm{def}}{=} \int_{t_0}^{\infty} U_n(s) \, \mathrm{d}s.$$

From the last inequality it follows

$$\|(Tg_n)(t) - (Tg)(t)\|_* = \sup_{t \in J} \|(Tg_n)(t) - (Tg)(t)\| \leq \sup_{t \in J} \left\{ \int_{t_0}^{\infty} U_n(s) \, \mathrm{d}s \right\}.$$

Since $U_n(s) \to 0$, $n \to \infty$, and $U_n(s) \leq \psi_1(s)\omega_1(2\lambda) + \psi_2(s)\omega_2(2\lambda)$, using the Lebesgue Dominated Convergence Theorem we obtain

$$\|(Tg_n)(t) - (Tg)(t)\|_* \to 0 \quad \text{as} \quad n \to \infty.$$

But this means that T is continuous on F.

Now, from the fact that $TF \subset F$ if follows that the functions of TF are uniformly bounded (in the norm). If $t_1, t_2 \in J$, $t_1 < t_2$, are two arbitrary points, then we have the following estimate for T:

$$\|(Tg)(t_2) - (Tg)(t_1)\| \leq \omega_1(\lambda) \int_{t_1}^{t_2} \psi_1(s) \, \mathrm{d}s + \omega_2(\lambda) \int_{t_1}^{t_2} \psi_2(s) \, \mathrm{d}s$$

$$+ \int_{t_1}^{t_2} \|P(s)f(s, 0, 0)\| \, \mathrm{d}s.$$

The right hand side of this inequality does not depend on g and therefore the set TF of functions is equicontinuous.

The Schauder Fixed Point Theorem implies that operator T has fixed point $\tilde{g} \in F$, which means that $\tilde{g}$ is a solution of the equations (5) and also a solution of the initial problem (1), (2) and

(6) $$\tilde g = z(t) = c - \int_t^\infty P(s) f(s, Q(s)\tilde g(s), R[h(s)]\tilde g[h(s)])\,\mathrm{d}s, \quad t\in J,$$

$$\tilde g = z(t) = \varphi(t) = c - \int_{t_0}^\infty P(s) f(s, Q(s)\tilde g(s), R[h(s)]\tilde g[h(s)])\,\mathrm{d}s, \quad t\in E_{t_0}.$$

From (6) it follows that $z(t)\to c$ as $t\to\infty$. This completes the proof.

Let $\bar\Phi\subset\Phi$ denote the set of all initial functions $\varphi\in\Phi$ having bounded norm $\|\cdot\|$ on E_{t_0}. Let $\bar Z\subset Z$ be a set of all solutions $z\in Z$ corresponding to $\varphi\in\bar\Phi$. It is evident that if $\inf_{t\in J} h(t) > -\infty$, then $\bar\Phi\equiv\Phi$ and $\bar Z\equiv Z$.

Theorem 2. *Suppose that*

(i) $$\psi_1(t),\ \psi_2(t)\in C(J, [0,\infty))$$

and

$$\int_{t_0}^\infty \psi_1(t)\,\mathrm{d}t = \bar K_1 < \infty, \quad \int_{t_0}^\infty \psi_2(t)\,\mathrm{d}t = \bar K_2 < \infty, \quad \bar K_1 + \bar K_2 < 1,$$

(ii) $$\|P(t)[f(t, Q(t)u, R[h(t)]v) - f(t, Q(t)\bar u, R[h(t)]\bar v)]\| \le$$
$$\le \psi_1(t)\|u-\bar u\| + \psi_2(t)\|v-\bar v\| \quad \text{on } D,$$

and (iv) *of Theorem 1 hold.*

Then for every $\varphi\in\bar\Phi$ there exists a unique solution $z\in\bar Z$ defined on J and for this solution there exists a unique constant vector $c\in R^n$ such that (V) *holds and, conversely, for every constant vector $c\in R^n$ there exists a unique solution $z\in\bar Z$ on J and a unique $\varphi\in\bar\Phi$ such that* (V) *holds.*

Proof. Let G equipeed with the norm $\|\cdot\|_*$ have the same meaning as in the proof of Theorem 1. Denote $\varrho(g_1, g_2) = \|g_1 - g_2\|_*$ the metrics of G.

a) (The proof of this part is similar as in [9]). Let $\varphi\in\bar\Phi$ be given. Define on G an operator S by

(7) $$(Sg)(t) = \varphi(t), \quad t\in E_{t_0},$$

$$(Sg)(t) = \varphi(t_0) + \int_{t_0}^t P(s) f(s, Q(s)g(s), R[h(s)]g[h(s)])\,\mathrm{d}s, \quad t\in J,$$

where $g[h(t)]\equiv\varphi[h(t)]$, for $h(t) < t_0$.

From the assumptions of the theorem and from (7), for $t\in J$ we get

$$\left\|\int_{t_0}^\infty P(t) f(t, Q(t)g(t), R[h(t)]g[h(t)])\,\mathrm{d}t\right\| \le \int_{t_0}^\infty \psi_1(t)\,\|g(t)\|\,\mathrm{d}t +$$

$$+ \int_{t_0}^\infty \psi_2(t)\,\|g[h(t)]\|\,\mathrm{d}t + \int_{t_0}^\infty \|P(t)f(t,0,0)\|\,\mathrm{d}t \le \|g\|_*\bar K_1 + \|g\|_*\bar K_2 + K.$$

Hence the operator S is defined on G and $SG\subset G$.

Let $g_1, g_2 \in G$. Then we have

$$\|(Sg_1)(t)-(Sg_2)(t)\| \leq \int_{t_1}^{\infty} \|P(t)[f(t, Q(t)g_1(t), R[h(t)]g_1[h(t)])-$$

$$-f(t, Q(t)g_2(t), R[h(t)]g_2[h(t)])]\| \, dt \leq \int_{t_0}^{\infty} \psi_1(t) \, \|g_1(t)-g_2(t)\| \, dt+$$

$$+\int_{t_0}^{\infty} \psi_2(t) \, \|g_1[h(t)]-g_2[h(t)]\| \, dt \leq \|g_1(t)-g_2(t)\|_* \int_{t_0}^{\infty} \psi_1(t) \, dt+$$

$$+\|g_1[h(t)]-g_2[h(t)]\|_* \int_{t_0}^{\infty} \psi_2(t) \, dt = \|g_1-g_2\|_* [\bar{K}_1+\bar{K}_2].$$

From the last inequality we obtain

$$\varrho(Sg_1, Sg_2) \leq [\bar{K}_1+\bar{K}_2]\varrho(g_1, g_2).$$

Thus the operator S is contractive. Using the Banach Fixed Point Theorem we obtain the existence of a unique solution $\tilde{g} \in G$ such that

$$\tilde{z}(t)=\tilde{g}(t)=\varphi(t), \quad t \in E_{t_0}, \tag{8}$$

$$\tilde{z}(t)=\tilde{g}(t)=\varphi(t_0)+\int_{t_0}^{t} P(s)f(s, Q(s)\tilde{g}(s), R[h(s)]\tilde{g}[h(s)]) \, ds, \quad t \in J.$$

The solution $\tilde{z}(t)$ is also the solution of initial problem (1), (2) on J, i.e. $\tilde{z} \in \bar{Z}$ and it is uniquely determined. From the equations (8) it follows that there is a unique constant vector $c \in R^n$ such that $\tilde{z}(t) \to c$ as $t \to \infty$. Therefore (V) holds.

b) Let $c \in R^n$ be a constant vector. Define on G an operator T by equations (5) of Theorem 1.

Similarly as in case *a*), from the assumptions of the theorem it follows that $TG \subset G$ and

$$\|(Tg_1)(t)-(Tg_2)(t)\|_* \leq [\bar{K}_1+\bar{K}_2] \, \|g_1-g_2\|_*, \quad g_1, g_2 \in G.$$

Thus T is contractive on G and therefore there exists a unique element $\tilde{g} \in G$ such that

$$\tilde{z}(t)=\tilde{g}(t)=\varphi(t)=c-\int_{t_0}^{\infty} P(s)f(s, Q(s)\tilde{g}(s), R[h(s)]\tilde{g}[h(s)]) \, ds, \quad t \in E_{t_0},$$

$$\tilde{z}(t)=\tilde{g}(t)=c-\int_{t}^{\infty} P(s)f(s, Q(s)\tilde{g}(s), R[h(s)]\tilde{g}[h(s)]) \, ds, \quad t \in J.$$

From the last equations it follows that (V) holds. This completes the proof.

II.

In this section, we will consider two systems

$$y'(t)=A(t)y(t)+f(t,\, y(t),\, y[h(t)]), \tag{9}$$

and

$$x'(t)=A(t)x(t), \tag{10}$$

where x, y are n — dimensional vectors and f is defined before. We assume hat $A(t)\in C(J,\, R^n)$ is an $n\times n$ matix.

Let every solution $y(t)$ of (9) satisfy the initial conditions

$$y(t_0)=\varphi(t_0),\;\; y[h(t)]\equiv\varphi[h(t)],\;\; h(t)<t_0, \quad \text{for each } \varphi\in\Phi, \tag{11}$$

where Φ is defined before.

Denote by Y the set of all solutions $y(t)$ of all initial problems (9), (11).

We will say that two systems (9) and (10) are asymptotically equivalent if for each solution $y\in Y$ defined on J there exists a solution $x(t)$ of (10) such that

$$\|y(t)-x(t)\|\to 0 \quad \text{as } t\to\infty, \tag{12}$$

and conversely, for each solution $x(t)$ of (10) there exists a solution $y\in Y$ on J such that (12) holds.

In particular, if the system (10) is of the form $x'(t)=0$ and (12) holds, we will say that the system (9) has an asymptotical equilibrium (cf. [2]).

Let $X(t)$ be a fundamental matrix for system (10) such that $X(t_0)=I$, where I denotes the identity matrix. Denote by $X^{-1}(t)$ the inverse matrix to $X(t)$.

Denote:

$$Q(t)=X(t),\;\; P(t)=X^{-1}(t), \quad \text{for } t\in J \text{ and}$$

$$R(t)=\begin{cases} X(t), & t\in J, \\ I, & t\in E_{t_0}. \end{cases}$$

Theorem 3. *Assume that for the matrices P, Q, R and the function f from* (9) *the hypotheses of Theorem 1 hold. Then every solution $y\in Y$ is defined on J and for this solution there exists a constant vector $c\in R^n$ such that*

$$y(t)=R(t)z(t), \quad z(t)\to c,\; t\to\infty, \tag{13}$$

holds. Conversely, for every constant vector $c\in R^n$ there exists a solution $y\in Y$ on J such that (13) *holds.*

Proof. By substitution ([3], pp. 96)

$$y(t)=R(t)z(t), \tag{14}$$

we can transform every initial problem (9), (11) to the initial problem (1), (2) with the property: $z(t_0) = R^{-1}(t_0)\varphi(t_0) = \varphi(t_0)$ and $z[h(t)] \equiv I\varphi[h(t)] \equiv \varphi[h(t)]$, for $h(t) < t_0$. Relationship between sets Y, Z is determined by (14).

From Theorem 1 and from (14) it follows that every solution $y \in Y$ is defined on J and for this solution there exists a constant vector $c \in R^n$ such that (13) holds, and conversely.

Theorem 4. *Assume that for the matrices P, Q, R and the function f from* (9) *the hypotheses of Theorem 2 hold. Then for every $\varphi \in \bar{\Phi}$ there exists a unique constant vector $c \in R^n$ and solution $y \in Y$ defined on J such that* (14) *holds and, conversely, for every constant vector $c \in R^n$ there exists a unique $\varphi \in \bar{\Phi}$ and a solution $y \in Y$ on J such that* (14) *holds.*

The proof follows from Theorem 2 similarly, as in Theorem 3.

Theorem 3 implies the following corollaries.

Corollary 1. *Assume that the hypotheses of Theorem 3 are satisfied and let all solutions of the system* (10) *be bounded on J. Then the systems* (9) *and* (10) *are asymptotically equivalent.*

Proof. Since all solution $x(t)$ of (10) are bounded on J, then for the fundamental matrix $X(t)$ of system (10) we have

$$\|X(t)\| \leq M, \quad t \in J,$$

where M is a suitable constant.

We know that if a is a constant vector then the vector valued function $x(t) = X(t)a$ is a solution of (10).

Let $y \in Y$ be a solution of (9) on J. Theorem 3 implies that for the solution y there exists a constant vector c such that for $t \in J$ (13) holds. Consider the solution $x(t)$ of (10) in the form $x(t) = X(t)c$. Then we get

$$\begin{aligned} \|y(t) - x(t)\| = \|X(t)z(t) - X(t)c\| \leq \|X(t)\|\ \|z(t) - c\| \leq \\ \leq M\|z(t) - c\| \to 0 \quad \text{as} \quad t \to \infty. \end{aligned} \tag{15}$$

Conversely, for each solution $x(t)$ of (10) there exists a constant vector c such that $x(t) = X(t)c$ $t \in J$, and for c there exists $y \in Y$ such that (13) holds. Therefore also (15) holds.

Corollary 2. *Assume that the hypotheses of Theorem 3 are satisfied and, furthermore, let the system* (10) *have an asymptotical equilibrium. Then the system* (9) *has an asymptotical equilibrium.*

Proof. Since the system (10) has an asymptotical equilibrium, we can find a fundamental matrix $X(t)$ such that $X(t) \to I$ as $t \to \infty$. Then from (13) we obtain

$$y(t) \to Ic = c, \quad t \to \infty,$$

and consequently $\|y(t) - c\| \to 0, \quad t \to \infty$.

Corollary 3. *Assume that the hypotheses of Theorem 3 are satisfied and let the solution $x=0$ of (10) be exponentially asymptotically stable. Then for every solution $y \in Y$ we have*

$$y(t) \to 0 \quad \text{as} \quad t \to \infty.$$

Proof. Since the solution $x=0$ of (10) is exponentially asymptotically stable there exists constants $M>0$, $m>0$ such that

$$\|X(t)\| \leqslant Me^{-m(t-t_0)}, \quad t \in J.$$

From (13) for $t \in J$ we get

$$\|y(t)\| \leqslant \|X(t)\| \; \|z(t)\| \leqslant Me^{-m(t-t_0)} \|z(t)\|$$

and it follows that $y(t) \to 0$ as $t \in \infty$.

Remark. If the solution $x(t)=0$ of (10) is asymptotically stable, then the assertion of Corollary 3 is true.

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Katedra matematiky
Strojno-elektrotechnickej fakulty VŠDS
Marxa a Engelsa 25
010 88 Žilina

ОБ АСИМПТОТИЧЕСКОМ ПОВЕДЕНИИ РЕШЕНИЙ СИСТЕМЫ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ С ЗАПАЗДЫВАЮЩИМ АРГУМЕНТОМ

Ян Футак

Резюме

В работе приведены достаточные условия для того, чтобы каждое решение начальной задачи

(1) $$z'(t) = P(t)f(t, Q(t)z(t), R[h(t)]),$$

(2) $$z(t_0) = \varphi(t_0), \quad z[h(t)] \equiv \varphi[h(t)], \quad h(t) < t_0,$$

существовало на $J = [t_0, \infty)$ и к этому решению существовал постоянный вектор $c \in R^n$ такой, что $z(t) \to c$ при $t \to \infty$, и наоборот.

Более того, на основании этих результатов показаны условия для асимптотической эквивалентности ситем (9), (10).