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MULTILINEAR OPERATORS ON $C(K,X)$ SPACES

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Abstract. Given Banach spaces X, Y and a compact Hausdorff space K, we use polymeasures to give necessary conditions for a multilinear operator from $C(K,X)$ into Y to be completely continuous (resp. unconditionally converging). We deduce necessary and sufficient conditions for X to have the Schur property (resp. to contain no copy of c_0), and for K to be scattered. This extends results concerning linear operators.

Keywords: completely continuous, unconditionally converging, multilinear operators, $C(K,X)$ spaces

MSC 2000: 46B25, 46G10

1. Introduction

It has been known for a long time that every linear operator from the space of vector valued continuous functions $C(K,X)$ (where X is a Banach space and K is a compact Hausdorff space) into another Banach space Y admits a representation via a measure defined on the Borel sets of K and taking values in the space $\mathscr{L}(X;Y^{**})$ of continuous linear operators from X into Y^{**} (see for instance [12, §19]). In several papers (see specially [3], [6], [14] and [23]), certain classes of these operators are studied in terms of their representing measure. Many results have been obtained in this direction; we want to mention here that, putting together the results of [3], [14] and [23], one gets the following result.

Let K be a compact Hausdorff space, X and Y Banach spaces, $T\colon C(K,X) \to Y$ a linear operator and $m\colon \Sigma \to \mathscr{L}(X;Y^{**})$ its representing measure (Σ is the σ-algebra of the Borel sets of K). If T is completely continuous (resp. unconditionally converging) then

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(i) for every $A \in \Sigma$, $m(A) \in \mathscr{L}(X;Y)$ and it is completely continuous, (resp.
(i$'$) for every $A \in \Sigma$, $m(A) \in \mathscr{L}(X;Y)$ and it is unconditionally converging.)
(ii) the semivariation of m, $|m|$, is continuous at $\emptyset$.

Moreover, the following are equivalent

1. X is a Schur space.
2. For every K and Y, T is completely continuous if and only if m satisfies (i) and (ii).

Also, the following are equivalent

1$'$. X does not contain isomorphic copies of c_0.
2$'$. For every K and Y, T is unconditionally converging if and only if m satisfies (i$'$) and (ii).

Finally, the following are equivalent

1$''$. K is scattered.
2$''$. For every X and Y, T is completely continuous (resp. unconditionally converging) if and only if m satisfies (i) and (ii) (resp. (i$'$) and (ii)).

In a series of papers (see [15], [16] and the references therein), Dobrakov developed a theory of *polymeasures* (set functions defined on a product of σ-algebras which are separately measures) that can be used to extend the classical Riesz representation theorem to a multilinear setting. With this theory, multilinear operators from a product of $C(K,X)$ spaces into Y can be represented as operator valued polymeasures. Using the representing polymeasures, we show in the present paper that the above mentioned result concerning completely continuous and unconditionally converging linear operators can be satisfactorily extended to the multilinear case, together with some related facts. In a forthcoming paper [6], we use these results to prove that if X is a Banach space such that every multilinear form defined on it is weakly sequentially continuous and K is scattered, then $C(K,X)$ also satisfies that every multilinear form defined on it is weakly sequentially continuous.

We shall first explain our notation and some well known facts which will be freely used along the paper. In what follows, K, K_i will be compact Hausdorff spaces, X, X_i, Y will be Banach spaces, $C(K)$ will be the space of continuous scalar valued functions on K, and $C(K,X)$ will be the space of continuous X-valued functions on K, both endowed with the supremum norm. It is well known (see, f.i., [9, Proposition 1.7.1 and Corollary 1.7.1]) that, if $(f_n) \subset C(K,X)$ is a bounded sequence, then (f_n) converges weakly to $f \in C(K,X)$ if and only if, for every $t \in K$, the sequence $(f_n(t))$ converges weakly to $f(t)$; analogously, the sequence (f_n) is weakly Cauchy if and only if, for every $t \in K$, the sequence $(f_n(t))$ is weakly Cauchy. We denote the topological dual of X by X^*. $\mathscr{L}^k(X_1,\dots,X_k;Y)$ will denote the space of continuous k-linear operators from $X_1 \times \dots \times X_k$ into Y. It is well known that $\mathscr{L}^k(X_1,\dots,X_k;Y)$ is isometricaly isomorphic to $\mathscr{L}(X_1 \hat{\otimes} \dots \hat{\otimes} X_k;Y)$, where $X_1 \hat{\otimes} \dots \hat{\otimes} X_k$ denotes the projective tensor product of the spaces. We use the notation $\overset{[i]}{\ldots}$ to mean that the i^{th} variable is not involved. We denote the support of a function f by $\operatorname{supp} f$.

Let Σ be a σ-algebra. We say that a set function

$$\lambda\colon\ \Sigma \to X$$

is a *measure* if it is finitely additive. We write $\lambda \in \mathrm{ca}(\Sigma; X)$ if λ is countably additive and we write $\lambda \in \mathrm{rca}(\Sigma; X)$ if λ is regular. If $X = \mathbb{K}$ we will omit it. Also, we write $\lambda \in \mathrm{rcabv}(\Sigma; X)$ if λ is regular and has bounded variation. If

$$\lambda\colon\ \Sigma \to \mathscr{L}(X; Y)$$

is a finitely additive set function, we say that λ is an *operator valued measure*, and in that case we consider its *semivariation* $|\lambda|$ to be defined as in [12, p. 51]. If $|\lambda|$ is bounded, we write $\lambda \in \mathrm{ba}(\Sigma; \mathscr{L}(X; Y))$. If λ is a measure or an operator valued measure, we denote its variation by $v(\lambda)$; it is well known, and very easy to check, that if $\lambda\colon\ \Sigma \to X^*$ is an operator valued measure, then $|\lambda| = v(\lambda)$.

If F is a Banach space and $\Sigma_1, \ldots, \Sigma_k$ are σ-algebras, following [15], we say that a set function $\Gamma\colon\ \Sigma_1 \times \ldots \times \Sigma_k \to F$ is a *polymeasure* if it is separately finitely additive. If $F = \mathscr{L}^k(X_1 \ldots, X_k; Y)$ then we say that Γ is an *operator valued polymeasure.* In this last case, we define its *semivariation*

$$|\Gamma|\colon\ \Sigma_1 \times \ldots \times \Sigma_k \to [0, +\infty]$$

by

$$|\Gamma|(A_1, \ldots, A_k) = \sup\left\{ \left\| \sum_{j_1=1}^{n_1} \cdots \sum_{j_k=1}^{n_k} \Gamma(A_{1,j_1}, \ldots, A_{k,j_k})(x_{1,j_1}, \ldots, x_{k,j_k}) \right\| \right\}$$

where $(A_{i,j_i})_{j_i=1}^{n_i}$ is a Σ_i-partition of A_i $(1 \leqslant i \leqslant k)$, $x_{i,j_i} \in X_i$ and $\|x_{i,j_i}\| \leqslant 1$. It is trivial to check that this definition generalizes the above mentioned semivariation of an operator valued measure. We denote by $\mathrm{pm}(\Sigma_1, \ldots, \Sigma_k; X)$ the set of polymeasures from $\Sigma_1 \times \ldots \times \Sigma_k$ into X. We say that $\Gamma \in \mathrm{pm}(\Sigma_1, \ldots, \Sigma_k; X)$ is *countably additive* (respectively *regular*) if it is separately countably additive (respectively separately regular), that is, if for every $i \in \{1, \ldots, k\}$ and every $(A_1, \overset{[i]}{\ldots}, A_k) \in \Sigma_1 \times \overset{[i]}{\ldots} \times \Sigma_k$, the measure

$$\Gamma(A_1, \ldots, A_{i-1}, \cdot, A_{i+1}, \ldots, A_k)\colon\ \Sigma_i \to X$$

is countably additive (resp. regular). In that case we write $\Gamma \in \mathrm{capm}(\Sigma_1, \ldots, \Sigma_k; X)$ (resp. $\Gamma \in \mathrm{rcapm}(\Sigma_1, \ldots, \Sigma_k; X)$).

If Γ is an operator valued polymeasure defined from $\Sigma_1 \times \ldots \times \Sigma_k$ into $\mathscr{L}^k(X_1, \ldots, X_k; Y)$ then, for every $(x_1, \ldots, x_k) \in X_1 \times \ldots \times X_k$ we define the polymeasure $\Gamma_{x_1,\ldots,x_k} \in \mathrm{pm}(\Sigma_1, \ldots, \Sigma_k; Y)$ by

$$\Gamma_{x_1,\ldots,x_k}(A_1, \ldots, A_k) = \Gamma(A_1, \ldots, A_k)(x_1, \ldots, x_k)$$

and, for every $y^* \in Y^*$, we can also define the operator valued polymeasure $\Gamma_{y^*} \in \mathrm{pm}(\Sigma_1, \ldots, \Sigma_k; \mathscr{L}^k(X_1, \ldots, X_k; \mathbb{K}))$ by

$$\Gamma_{y^*}(A_1, \ldots, A_k)(x_1, \ldots, x_k) = \langle \Gamma(A_1, \ldots, A_k)(x_1, \ldots, x_k), y^* \rangle.$$

Given $y^* \in Y^*$, if, for every $(x_1, \ldots, x_k) \in X_1 \times \ldots \times X_k$, the polymeasure $\Gamma_{y^*,x_1,\ldots,x_k} \colon \Sigma_1 \times \ldots \times \Sigma_k \to \mathbb{K}$ defined by

$$\Gamma_{y^*,x_1,\ldots,x_k}(A_1, \ldots, A_k) = \Gamma_{y^*}(A_1, \ldots, A_k)(x_1, \ldots, x_k)$$

is regular, then we say that Γ_{y^*} is *weak-star regular*. We shall use analogous notation for measures. For definitions, notation and basic concepts concerning polymeasures, see [5], [15].

From now on, Σ, Σ_i will be the σ-algebras of the Borel sets of K, K_i. $S(\Sigma)$ will be the normed space of the scalar Σ-simple functions defined on K endowed with the supremum norm and $S(\Sigma, X)$ will be the normed space of the X-valued Σ-simple functions defined on K endowed also with the supremum norm. $B(\Sigma)$ and $B(\Sigma, X)$ denote the completion of $S(\Sigma)$ and $S(\Sigma, X)$ respectively. Given a sequence (A_n) of subsets of K and a set $A \subset K$, if, for every $t \in K$, $\chi_{A_n}(t) \to \chi_A(t)$, then we say that (A_n) converges to A, and we write $A_n \to A$. If (A_n) is a non increasing sequence, then we write the previous relation as $A_n \searrow A$.

If $s_i = \sum\limits_{j_i=1}^{n_i} \chi_{A_{i,j_i}} x_{i,j_i} \in S(\Sigma_i, X_i)$ then, for every polymeasure $\Gamma \in \mathrm{pm}(\Sigma_1, \ldots, \Sigma_k; \mathscr{L}^k(X_1, \ldots, X_k; Y))$, the formula

$$T_\Gamma(s_1, \ldots, s_k) = \sum_{j_1=1}^{n_1} \cdots \sum_{j_k=1}^{n_k} \Gamma(A_{1,j_1}, \ldots, A_{k,j_k})(x_{1,j_1}, \ldots, x_{k,j_k})$$

defines a multilinear map $T_\Gamma \colon S(\Sigma_1, X_1) \times \ldots \times S(\Sigma_k, X_k) \to Y$ such that $\|T_\Gamma\| = |\Gamma|(K_1, \ldots, K_k)$ ($\overset{\text{def}}{=} |\Gamma|$).

So, if $|\Gamma| < \infty$, i.e., if Γ has *finite semivariation*, then T_Γ can be uniquely extended (with the same norm) to $B(\Sigma_1, X_1) \times \ldots \times B(\Sigma_k, X_k)$. We still denote this extension by T_Γ and we write also

$$T_\Gamma(g_1, \ldots, g_k) \overset{def}{=} \int (g_1, \ldots, g_k)\, \mathrm{d}\Gamma.$$

It is easily seen that the correspondence $\Gamma \leftrightarrow T_\Gamma$ is an isometric isomorphism between the Banach space $\mathrm{bpm}(\Sigma_1, \dots, \Sigma_k; \mathscr{L}^k(X_1, \dots, X_k; Y))$ of all $\mathscr{L}^k(X_1, \dots, X_k; Y)$-valued polymeasures of finite semivariation endowed with the semivariation norm, and $\mathscr{L}^k(B(\Sigma_1, X_1), \dots, B(\Sigma_k, X_k); Y)$ endowed with the usual multilinear operator norm. For a quite exhaustive presentation of the integral with respect to polymeasures, see [16] and the references therein. See also [20], [21] and [13] for integration with respect to certain particular classes of polymeasures.

If $m\colon \Sigma \to \mathscr{L}(X; Y)$ is an operator valued measure of bounded semivariation, $g \in B(\Sigma, X)$ and $A \subset K$, the following relation

$$\left\| \int_A g \, \mathrm{d}m \right\| \stackrel{\text{def}}{=} \left\| \int \chi_A g \, \mathrm{d}m \right\| \leqslant \|g\| \, |m|(A)$$

is well known and we shall often use it without explicit mention.

In [16, Theorem 5], a representation of the multilinear operators from $C(K_1, X_1) \times C(K_2, X_2) \times \ldots \times C(K_k, X_k)$ into Y in terms of *Baire* operator valued polymeasures is obtained (although the result is not complete as stated in that paper). Using the representation of multilinear operators on $C(K)$ spaces described in [5, Theorem 3], an analogous theorem can be obtained for *Borel* polymeasures. We presently state without proof such a theorem for reference purposes. A more detailed exposition on this subject can be found in [24].

Theorem 1.1. *Let $T \in \mathscr{L}^k(C(K_1, X_1), \dots, C(K_k, X_k); Y)$. Then T has a unique extension $\overline{T} \in \mathscr{L}^k(B(\Sigma_1, X_1), \dots, B(\Sigma_k, X_k); Y^{**})$ with the same norm and separately weak-star to weak-star continuous, where the weak-star topology that we consider in each of the $B(\Sigma_i, X_i)$ is the one induced by the canonical isometric inclusion $B(\Sigma, X) \hookrightarrow C(K, X)^{**}$.*

If we now define the operator valued polymeasure

$$\Gamma\colon \Sigma_1 \times \ldots \times \Sigma_k \to \mathscr{L}^k(X_1, \dots, X_k; Y^{**})$$

by

$$\Gamma(A_1, \dots, A_k)(x_1, \dots, x_k) = \overline{T}(x_1 \chi_{A_1}, \dots, x_k \chi_{A_k})$$

then it satisfies:

(i) *Γ has bounded semivariation.*

(ii) *For every $y^* \in Y^* (\subset Y^{***})$, Γ_{y^*} is weak-star regular.*

(iii) *If Z is the Banach space of weak-star regular polymeasures of bounded semivariation from $\Sigma_1 \times \ldots \times \Sigma_k$ into $\mathscr{L}^k(X_1, \dots, X_k; \mathbb{K})$, then the mapping*

$$Y^* \to Z, \qquad y^* \mapsto \Gamma_{y^*}$$

is weak-star to weak-star continuous, where we consider a weak-star topology on the range span taking into account that from this theorem in the scalar valued case it follows that

$$(C(K_1, X_1)\hat{\otimes} \dots \hat{\otimes} C(K_k, X_k))^* = Z.$$

(iv) $T(f_1, \dots, f_k) = \int (f_1, \dots, f_k)\, \mathrm{d}\Gamma$, *for every* $f_i \in C(K_i, X_i)$.

(v) $|\Gamma|(K_1, \dots, K_k) = \|T\|$.

Conversely, every operator valued polymeasure

$$\Gamma\colon\ \Sigma_1 \times \dots \times \Sigma_k \to \mathscr{L}^k(X_1, \dots, X_k; Y^{**})$$

that satisfies (i), (ii) *and* (iii) *defines via* (iv) *a k-linear operator*

$$T\colon\ C(K_1, X_1) \times \dots \times C(K_k, X_k) \to Y$$

for which (v) *holds.*

Remark 1.2. If T, $\overline{T}$ and Γ are as in the theorem and $(f_1, \overset{[i]}{\dots}, f_k) \in C(K_1, X_1) \times C(K_2, X_2) \times \overset{[i]}{\dots} \times C(K_k, X_k)$ then we can consider the operator

$$T_{f_1, \overset{[i]}{\dots}, f_k}\colon\ C(K_i, X_i) \to Y$$

given by

$$T_{f_1, \overset{[i]}{\dots}, f_k}(f_i) = T(f_1, \dots, f_k).$$

If $\Gamma_{f_1, \overset{[i]}{\dots}, f_k}: \Sigma_i \to \mathscr{L}(X_i; Y^{**})$ is its associated measure, it is clear that

$$\Gamma_{f_1, \overset{[i]}{\dots}, f_k}(A_i)(x_i) = \overline{T}(f_1, \dots, f_{i-1}, x_i\chi_{A_i}, f_{i+1}, \dots, f_k).$$

Similarly, for $(g_1, \overset{[i]}{\dots}, g_k) \in B(\Sigma_1, X_1) \times \overset{[i]}{\dots} \times B(\Sigma_k, X_k)$, if the operator

$$T_{g_1, \overset{[i]}{\dots}, g_k}\colon\ C(K_i, X_i) \to Y^{**}$$

defined by

$$T_{g_1, \overset{[i]}{\dots}, g_k}(f_i) = \overline{T}(g_1, \dots, g_{i-1}, f_i, g_{i+1}, \dots, g_k)$$

takes its values in Y and $\Gamma_{g_1, \overset{[i]}{\dots}, g_k}\colon\ \Sigma_i \to \mathscr{L}(X_i; Y^{**})$ is its associated measure then, easily, for every $A_i \in \Sigma_i$ and every $x_i \in X_i$,

$$\Gamma_{g_1, \overset{[i]}{\dots}, g_k}(A_i)(x_i) = \overline{T}(g_1, \dots, g_{i-1}, x_i\chi_{A_i}, g_{i+1}, \dots, g_k).$$

We shall use these operators and measures, and this notation, throughout the paper.

Remark 1.3. For every $\overline{x} = (x_1, \ldots, x_k) \in X_1 \times \ldots \times X_k$, let us consider the multilinear operator

$$T_{\overline{x}}\colon\ C(K_1) \times \ldots \times C(K_k) \to Y$$

defined by

$$T_{\overline{x}}(\varphi_1, \ldots, \varphi_k) = T(x_1\varphi_1, \ldots, x_k\varphi_k).$$

Then, if $\Gamma_{\overline{x}}\colon\ \Sigma_1 \times \ldots \times \Sigma_k \to Y^{**}$ is the representing polymeasure of $T_{\overline{x}}$ (in the sense of [5, Theorem 3]), then it is easy to see that

$$\Gamma_{\overline{x}}(A_1, \ldots, A_k) = \Gamma(A_1, \ldots, A_k)(x_1, \ldots, x_k).$$

Remark 1.4. With the notation of Theorem 1.1, it is easy to check that, as in the linear case, $\overline{T}$ is Y-valued if and only if Γ is $\mathscr{L}^k(X_1, \ldots, X_k; Y)$-valued.

A linear operator between Banach spaces is *completely continuous* if it takes weakly convergent sequences into norm convergent sequences. This happens if and only if the operator takes weakly Cauchy sequences into norm convergent sequences [11, p. 49].

We say that $T \in \mathscr{L}^k(E_1, \ldots, E_k; X)$ is *completely continuous* if, given weakly Cauchy sequences $(x_i^n)_{n\in\mathbb{N}} \subset E_i$ $(1 \leqslant i \leqslant k)$, the sequence $(T(x_1^n, \ldots, x_k^n))_n$ is norm convergent in X.

A formal series $\sum x^n$ in a Banach space E is *weakly unconditionally Cauchy* (*w.u.C.*, for short) if there is $C > 0$ such that, for any finite subset Δ of $\mathbb{N}$ and any signs $\pm$, we have $\Big\| \sum\limits_{n\in\Delta} \pm x^n \Big\| \leqslant C$. For other equivalent definitions, see [10, Theorem V.6]. The series $\sum x^n$ is *unconditionally convergent* if every subseries is norm convergent. Other equivalent definitions may be seen in [11, Theorem 1.9].

A linear operator between Banach spaces is *unconditionally converging* if it takes w.u.C. series into unconditionally convergent series.

Following [17], we say that $T \in \mathscr{L}^k(E_1, \ldots, E_k; X)$ is *unconditionally converging* if, given w.u.C. series $\sum\limits_{n\in\mathbb{N}} x_i^n$ in E_i $(1 \leqslant i \leqslant k)$, the sequence

$$(T(s_1^m, \ldots, s_k^m))_m$$

is norm convergent in X, where $s_i^m = \sum\limits_{n=1}^{m} x_i^n$. Since a linear operator fails to be unconditionally converging if and only if it preserves a copy of c_0 [10, Exercise V.8], it is clear that the definition of unconditionally converging k-linear operators agrees for $k = 1$ with that of unconditionally converging linear operators. This seems to be the "right" definition of unconditionally converging multilinear operators, and it has already been used for several purposes (see [4], [17], [18], [19]).

It is a well known fact (and it follows easily from the definitions) that every completely continuous linear operator is unconditionally converging. The same relation is easily seen to be true for multilinear operators.

Along the paper, parallel results, with parallel proofs, will be obtained for completely continuous and unconditionally converging multilinear operators. The reason for this parallelism can be found essentially in the following two lemmas. The first of them is well known, and its proof is contained in [1, Theorem 2.3 and Lemma 2.4]. The second one can be found in [4, Theorem 7].

Lemma 1.5. *Let $T \in \mathscr{L}^k(E_1, \ldots, E_k; X)$. Then T is completely continuous if and only if, for all weak Cauchy sequences $(x_j^n)_{n\in\mathbb{N}} \subset E_j$ $(1 \leqslant j \leqslant k)$, at least one of which converges weakly to zero, we have*

$$\lim_{n\to\infty} \|T(x_1^n, \ldots, x_k^n)\| = 0.$$

Lemma 1.6. *Let $T \in \mathscr{L}^k(E_1, \ldots, E_k; X)$. Then T is unconditionally converging if and only if, for all weakly unconditionally Cauchy series $\sum\limits_{n=1}^{\infty} x_j^n \subset E_j$ $(1 \leqslant j \leqslant k)$, such that there exists an $i \in \{1, \ldots, k\}$ for which the sequence $\Big(\sum\limits_{n=1}^{m} x_i^n\Big)_m$ converges weakly to zero, we have*

$$\lim_{m\to\infty} \left\| T\left(\sum_{n=1}^{m} x_1^n, \ldots, \sum_{n=1}^{m} x_k^n\right) \right\| = 0.$$

If $m \in \mathrm{ba}(\Sigma; \mathscr{L}(X;Y))$, for every $y^* \in Y^*$ we can define the measure $m_{y^*} \in \mathrm{ba}(\Sigma, X^*)$ by $m_{y^*}(A)(x) = \langle m(A)(x), y^* \rangle$. With this notation, we state here for reference purposes the following well known lemma.

Lemma 1.7. *Let $(m_n) \subset \mathrm{ba}(\Sigma; \mathscr{L}(X;Y))$ be a sequence of bounded operator valued measures. Then the following are equivalent:*

a) *$(|m_n|)_n$ are equicontinuous at $\emptyset$, i.e., if $(A_p) \subset \Sigma$ is a sequence such that $A_p \searrow \emptyset$, then*

$$\lim_{p\to\infty} \sup_{n\in\mathbb{N}} |m_n|(A_p) = 0.$$

(In other words, the measures have equicontinuous semivariation at $\emptyset$.)

b) *The set $\{|m_{n,y^*}|\colon\ n \in \mathbb{N}, y^* \in B_{Y^*}\}$ is equicontinuous at $\emptyset$.*

c) *The set $\{|m_{n,y^*}|\colon\ n \in \mathbb{N}, y^* \in B_{Y^*}\}$ is uniformly countably additive.*

d) *The semivariations* $(|m_n|)$ *have a uniform control measure, i.e., there exists* $\lambda \in \mathrm{ca}(\Sigma; [0, +\infty))$ *such that*

$$\lim_{\lambda(A)\to 0} \sup_{n\in\mathbb{N}} |m_n|(A) = 0.$$

2. The results

If $T \in \mathscr{L}(C(K, X); Y)$ and m is its associated measure, the fact that $|m|$ is continuous at $\emptyset$ is a "technical" condition needed for T to be either completely continuous or unconditionally converging ([23, Section 2 and 3]). In the multilinear setting, the following seem to be the necessary technical conditions which replace that one.

Lema 2.1. *Let* $T \in \mathscr{L}^k(C(K_1, X_1), \ldots, C(K_k, X_k); Y)$ *and let* $\overline{T}$ *be its extension defined in Theorem 1.1. If* T *is completely continuous and* $(f_1^n)_n \subset C(K_1, X_1), \ldots, (f_k^n)_n \subset C(K_k, X_k)$ *are weak Cauchy sequences then, for each* $i \in \{1, \ldots, k\}$, *the measures*

$$\left\{\Gamma_n = \Gamma_{f_1^n,\overset{[i]}{\ldots},f_k^n} :\ n \in \mathbb{N}\right\} \subset \mathrm{ba}(\Sigma_i; \mathscr{L}(X_i; Y))$$

defined by

$$\Gamma_n(A_i)(x_i) = \overline{T}(f_1^n, \ldots, f_{i-1}^n, x_i\chi_{A_i}, f_{i+1}^n, \ldots, f_k^n)$$

have equicontinuous semivariation at $\emptyset$.

If T *is unconditionally converging and* $\sum_n f_1^n \subset C(K_1, X_1), \ldots, \sum_n f_k^n \subset C(K_k, X_k)$ *are w.u.C. series, then, for each* $i \in \{1, \ldots, k\}$, *the measures*

$$\left\{\Gamma_n = \Gamma_{s_1^n,\overset{[i]}{\ldots},s_k^n} :\ n \in \mathbb{N}\right\} \subset \mathrm{ba}(\Sigma_i; \mathscr{L}(X_i; Y))$$

defined by

$$\Gamma_n(A_i)(x_i) = \overline{T}(s_1^n, \ldots, s_{i-1}^n, x_i\chi_{A_i}, s_{i+1}^n, \ldots, s_k^n), \quad \textit{where } s_j^n = \sum_{m=1}^{n} f_j^m,$$

have equicontinuous semivariation at $\emptyset$.

P r o o f. We give the proof for unconditionally converging multilinear operators. The other case is a little easier. First, let us observe that, if T is unconditionally converging then, for every $\overline{x} = (x_1, \ldots, x_k) \in X_1 \times \ldots \times X_k$, the mapping

$T_{\overline{x}} \in \mathscr{L}^k(C(K_1), \ldots, C(K_k); Y)$ defined by $T_{\overline{x}}(\varphi_1, \ldots, \varphi_k) = T(x_1\varphi_1, \ldots, x_k\varphi_k)$ is unconditionally converging. Now, using [19, Corollary 5.4] and Remark 1.3 we obtain that the representing polymeasure Γ of T is $\mathscr{L}^k(X_1, \ldots, X_k; Y)$-valued. Let us suppose without loss of generality that $i = k$. If the result is not true, Lemma 1.7 tells us that the family $\{|\Gamma_{n,y^*}|\colon\ n \in \mathbb{N}, y^* \in B_{Y^*}\}$ is not uniformly countably additive. Therefore, there exist $\varepsilon > 0$, an increasing sequence of indexes $(n(l))_l$ with $n(1) = 0$, a sequence $(y_l^*)_l \subset B_{Y^*}$ and a sequence $(A_k^l)_l \subset \Sigma_k$ of disjoint open sets such that, for every $l > 1$,

$$|\Gamma_{n(l),y_l^*}|(A_k^l) > \varepsilon.$$

Clearly, for every $n \in \mathbb{N}$, the operator $T_n\colon\ C(K_k, X_k) \to Y$ defined by

$$T_n(f) = T(s_1^n, \ldots, s_{k-1}^n, f)$$

is unconditionally converging and Γ_n is its representing measure. It is well known that, in that case, for every $y^* \in Y^*$, the scalar valued measure $|\Gamma_{n,y^*}|$ is regular.

So, for every $l \in \mathbb{N}$ there exists $f_k^l \in C(K_k, X_k)$ with $\|f_k^l\| \leqslant 1$ and $\operatorname{supp} f_k^l \subset A_k^l$ such that

$$\left|\left\langle T(s_1^{n(l)}, \ldots, s_{k-1}^{n(l)}, f_k^l), y_l^*\right\rangle\right| = \left|\int f_i^l \, \mathrm{d}\Gamma_{n(l),y_l^*}\right| > \varepsilon$$

and therefore

$$\|T(s_1^{n(l)}, \ldots, s_{k-1}^{n(l)}, f_k^l)\| > \varepsilon.$$

For each $i \in \{1, \ldots, k-1\}$ and $l \in \mathbb{N}$ let

$$y_i^l = \sum_{m=n(l)+1}^{n(l+1)} f_i^m$$

and

$$y_k^1 = f_k^1, \quad y_k^l = f_k^l - f_k^{l-1} \quad \text{if } l \geqslant 2.$$

Then, for every $i \in \{1, \ldots, k\}$, $\sum_l y_i^l$ is clearly a w.u.C. series and, moreover, $\sum_{l=1}^m y_k^l = f_k^m$ converges weakly to zero as $m \to \infty$, by the characterization of weak convergence in $C(K, X)$ mentioned in the introduction. Moreover, for every $m \in \mathbb{N}$,

$$\left\|T\left(\sum_{l=1}^m y_1^l, \ldots, \sum_{l=1}^m y_k^l\right)\right\| = \|T\big(s_1^{n(m)}, \ldots, s_{k-1}^{n(m)}, f_k^m\big)\| > \varepsilon$$

which is in contradiction to Lemma 1.6 and the fact that T is unconditionally converging. $\square$

Although these conditions may seem artificial, we want to point out that when one looks at [19, Lemmas 3.2 and 4.1] then they begin to look somehow "natural". For example, the condition concerning completely continuous multilinear operators is related to the fact that if T is completely continuous then the operators $T_{f_1^n,\overset{[i]}{\ldots},f_k^n}$, whose associated measures are the $\Gamma_{f_1^n,\overset{[i]}{\ldots},f_k^n}$ of the lemma, are "uniformly completely continuous" (see [19], Lemma 3.2] for details).

With analogous reasonings, the following refinement of Lemma 2.1 can be easily proved. $\square$

Lemma 2.2. *Let $T \in \mathscr{L}^k(C(K_1, X_1), \ldots, C(K_k, X_k); Y)$, let $\overline{T}$ be its extension mentioned in Theorem* 1.1 *and, for every $i = 1, \ldots, k$, let $\overline{T}_i$ be the restriction of $\overline{T}$ to $B(\Sigma_1, X_1) \times \ldots \times B(\Sigma_i, X_i) \times C(K_{i+1}, X_{i+1}) \times \ldots \times C(K_k, X_k)$. Let $(g_j^n)_{n\in\mathbb{N}} \subset B(\Sigma_j, X_j)$ $(1 \leqslant j \leqslant i-1)$ and $(g_j^n)_{n\in\mathbb{N}} \subset C(K_j, X_j)$ $(i+1 \leqslant j \leqslant k)$ be weakly Cauchy sequences (respectively, let $\sum\limits_n g_j^n \subset B(\Sigma_j, X_j)$ $(1 \leqslant j \leqslant i-1)$ and $\sum\limits_n g_j^n \subset C(K_j, X_j)$ $(i + 1 \leqslant j \leqslant k)$ be w.u.C. series). If $\overline{T}_i$ is completely continuous (respectively unconditionally converging) then the measures*

$$\left\{\Gamma_n = \Gamma_{g_1^n,\overset{[i]}{\ldots},g_k^n} \colon n \in \mathbb{N}\right\} \subset \mathrm{ba}(\Sigma_i; \mathscr{L}(X_i; Y))$$

(respectively

$$\left\{\Gamma_n = \Gamma_{s_1^n,\overset{[i]}{\ldots},s_k^n} \colon n \in \mathbb{N}\right\} \subset \mathrm{ba}(\Sigma_i; \mathscr{L}(X_i; Y)), \text{ where } s_j^n = \sum_{m=1}^{n} g_j^m)$$

have equicontinuous semivariation at $\emptyset$.

Proof. All the reasonings are analogous to those in the proof of Lemma 2.1. Let us just mention that if $\overline{T}_i$ is either completely continuous or unconditionally converging, then so is T and therefore $\overline{T}$ is Y-valued, and the measures Γ_n in the lemma are $\mathscr{L}(X_i; Y)$-valued. $\square$

We present now a result which extends [3, Theorem 3].

Proposition 2.3. *Let $T \in \mathscr{L}^k(C(K_1, X_1), \ldots, C(K_k, X_k); Y)$. Then T is completely continuous (respectively unconditionally converging) if and only if its extension $\overline{T}$ is completely continuous (respectively unconditionally converging).*

Proof. We prove the result only for completely continuous mappings, since the other case is treated similarly. One direction is clear. For the converse let us first note that, as mentioned in the proof of Lemma 2.1, if T is completely continuous then $\overline{T}$ is Y-valued. We proceed now in k steps. Let us first see that $\overline{T}_1$, defined as in

Lemma 2.2, is completely continuous: otherwise, there exist $\varepsilon > 0$ and weakly Cauchy sequences $(g_1^n) \subset B(\Sigma_1, X_1)$, $(f_2^n) \subset C(K_2, X_2), \ldots, (f_k^n) \subset C(K_k, X_k)$ contained in the respective unit balls, one of which weakly converges to zero, and such that

$$\|\overline{T}_1(g_1^n, f_2^n, \ldots, f_k^n)\| > \varepsilon.$$

For every $n \in \mathbb{N}$, let $T_n \in \mathscr{L}(C(K_1, X_1); Y)$ be defined by

$$T_n(f_1) = T(f_1, f_2^n, \ldots, f_k^n)$$

and let Γ_n be its associated measure. Lemma 2.1 tells us that these measures have equicontinuous semivariation at $\emptyset$ and therefore, by Lemma 1.7, there exists $\lambda \in \mathrm{ca}(\Sigma; [0, +\infty))$ and $\delta > 0$ such that, if $\lambda(A) < \delta$, then

$$\sup_{n\in\mathbb{N}} |\Gamma_n|(A) < \frac{\varepsilon}{4}.$$

According to Luzin's Theorem, there exists $K_1' \subset K_1$ with $\lambda(K_1 \setminus K_1') < \delta$ and such that, for every $n \in \mathbb{N}$,

$$g_1^n|_{K_1'} := h_1^n \in C(K_1', X_1).$$

Let $H = \overline{[(h_1^n)_{n\in\mathbb{N}}]} \subset C(K_1', X_1)$. Theorem 1 in [3] allows us to state the existence of an isometric extension operator $S\colon H \to C(K_1, X_1)$; let us call $S(h_1^n) = f_1^n$. Since (g_1^n) is a weakly Cauchy sequence, so are (f_1^n) and (h_1^n). Moreover, if (g_1^n) was the sequence weakly converging to zero, then (f_1^n) also converges weakly to zero. Therefore, the sequences $(f_1^n) \subset C(K_1, X_1)$, $(f_2^n) \subset C(K_2, X_2), \ldots, (f_k^n) \subset C(K_k, X_k)$ are all weakly Cauchy and at least one of them converges weakly to 0. Hence, since T is completely continuous, there exists an $n_0 \in \mathbb{N}$ such that, for every $n > n_0$,

$$\|T(f_1^n, f_2^n, \ldots, f_k^n)\| < \frac{\varepsilon}{2}.$$

But, on the other hand, for every $n \in \mathbb{N}$,

$$\begin{aligned}
\|T(f_1^n, f_2^n, \ldots, f_k^n)\| &\geqslant \left\|\int_{K_1'} f_1^n \,\mathrm{d}\Gamma_n\right\| - \left\|\int_{K_1\setminus K_1'} f_1^n \,\mathrm{d}\Gamma_n\right\| \\
&\geqslant \left\|\int_{K_1'} g_1^n \,\mathrm{d}\Gamma_n\right\| - \sup_{n\in\mathbb{N}} |\Gamma_n|(K_1 \setminus K_1') \\
&\geqslant \left\|\int_{K_1} g_1^n \,\mathrm{d}\Gamma_n\right\| - \left\|\int_{K_1\setminus K_1'} g_1^n \,\mathrm{d}\Gamma_n\right\| - \sup_{n\in\mathbb{N}} |\Gamma_n|(K_1 \setminus K_1') \\
&= \|T(g_1^n, f_2^n, \ldots, f_k^n)\| - 2\sup_{n\in\mathbb{N}} |\Gamma_n|(K_1 \setminus K_1') \geqslant \frac{\varepsilon}{2},
\end{aligned}$$

a contradiction which proves that $\overline{T}_1$ is completely continuous.

Now, the proof that $\overline{T}_2$ is completely continuous is analogous, using Lemma 2.2 instead of Lemma 2.1. Continuing this reasoning finishes the proof. □

The following proposition extends to the multilinear setting well known results about linear operators (see [14, Theorem 3] and [23, p. 107]).

Proposition 2.4. *Let $T \in \mathscr{L}^k(C(K_1, X_1), \ldots, C(K_k, X_k); Y)$ and let Γ be its representing polymeasure.*

If T is completely continuous then the following conditions hold:

(i) *For every $(A_1, \ldots, A_k) \in \Sigma_1 \times \ldots \times \Sigma_k$, the multilinear operator $\Gamma(A_1, \ldots, A_k) \in \mathscr{L}^k(X_1, \ldots, X_k; Y)$ is completely continuous.*

(ii) *For every $i \in \{1, \ldots, k\}$ and for every weakly Cauchy sequences $(f_1^n)_n \subset C(K_1, X_1), \overset{[i]}{\ldots}, (f_k^n)_n \subset C(K_k, X_k)$, the measures $\{\Gamma_n \colon n \in \mathbb{N}\} \subset \mathrm{ba}(\Sigma_i; \mathscr{L}(X_i; Y))$ defined by*

$$\Gamma_n(A_i)(x_i) = \overline{T}(f_1^n, \ldots, f_{i-1}^n, x_i\chi_{A_i}, f_{i+1}^n, \ldots, f_k^n)$$

have equicontinuous semivariation at $\emptyset$.

If T is unconditionally converging then thefollowing conditions hold:

(i′) *For every $(A_1, \ldots, A_k) \in \Sigma_1 \times \ldots \times \Sigma_k$, the multilinear operator $\Gamma(A_1, \ldots, A_k) \in \mathscr{L}^k(X_1, \ldots, X_k; Y)$ is unconditionally converging.*

(ii′) *For every $i \in \{1, \ldots, k\}$ and for every w.u.C. series $\sum_n f_1^n \subset C(K_1, X_1), \overset{[i]}{\ldots}, \sum_n f_k^n \subset C(K_k, X_k)$, the measures $\{\Gamma_n \colon n \in \mathbb{N}\} \subset \mathrm{ba}(\Sigma_i; \mathscr{L}(X_i; Y))$ defined by*

$$\Gamma_n(A_i)(x_i) = \overline{T}\bigg(\sum_{m=1}^n f_1^m, \ldots, \sum_{m=1}^n f_{i-1}^m, x_i\chi_{A_i}, \sum_{m=1}^n f_{i+1}^m, \ldots, \sum_{m=1}^n f_k^m\bigg)$$

have equicontinuous semivariation at $\emptyset$.

P r o o f. To prove (i), let $(A_1, \ldots, A_k) \in \Sigma_1 \times \ldots \times \Sigma_k$ be nonempty sets, and let $(x_1^n) \subset X_1, \ldots, (x_k^n) \subset X_k$ be weakly Cauchy sequences. For every $i \in \{1, \ldots, k\}$ the mapping

$$\begin{aligned} \varphi_i \colon X_i &\to B(\Sigma_i, X_i), \\ x_i &\mapsto x_i\chi_{A_i} \end{aligned}$$

is clearly an isometric embedding, and therefore the sequences $(x_1^n\chi_{A_1}) \subset B(\Sigma_1, X_1)$, $(x_2^n\chi_{A_2}) \subset (\Sigma_2, X_2), \ldots, (x_k^n\chi_{A_k}) \subset B(\Sigma_k, X_k)$ are also weakly Cauchy. Since Proposition 2.3 states that $\overline{T}$ is completely continuous, we get that the sequence

$$\Gamma(A_1, \ldots, A_k)(x_1^n, \ldots x_k^n) = \overline{T}(\varphi_1(x_1^n), \ldots, \varphi_k(x_k^n))$$

is norm converging, and hence $\Gamma(A_1, \ldots, A_k)$ is completely continuous. The proof of (i′) is analogous.

Conditions (ii) and (ii′) are just Lemma 2.1. □

If $T\colon C(K,X)\to Y$ is an operator and m is its representing measure, then m is $\mathscr{L}(X;Y)$-valued if and only if, for every $x\in X$, the measure m_x defined by $m_x(A)=m(A)x$ is countably additive. It is well known that if $|m|$ is continuous at $\emptyset$ (i.e., for every sequence $(A_n)\subset\Sigma$ such that $A_n\searrow\emptyset$, $\lim\limits_{n\to\infty}|m|(A_n)=0$) then m is $\mathscr{L}(X;Y)$-valued. In the multilinear case we have

Proposition 2.5. *If $T\in\mathscr{L}^k(C(K_1,X_1),\dots,C(K_k,X_k);Y)$ is a multilinear operator such that its representing polymeasure Γ satisfies either condition* (ii) *or condition* (ii′) *in Proposition* 2.4, *then Γ is $\mathscr{L}^k(X_1,\dots,X_k;Y)$-valued.*

Proof. Let us first suppose that Γ satisfies condition (ii). Clearly, we just need to prove that for every $\overline{x}=(x_1,\dots,x_k)\in X_1\times\ldots\times X_k$, the polymeasure $\Gamma_{\overline{x}}$ is Y-valued (see Remarks 1.3 and 1.4). But according to [25, Theorem 5], this is equivalent to seeing that $T_{\overline{x}}$ is completely continuous. By [25, Lemma 2], $T_{\overline{x}}$ is completely continuous if and only if, for every weakly Cauchy sequences $(\varphi_1^n)\subset C(K_1),\overset{[i]}{\ldots},(\varphi_k^n)\subset C(K_k)$, the measures

$$\left\{\Gamma_{\overline{x},\varphi_1^n,\overset{[i]}{\ldots},\varphi_k^n}\colon\ n\in\mathbb{N}\right\}\subset\mathrm{ca}(\Sigma_i;Y)$$

are uniformly countably additive, where each

$$\Gamma_{\overline{x},\varphi_1^n,\overset{[i]}{\ldots},\varphi_k^n}\colon\ \Sigma_i\to Y$$

is given by

$$\Gamma_{\overline{x}\,\varphi_1^n,\overset{[i]}{\ldots},\varphi_k^n}(A_i)=\overline{T}(x_1\varphi_1^n,\dots,x_{i-1}\varphi_{i-1}^n,x_i\chi_{A_i},x_{i+1}\varphi_{i+1}^n,\dots,x_k\varphi_k^n).$$

(Note that each of these measures is countably additive and Y-valued, as follows from the paragraph preceding this proposition). So, let us suppose without loss of generality that $i=k$ and let us choose weakly Cauchy sequences $(\varphi_1^n)\subset C(K_1),\dots,(\varphi_{k-1}^n)\subset C(K_{k-1})$. If the measures

$$\left\{\Gamma_{\overline{x},\varphi_1^n,\dots,\varphi_{k-1}^n}\colon\ n\in\mathbb{N}\right\}\subset\mathrm{ca}(\Sigma_k;Y)$$

are not uniformly countably additive then, considering subsequences if neccesary, there exist an $\varepsilon>0$ and a sequence $(A_k^n)\subset\Sigma_k$, $A_k^n\searrow\emptyset$, such that, for every $n\in\mathbb{N}$,

$$\left\|\Gamma_{\overline{x},\varphi_1^n,\dots,\varphi_{k-1}^n}(A_k^n)\right\|>\varepsilon.$$

But from here it follows that

$$\left|\Gamma_{x_1\varphi_1^n,\dots,x_{k-1}\varphi_{k-1}}\right|(A_k^n)>\varepsilon,$$

a contradiction to the fact that Γ satisfies condition (ii).

In case Γ satisfies condition (ii$'$) above, again we just need to prove that for every $\overline{x} = (x_1, \ldots, x_k) \in X_1 \times \ldots \times X_k$, $\Gamma_{\overline{x}}$ is Y-valued. But using [25, Theorem 5] and [19, Corollary 5.4], we just need to show that $T_{\overline{x}}$ is unconditionally converging. Using [19, Lemma 4.1], a result similar to [25, Lemma 2] can be proved stating that $T_{\overline{x}}$ is unconditionally converging if and only if, for all w.u.C. series $\sum_n \varphi_1^n \subset C(K_1), \overset{[i]}{\ldots}, \sum \varphi_k^n \subset C(K_k)$, the measures

$$\left\{\Gamma_{\overline{x}, s_1^n, \overset{[i]}{\ldots}, s_k^n} : n \in \mathbb{N}\right\} \subset \mathrm{ca}(\Sigma_i; Y)$$

are uniformly countably additive, where $s_j^n = \sum_{m=1}^{n} \varphi_j^m$ (for the same reasons as mentioned above each of these measures is countably additive and Y-valued). See [8] for a full statement and proof of this result. So, let us choose w.u.C. series $\sum_n \varphi_1^n \subset C(K_1), \ldots, \sum_n \varphi_{k-1}^n \subset C(K_{k-1})$. If the measures

$$\left\{\Gamma_{\overline{x}, s_1^n, \ldots, s_{k-1}^n} : n \in \mathbb{N}\right\} \subset \mathrm{ca}(\Sigma_k; Y)$$

are not uniformly countably additive, then there exist $\varepsilon > 0$, a sequence $(A_k^n) \subset \Sigma_k$, $A_k^n \searrow \emptyset$ and an increasing sequence of integers $(m(n))_n \subset \mathbb{N}$ such that, for every $n \in \mathbb{N}$,

$$\left\|\Gamma_{\overline{x}, s_1^{m(n)}, \ldots, s_{k-1}^{m(n)}}(A_k^n)\right\| > \varepsilon. \tag{1}$$

If, for every $i \in \{1, \ldots, k-1\}$ and $n \in \mathbb{N}$, we consider $f_i^n = x_i \varphi_i^n$, it is clear that the series $\sum_n f_i^n \subset C(K_i, X_i)$ is w.u.C. If we call $\sigma_i^n = \sum_{m=1}^{n} f_i^m$, then (1) implies that

$$\left|\Gamma_{s_1^{m(n)}, \ldots, s_{k-1}^{m(n)}}\right|(A_k^n) > \varepsilon,$$

a contradiction to the fact that Γ satisfies condition (ii$'$). $\square$

We are now ready for two of our main results. It is known ([23, Theorem 3.1]) that X is a Schur space if and only if, for every K and Y, an operator $T \in \mathscr{L}(C(K, X); Y)$ is completely continuous if and only if its representing measure satisfies the linear version of conditions (i) and (ii) in Proposition 2.4. It is also known ([23, Theorem 2.1]) that X does not contain an isomorphic copy of c_0 if and only if, for every K and Y, an operator $T \in \mathscr{L}(C(K, X); Y)$ is unconditionally converging if and only if its representing measure satisfies the linear version of conditions (i$'$) and (ii$'$). In the next two theorems we extend those results to multilinear operators.

Theorem 2.6. *Let $X_1, \dots, X_k$ be Banach spaces. Then the following assertions are equivalent:*

(1) *For every $i \in \{1, \dots, k\}$, X_i is a Schur space.*

(2) *For all compact spaces $K_1, \dots, K_k$ and every Banach space Y, a multilinear operator $T \in \mathscr{L}^k(C(K_1, X_1), \dots, C(K_k, X_k); Y)$ is completely continuous if and only if its representing polymeasure Γ satisfies conditions* (i) *and* (ii) *of Proposition* 2.4.

Proof. Let us first show that (1) implies (2): Using Proposition 2.4, it is clear that we only have to prove that if a multilinear operator $T \in \mathscr{L}^k(C(K_1, X_1), \dots, C(K_k, X_k); Y)$ satisfies (i) and (ii) then it is completely continuous. Let us then choose one such T and let $(f_1^n) \subset C(K_1, X_1), \dots, (f_k^n) \subset C(K_k, X_k)$ be weakly Cauchy sequences in the respective unit balls such that one of them converges weakly to zero. We shall assume without loss of generality that (f_1^n) converges weakly to zero. Hence, for every $t \in K_1$, $f_1^n(t)$ converges weakly to zero and, since X_1 is Schur, we obtain that, for every $t \in K_1$,

$$\|f_1^n(t)\| \to 0.$$

For every $n \in \mathbb{N}$, let $\Gamma_n = \Gamma_{f_2^n, \dots, f_k^n} \in \mathrm{ba}(\Sigma_1; \mathscr{L}(X_1; Y))$ and choose $\varepsilon > 0$. According to Lemma 1.7, there exist $\lambda \in \mathrm{ca}(\Sigma_1, [0, +\infty))$ and $\delta > 0$ such that, if $\lambda(A) < \delta$, then

$$\sup_{n \in \mathbb{N}} |\Gamma_n|(A) < \frac{\varepsilon}{2}.$$

By Egoroff's theorem, there exists $K_1' \subset K_1$ with $\lambda(K_1 \setminus K_1') < \delta$ such that

$$\lim_{n \to \infty} \sup_{t \in K_1'} \|f_1^n(t)\| = 0.$$

Therefore, there exists $n_0 \in \mathbb{N}$ such that, for every $n > n_0$,

$$\sup_{t \in K_1'} \|f_1^n(t)\| < \frac{\varepsilon}{2|\Gamma|}.$$

Then, for every $n > n_0$,

$$\|T(f_1^n, \dots, f_k^n)\| = \left\| \int_{K_1} f_1^n \, \mathrm{d}\Gamma_n \right\| \leqslant \left\| \int_{K_1'} f_1^n \, \mathrm{d}\Gamma_n \right\| + \left\| \int_{K_1 \setminus K_1'} f_1^n \, \mathrm{d}\Gamma_n \right\| < \varepsilon.$$

Hence T is completely continuous.

The proof that (2) implies (1) is based on the proofs of [2, Teorema III.5.6] and [23, Theorem 3.1]: Let us suppose that X_1 is not a Schur space (the other cases are similar). Then, there exist $\delta > 0$ and a sequence weakly converging to zero $(x_n) \subset X_1$ such that $\|x_n\| > \delta$ for every $n \in \mathbb{N}$. Then, there exists another sequence $(x_n^*) \subset B_{X_1^*}$ such that for every $n \in \mathbb{N}$, $|\langle x_n^*, x_n\rangle| > \delta$. Let $K_1 = [0,1]$ and take non empty compact Hausdorff spaces $K_2, \ldots, K_k$. Let λ be the Lebesgue measure on $[0,1]$. Let $(r_n)_n \subset C(K_1)$ be a bounded sequence which forms an orthonormal system with respect to the inner product induced by $L_2(\lambda)$ (for example $r_n = \sqrt{2}\sin 2\pi nt$). Clearly, (r_n) converges weakly to zero in $L_2(\lambda)$ and therefore it also converges weakly to zero in $L_1(\lambda)$. For each $j \in \{2, \ldots, k\}$, let $\alpha_j \in C(K_j, X_j)^*$ be norm one. Now we can define

$$\begin{aligned} T\colon\ & C(K_1, X_1) \times \ldots \times C(K_k, X_k) \to c_0 \\ & (f_1 \quad, \ldots, \quad f_k) \mapsto \left(\left(\prod_{j=2}^{k} \alpha_j(f_j)\right) \int_{K_1} \langle f_1(t), x_n^*\rangle r_n(t)\,\mathrm{d}\lambda(t)\right)_n. \end{aligned}$$

It is not difficult to see that T is well defined and that its associated polymeasure Γ satisfies conditions (i) and (ii). Yet T is not completely continuous. To see this, for every $i \in \{2, \ldots, k\}$, choose $f_i \in C(K_i, X_i)$ with $\alpha_i(f_i) = 1$, and let us consider the constant sequence $f_i^n = f_i$ for every $n \in \mathbb{N}$. Let us also consider the sequence $(r_n x_n) \subset C(K_1, X_1)$. Clearly $(r_n x_n)$ converges weakly to zero and, if $(e_n)_n$ denotes the usual basis of c_0,

$$\|T(r_n x_n, f_2^n, \ldots, f_k^n)\| = \|\langle x_n, x_n^*\rangle e_n\| > \delta,$$

which implies that T is not completely continuous. □

Theorem 2.7. *Let $X_1, \ldots, X_k$ be Banach spaces. Then the following assertions are equivalent:*

(1) *For every $i \in \{1, \ldots, k\}$, X_i does not contain isomorphic copies of c_0 (we abbreviate this by $X_i \not\supset c_0$).*
(2) *For all compact spaces $K_1, \ldots, K_k$ and for every Banach space Y, a multilinear operator $T \in \mathscr{L}^k(C(K_1, X_1), \ldots, C(K_k, X_k); Y)$ is unconditionally converging if and only if its representing polymeasure Γ satisfies conditions* (i′) *and* (ii′) *of Proposition* 2.4.

P r o o f. The proof that (1) implies (2) is very similar to that of Theorem 2.6, but instead of using that X_i is a Schur space, we use that $X_i \not\supset c_0$ to obtain, via the Bessaga-Pełczyński Theorem, that, if $\sum_n f_i^n \subset C(K_i, X_i)$ is a w.u.C. series such that $s_i^m = \sum_{n=1}^{m} f_i^n$ converges weakly to zero as $m \to \infty$, then, for every $t \in K_i$,

$$\lim_{m\to\infty} \|s_i^m(t)\| = 0.$$

Let us now show that (2) implies (1) (the proof is again very similar to that of Theorem 2.6): Let us suppose that there exists an injective isomorphism $\theta\colon\ c_0 \to X_1$. Let $x_n = \theta(e_n)$ and, for each $n \in \mathbb{N}$, choose $x_n^* \in B_{X_1^*}$ with $\langle x_n, x_n^*\rangle = \|x_n\|$. Let $K_1, \ldots, K_k,\ \lambda,\ \alpha_2, \ldots, \alpha_k$ and (r_n) be as in the proof of Theorem 2.6, and let us consider the multilinear operator

$$
\begin{aligned}
T\colon\ & C(K_1, X_1) \times \ldots \times C(K_k, X_k) \to c_0 \\
& (f_1 \quad , \ldots , \quad f_k) \mapsto \left(\left(\prod_{j=2}^{k} \alpha_j(f_j)\right) \int_{K_1} \langle f_1(t), x_n^*\rangle r_n(t)\, \mathrm{d}\lambda(t)\right)_n .
\end{aligned}
$$

Again, it is easy to see that T is well defined and that its representing polymeasure Γ satisfies conditions (i′) and (ii′) of Proposition 2.4, so we just have to see that T is not unconditionally converging: let us consider the w.u.C. series $\sum_n x_n r_n$. For every $i \in \{2, \ldots, k\}$, choose $f_i \in C(K_i, X_i)$ with $\alpha_i(f_i) = 1$, and consider the w.u.C. series $\sum_n g_i^n$ where $g_i^1 = f_i$ and $g_i^n = 0$ if $n > 1$. Then

$$
T\left(\sum_{n=1}^{m} x_1^n r_n, \sum_{n=1}^{m} g_2^n, \ldots, \sum_{n=1}^{m} g_k^n\right) = T\left(\sum_{n=1}^{m} x_1^n r_n, f_2, \ldots, f_k\right) = \sum_{n=1}^{m} \|x_1^n\| e_n
$$

which is clearly not a Cauchy sequence, and hence T is not unconditionally converging. □

3. Scattered compact spaces

Let us recall that a compact space is *scattered* (or *dispersed*) if it does not contain any non void perfect set. In [3, Theorem 9′] the authors prove that K is a scattered compact Hausdorff space if and only if, for every X and Y, a necessary and sufficient condition for an operator $T \in \mathscr{L}(C(K, X); Y)$ to be completely continuous is that its representing measure satisfies the linear version of conditions (i) and (ii) in Proposition 2.4. In [3, Theorem 9] they also prove that K is scattered if and only if, for every X and Y, a necessary and sufficient condition for an operator $T \in \mathscr{L}(C(K, X); Y)$ to be unconditionally converging is that its representing measure satisfies the linear version of conditions (i′) and (ii′) in Proposition 2.4 (that is the equivalence between (1″) and (2″) in the result stated at the beginning of the introduction).

In this section we prove that these results can also be extended to multilinear operators. We first need the following technical result.

Proposition 3.1. *Let $T \in \mathscr{L}^k(C(K_1, X_1), \ldots, C(K_k, X_k); Y)$, let Γ be its associated polymeasure and let $\overline{T}$ be its extension defined in Theorem* 1.1.

(a) *If Γ satisfies condition* (ii) *of Proposition* 2.4 *and $(g_1^n) \subset B(\Sigma_1, X_1), \ldots, (g_k^n) \subset B(\Sigma_k, X_k)$ are bounded sequences such that, for every $i \in \{1, \ldots, k\}$ and $t \in K_i$, the sequence $(g_i^n(t))_n \subset X_i$ is weakly Cauchy, then, for each $j \in \{1, \ldots, k\}$, the measures*

$$\Big\{\Gamma_n = \Gamma_{g_1^n, \overset{[j]}{\ldots}, g_k^n} \colon n \in \mathbb{N}\Big\} \subset \mathrm{ba}(\Sigma_j; \mathscr{L}(X_j; Y))$$

have equicontinuous semivariation at $\emptyset$.

(b) *If Γ satisfies condition* (ii$'$) *of Proposition* 2.4 *and $\Big(\sum\limits_n g_1^n, \ldots, \sum\limits_n g_k^n\Big) \subset B(\Sigma_1, X_1) \times \ldots \times B(\Sigma_k, X_k)$ are series such that, for every $i \in \{1, \ldots, k\}$ and $t \in K_i$, the series $\sum\limits_n g_i^n(t) \subset X_i$ is w.u.C., then, for each $j \in \{1, \ldots, k\}$, the measures*

$$\Big\{\Gamma_n = \Gamma_{\sum\limits_{m=1}^{n} g_1^m, \overset{[j]}{\ldots}, \sum\limits_{m=1}^{n} g_k^m} \colon n \in \mathbb{N}\Big\} \subset \mathrm{ba}(\Sigma_j; \mathscr{L}(X_j; Y))$$

have equicontinuous semivariation at $\emptyset$.

Proof. We give the proof for polymeasures satisfying condition (ii), the other case being similar. Note that, by Proposition 2.5, Γ is $\mathscr{L}^k(X_1, \ldots, X_k; Y)$-valued; hence $\overline{T}$ takes values in Y and therefore the measures Γ_n are indeed $\mathscr{L}(X_j; Y)$-valued. Without loss of generality, we assume that $j = k$. Consider $(g_1^n)_n, \ldots, (g_k^n)_n$ as in the hypothesis and suppose that $(f_2^n) \subset C(K_2, X_2), \ldots, (f_{k-1}^n) \subset C(K_{k-1}, X_{k-1})$ are weakly Cauchy sequences. We can assume that these functions are in the respective unit balls. First we show that the measures

$$\big\{\Gamma_n = \Gamma_{g_1^n, f_2^n, \ldots, f_{k-1}^n} \colon n \in \mathbb{N}\big\} \subset \mathrm{ba}(\Sigma_k; \mathscr{L}(X_k; Y))$$

have equicontinuous semivariation at $\emptyset$. Otherwise, reasoning as in the proof of Lemma 2.1 passing to subsequences if neccesary, we can suppose the existence of an $\varepsilon > 0$ and a sequence $(f_k^n)_n \subset C(K_k, X_k)$ of functions of norm one and disjoint supports, such that

$$\left\|\int f_k^n \,\mathrm{d}\Gamma_n\right\| > \varepsilon \quad \text{or equivalently,} \quad \|\overline{T}(g_1^n, f_2^n, \ldots f_k^n)\| > \varepsilon.$$

We consider now the measures $(\Gamma_{f_2^n, \ldots, f_k^n}) \subset \mathrm{ba}(\Sigma_1; \mathscr{L}(X_1; Y))$. By the hypothesis, they have equicontinuous semivariation at $\emptyset$. As in the proof of Proposition 2.3, using Luzin's theorem we can assume the existence of a weakly Cauchy sequence $(f_1^n)_n$ in the unit ball of $C(K_1, X_1)$ and a compact set $K_1' \subset K_1$ such that, for all $n \in \mathbb{N}$,

$g_1^n|_{K_1'} = f_1^n|_{K_1'}$ and $\sup\limits_n \bigl|\Gamma_{f_2^n,\ldots,f_k^n}\bigr|(K_1 \setminus K_1') < \frac{\varepsilon}{4}$. By the hypothesis, $\Gamma_{f_1^n,\ldots,f_{k-1}^n}$ are equicontinuous at $\emptyset$, and since the functions (f_k^n) have disjoint supports, there exists $n_0 \in \mathbb{N}$ such that, for all $n > n_0$,

$$\|T(f_1^n,\ldots,f_k^n)\| < \frac{\varepsilon}{2}.$$

But

$$\begin{aligned}\|T(f_1^n,\ldots,f_k^n)\| = \left\|\int_{K_1} f_1^n \,\mathrm{d}\Gamma_{f_2^n,\ldots,f_k^n}\right\| \geqslant \left\|\int_{K_1} g_1^n \,\mathrm{d}\Gamma_{f_2^n,\ldots,f_k^n}\right\| \\ - \left\|\int_{K_1\setminus K_1'} g_1^n \,\mathrm{d}\Gamma_{f_2^n,\ldots,f_k^n}\right\| - \left\|\int_{K_1\setminus K_1'} f_1^n \,\mathrm{d}\Gamma_{f_2^n,\ldots,f_k^n}\right\| \geqslant \frac{\varepsilon}{2}.\end{aligned}$$

This contradiction proves that the measures $(\Gamma_{g_1^n,f_2^n,\ldots,f_{k-1}^n})$ have equicontinuous semivariation at $\emptyset$. Actually, a change in the order of the variables proves that, whenever $(f_2^n)_n \subset C(K_2,X_2),\ldots,(f_k^n)_n \subset C(K_k,X_k)$ are weakly Cauchy sequences, the measures $\left(\Gamma_{g_1^n,f_2^n,\overset{[i]}{\ldots},f_k^n}\right) \subset \mathrm{ba}(\Sigma_i;\mathscr{L}(X_i;Y))$ have equicontinuous semivariation at $\emptyset$.

Let us now see that the measures $\Gamma_{g_1^n,g_2^n,f_3^n,\ldots,f_{k-1}^n}$ have equicontinuous semivariation at $\emptyset$. If not, we can choose as before a sequence $(f_k^n) \subset C(K_k,X_k)$ of functions of norm one and disjoint supports such that, for each $n \in \mathbb{N}$,

$$\|T(g_1^n,g_2^n,f_3^n,\ldots,f_k^n)\| > \varepsilon > 0.$$

The previous paragraph proves that the measures $(\Gamma_{g_1^n,f_3^n,\ldots,f_k^n})$ have equicontinuous semivariation at $\emptyset$ and, reasoning as before, we get again a contradiction.

Continuing these reasonings we finish the proof. □

Now we are ready to state our last two main results. The following one extends [3, Theorem 9′] to the multilinear setting.

Theorem 3.2. *Let $K_1,\ldots,K_k$ be compact Hausdorff spaces. Then the following are equivalent:*

(1) *For every $i \in \{1,\ldots,k\}$, K_i is scattered.*

(2) *For all Banach spaces $X_1,\ldots,X_k,Y$, a multilinear operator $T \in \mathscr{L}^k(C(K_1,X_1), C(K_2,X_2),\ldots,C(K_k,X_k);Y)$ is completely continuous if and only if its associated polymeasure Γ satisfies conditions* (i) *and* (ii) *of Proposition* 2.4.

P r o o f. To see that (1) implies (2), let us suppose that $K_1,\ldots,K_k$ are scattered compact spaces and $T \in \mathscr{L}^k(C(K_1,X_1),\ldots,C(K_k,X_k);Y)$. We just have to prove that if Γ satisfies (i) and (ii) then T is completely continuous.

Let $(f_1^n) \subset C(K_1, X_1), \ldots, (f_k^n) \subset C(K_k, X_k)$ be weakly Cauchy sequences contained in the respective unit balls and such that at least one of them converges weakly to zero. Using Lemma 1.5 we just have to check that

$$\lim_{n\to\infty} \|T(f_1^n, \ldots, f_k^n)\| = 0.$$

We suppose without losing generality that (f_1^n) converges weakly to zero.

Let us fix an $\varepsilon > 0$. The measures $\Gamma_{f_2^n,\ldots,f_k^n}$ have equicontinuous semivariation at $\emptyset$, so, by Lemma 1.7 there exists $\lambda \in \mathrm{ca}(\Sigma_1; [0, +\infty))$ such that

$$\lim_{\lambda(A)\to 0} \sup_{n\in\mathbb{N}} \left|\Gamma_{f_2^n,\ldots,f_k^n}\right|(A) = 0.$$

Since K is scattered, λ is purely atomic ([22]), so it has countable support, i.e., there exists a countable $K_1' = \{t_1^n \colon n \in \mathbb{N}\} \subset K_1$ such that, for any $A \subset K_1 \setminus K_1'$, $\lambda(A) = 0$ (and hence, $\sup_{n\in\mathbb{N}} \left|\Gamma_{f_2^n,\ldots,f_k^n}\right|(A) = 0$). Since $\bigcap_{r=1}^{\infty} \{t_1^n \colon n > r\} = \emptyset$, there exists $B_1 = \{t_1^n \colon n \leqslant r_1\} \subset K_1$ such that

$$\sup_{n\in\mathbb{N}} \left|\Gamma_{f_2^n,\ldots,f_k^n}\right|(K \setminus B_1) < \varepsilon.$$

So,

$$\begin{aligned}
\|T(f_1^n, \ldots, f_k^n)\| &= \left\| \int_{K_1} f_1^n \, \mathrm{d}\Gamma_{f_2^n,\ldots,f_k^n} \right\| \\
&\leqslant \left\| \int_{K_1\setminus B_1} f_1^n \, \mathrm{d}\Gamma_{f_2^n,\ldots,f_k^n} \right\| + \left\| \int_{B_1} f_1^n \, \mathrm{d}\Gamma_{f_2^n,\ldots,f_k^n} \right\| \\
&\leqslant \varepsilon + \left\| \int_{K_1} f_1^n \chi_{B_1} \, \mathrm{d}\Gamma_{f_2^n,\ldots,f_k^n} \right\| \\
&= \varepsilon + \left\| \int_{K_2} f_2^n \, \mathrm{d}\Gamma_{f_1^n\chi_{B_1},f_3^n,\ldots,f_k^n} \right\|.
\end{aligned}$$

Proposition 3.1 tells us that the measures $\Gamma_{f_1^n\chi_{B_1},f_3^n,\ldots,f_k^n}$ have equicontinuous semivariation at $\emptyset$ and, therefore, reasoning as before, we can assure the existence of a set $B_2 = \{t_2^n \colon n \leqslant r_2\} \subset K_2$ such that

$$\sup_{n\in\mathbb{N}} \left|\Gamma_{f_1^n\chi_{B_1},f_3^n,\ldots,f_k^n}\right|(K \setminus B_2) < \varepsilon.$$

Then,

$$\begin{aligned}\|T(f_1^n,\dots,f_k^n)\| &\leqslant \varepsilon + \left\|\int_{K_2} f_2^n \,\mathrm{d}\Gamma_{f_1^n\chi_{B_1},f_3^n,\dots,f_k^n}\right\| \\ &\leqslant \varepsilon + \left\|\int_{K_2\setminus B_2} f_2^n \,\mathrm{d}\Gamma_{f_1^n\chi_{B_1},f_3^n,\dots,f_k^n}\right\| + \left\|\int_{B_2} f_2^n \,\mathrm{d}\Gamma_{f_1^n\chi_{B_1},f_3^n,\dots,f_k^n}\right\| \\ &\leqslant \varepsilon + \varepsilon + \left\|\int_{K_2} f_2^n\chi_{B_2} \,\mathrm{d}\Gamma_{f_1^n\chi_{B_1},f_3^n,\dots,f_k^n}\right\| \\ &= 2\varepsilon + \left\|\int_{K_3} f_3^n \,\mathrm{d}\Gamma_{f_1^n\chi_{B_1},f_2^n\chi_{B_2},f_4^n,\dots,f_k^n}\right\|.\end{aligned}$$

Continuing this way we get that there exist sets $B_i = \{t_i^n\colon n \leqslant r_i\} \subset K_i$, $(1 \leqslant i \leqslant k)$ such that

$$\begin{aligned}\|T(f_1^n,\dots,f_k^n)\| &\leqslant k\varepsilon + \left\|\int (f_1^n\chi_{B_1}, f_2^n\chi_{B_2},\dots,f_k^n\chi_{B_k}) \,\mathrm{d}\Gamma\right\| \\ &= k\varepsilon + \sum_{m_1=1}^{r_1}\dots\sum_{m_k=1}^{r_k} \Gamma(\{t_1^{m_1}\},\dots,\{t_k^{m_k}\})(f_1^n(t_1^{m_1}),\dots,f_k^n(t_k^{m_k})).\end{aligned}$$

Now let us observe that, for every $(m_1,\dots,m_k)$, the sequences $(f_1^n(t_1^{m_1}))_n \subset X_1,\dots,(f_k^n(t_k^{m_k}))_n \subset X_k$ are weakly Cauchy, $(f_1^n(t_1^{m_1}))_n$ converges weakly to zero and, according to condition (i), $\Gamma(\{t_1^{m_1}\},\dots,\{t_k^{m_k}\})$ is completely continuous. Therefore it is clear that we can find $n_0 \in \mathbb{N}$ such that for every $n > n_0$,

$$\|T(f_1^n,\dots,f_k^n)\| \leqslant (k+1)\varepsilon$$

which proves that T is completely continuous.

Let us now prove that (2) implies (1): Suppose that K_1 is not scattered, and choose X_1 a Banach space without the Schur property. Then, let $\lambda \in \mathrm{rca}(\Sigma_1)$, $(A_1^{i,n})_{i,n} \subset \Sigma_1$, $(x_n)_n \subset X_1$, $(x_n^*)_n \subset X_1^*$ and $(r_n)_n \subset L^2(\lambda)$ be as λ, $(A_i^n)_{i,n}$, $(x_n)_n$, $(x_n^*)_n$ and $(r_n)_n$ in [3, Theorem 11] and let $(\alpha_2,\dots,\alpha_k) \in X_2^* \times \dots \times X_k^*$ be norm one forms. We define now $T \in \mathscr{L}^k(C(K_1,X_1),\dots,C(K_k,X_k);c_0)$ by

$$T(f_1,\dots,f_k) = \left(\left(\prod_{i=2}^k \alpha_i(f_i)\right)\int_{K_1}\langle f_1(t),x_n^*\rangle r_n(t)\,\mathrm{d}\lambda(t)\right)_n.$$

It is easily seen that T is well defined and that, if Γ is the representing polymeasure of T, for all Borel sets $(A_1,\dots,A_k) \in \Sigma_1 \times \dots \times \Sigma_k$

$$\Gamma(A_1,\dots,A_k)(x_1,\dots,x_k) = \left(\left(\prod_{i=2}^k \langle x_i\chi_{A_i},\alpha_i\rangle\right)\langle x_1,x_n^*\rangle\int_{A_1} r_n(t)\,\mathrm{d}\lambda(t)\right)_n.$$

Now, it is easy to prove that Γ satisfies conditions (i) and (ii). Yet T is not completely continuous. To see this, for every $i \in \{2, \ldots, k\}$ let $x_i \in X_i$ be such that $\alpha_i(x_i) = 1$ and let us consider the constant sequence $x_i^n = x_i$ for each $n \in \mathbb{N}$ and the sequence $f_1^n \subset C(K_1, X_1)$ as the sequence (f_n) at the end of the proof of [3, Theorem 11]. Then applying Lemma 1.5 and reasoning as in the end of the proof of [3, Theorem 11] we obtain what we wanted. $\square$

Our next theorem, which generalizes [3, Theorem 9], can now be proved by the reader.

Theorem 3.3. *Let $K_1, \ldots, K_k$ be compact Hausdorff spaces. Then the following are equivalent:*

(1) *For every $i \in \{1, \ldots, k\}$, K_i is scattered.*
(2) *For all Banach spaces $X_1, \ldots, X_k, Y$, a multilinear operator $T \in \mathscr{L}^k(C(K_1, X_1), C(K_2, X_2), \ldots, C(K_k, X_k); Y)$ is unconditionally converging if and only if its associated polymeasure Γ satisfies conditions (i′) and (ii′) of Proposition 2.4.*

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References

[1] *R. M. Aron, C. Hervés and M. Valdivia*: Weakly continuous mappings on Banach spaces. J. Funct. Anal. *52* (1983), 189–204.
[2] *F. Bombal*: Medidas vectoriales y espacios de funciones continuas. Publicaciones del Departamento de Análisis Matemático, Sección 1, No. 3. Fac. de Matemáticas, Universidad Complutense de Madrid, 1984.
[3] *F. Bombal and P. Cembranos*: Characterization of some classes of operators on spaces of vector valued continuous functions. Math. Proc. Cambridge Phil. Soc. *97* (1985), 137–146.
[4] *F. Bombal, M. Fernández and I. Villanueva*: Unconditionally converging multilinear operators. Math. Nachr. *226* (2001), 5–15.
[5] *F. Bombal and I. Villanueva*: Multilinear operators in spaces of continuous functions. Funct. Approx. Comment. Math. *XXVI* (1998), 117–126.
[6] *F. Bombal and I. Villanueva*: Polynomial sequential continuity on $C(K, E)$ spaces. J. Math. Anal. Appl. *282* (2003), 341–355.
[7] *J. Brooks and P. Lewis*: Linear operators and vector measures. Trans. Amer. Math. Soc. *192* (1974), 39–162.
[8] *F. Cabello, R. García and I. Villanueva*: Regularity and extension of multilinear forms on Banach spaces. Extracta Mathematicae (2000).
[9] *P. Cembranos and J. Mendoza*: Banach Spaces of Vector-valued Functions. Lecture Notes in Math. Vol. 1676. Springer, Berlin, 1997.
[10] *J. Diestel*: Sequences and Series in Banach Spaces. Graduate Texts in Math. Vol. 92. Springer, Berlin, 1984.

[11] *J. Diestel, H. Jarchow and A. Tonge*: Absolutely Summing Operators. Cambridge Stud. Adv. Math. Vol. 43. Cambridge Univ. Press, Cambridge, 1995.
[12] *N. Dinculeanu*: Vector Measures. Pergamon Press, 1967.
[13] *N. Dinculeanu and M. Muthiah*: Bimeasures in Banach spaces. Preprint.
[14] *I. Dobrakov*: On representation of linear operators on $C_0(T, X)$. Czechoslovak Math. J. *21(96)* (1971), 13–30.
[15] *I. Dobrakov*: On integration in Banach spaces. VIII (polymeasures). Czechoslovak Math. J. *37(112)* (1987), 487–506.
[16] *I. Dobrakov*: Representation of multilinear operators on $\times C_0(T_i, X_i)$, I. Atti Sem. Mat. Fis. Univ. Modena *XXXIX* (1991), 131–138.
[17] *M. Fernández Unzueta*: Unconditionally convergent polynomials in Banach spaces and related properties. Extracta Math. *12* (1997), 305–307.
[18] *M. González and J. Gutiérrez*: Orlicz-Pettis polynomials on Banach spaces. Monats. Math. *129* (2000), 341–350.
[19] *J. Gutiérrez and I. Villanueva*: Aron-Berner extensions and Banach space properties. Preprint.
[20] *B. Jefferies*: Radon polymeasures. Bull. Austral. Math. Soc. *32* (1985), 207–215.
[21] *B. Jefferies and W. Ricker*: Integration with respect to vector valued radon polymeasures. J. Austral. Math. Soc. (Series A) *56* (1994), 17–40.
[22] *H. E. Lacey*: The Isometric Theory of Classical Banach Spaces. Springer-Verlag, 1974.
[23] *P. Saab*: Weakly compact, unconditionally converging, and Dunford-Pettis operators on spaces of vector-valued continuous functions. Math. Proc. Camb. Phil. Soc. *95* (1984), 101–108.
[24] *I. Villanueva*: Representación de operadores multilineales en espacios de funciones continuas. PhD. Thesis. Universidad Complutense de Madrid, 1999.
[25] *I. Villanueva*: Completely continuous multilinear operators on $C(K)$ spaces. Proc. Amer. Math. Soc. *128* (1984), 793–801.

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