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# Tests in Weakly Nonlinear Regression Model *

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**Abstract**

In weakly nonlinear regression model a weakly nonlinear hypothesis can be tested by linear methods if an information on actual values of model parameters is at our disposal and some condition is satisfied. In other words we must know that unknown parameters are with sufficiently high probability in so called linearization region. The aim of the paper is to determine this region.

**Key words:** Regression model, nonlinear hypothesis, linearization.

**2000 Mathematics Subject Classification:** 62F03, 62J05

## 0 Introduction

A nonlinear hypothesis on model parameters in nonlinear regression model can be tested by linear methods if some conditions are satisfied. This condition is given in the form of the inclusion $\mathcal{E} \subset \mathcal{L}_T$ which must occur with sufficiently high probability. Here $\mathcal{E}$ is the $(1-\alpha)$-confidence region of the model parameters (for sufficiently small $\alpha$) and $\mathcal{L}_T$ is a special set in parameter space. The aim of the paper is to determine the set $\mathcal{L}_T$ (linearization region).

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# 1 Notation

Let $\mathbf{Y} \sim N_n[\mathbf{f}(\boldsymbol{\beta}), \sigma^2\mathbf{V}]$ be the regression model under consideration. Here $\mathbf{Y}$ is the $n$-dimensional normally distributed observation vector, $\mathbf{f}(\boldsymbol{\beta})$ is the mean value of the vector $\mathbf{Y}$, $\boldsymbol{\beta}$ is an unknown $k$-dimensional parameter, $\sigma^2\mathbf{V}$ is the covariance matrix of the vector $\mathbf{Y}$, $\sigma^2$ is known/unknown parameter and $\mathbf{V}$ is a given $n \times n$ positive definite matrix. The null hypothesis $H_0$ is given in the form $\mathbf{t}(\boldsymbol{\beta}) = \mathbf{0}$ and the alternative is $H_a : \mathbf{t}(\boldsymbol{\beta}) \neq \mathbf{0}$.

The functions $\mathbf{f}(\cdot)$ and $\mathbf{t}(\cdot)$ can be given in the form

$$\mathbf{f}(\boldsymbol{\beta}) = \mathbf{f}(\boldsymbol{\beta}^{(0)}) + \mathbf{F}\delta\boldsymbol{\beta} + \frac{1}{2}\boldsymbol{\kappa}(\delta\boldsymbol{\beta}), \quad \mathbf{t}(\boldsymbol{\beta}) = \mathbf{t}(\boldsymbol{\beta}^{(0)}) + \mathbf{T}\delta\boldsymbol{\beta} + \frac{1}{2}\boldsymbol{\tau}(\delta\boldsymbol{\beta}),$$

where $\delta\boldsymbol{\beta} = \boldsymbol{\beta} - \boldsymbol{\beta}^{(0)}$, $\boldsymbol{\beta}^{(0)}$ is an approximate value of the parameter $\boldsymbol{\beta}$,

$$\begin{aligned}
\mathbf{F} &= \left.\frac{\partial \mathbf{f}(\mathbf{u})}{\partial \mathbf{u}}\right|_{u=\beta^{(0)}}, \qquad \mathbf{T} = \left.\frac{\partial \mathbf{t}(\mathbf{u})}{\partial \mathbf{u}}\right|_{u=\beta^{(0)}},\\
\boldsymbol{\kappa}(\delta\boldsymbol{\beta}) &= [\kappa_1(\delta\boldsymbol{\beta}), \dots, \kappa_n(\delta\boldsymbol{\beta})]',\\
\kappa_i(\delta\boldsymbol{\beta}) &= (\delta\boldsymbol{\beta})' \left.\frac{\partial^2 f_i(\mathbf{u})}{\partial \mathbf{u}\partial \mathbf{u}'}\right|_{u=\beta^{(0)}} \delta\boldsymbol{\beta}, \quad i = 1, \dots, n,\\
\boldsymbol{\tau}(\delta\boldsymbol{\beta}) &= [\tau_1(\delta\boldsymbol{\beta}), \dots, \tau_q(\delta\boldsymbol{\beta})]',\\
\tau_i(\delta\boldsymbol{\beta}) &= (\delta\boldsymbol{\beta})' \left.\frac{\partial^2 t_i(\mathbf{u})}{\partial \mathbf{u}\partial \mathbf{u}'}\right|_{u=\beta^{(0)}} \delta\boldsymbol{\beta}, \quad i = 1, \dots, q.
\end{aligned}$$

Let the rank of the $n \times k$ matrix $\mathbf{F}$ be $r(\mathbf{F}) = k < n$ and the rank of the $q \times k$ matrix $\mathbf{T}$ be $r(\mathbf{T}) = q < k$.

# 2 Determination of the region $\mathcal{L}_T$

The linearized form of the model and the hypothesis is

$$\mathbf{Y} - \mathbf{f}_0 \sim N_n(\mathbf{F}\delta\boldsymbol{\beta}, \sigma^2\mathbf{V}), \quad \mathbf{T}\delta\boldsymbol{\beta} = \mathbf{0}. \tag{1}$$

(The vector $\boldsymbol{\beta}^{(0)}$ should be chosen such that $\mathbf{t}(\boldsymbol{\beta}^{(0)}) = \mathbf{0}$.)

The quadratized form of the model and the hypothesis is

$$\mathbf{Y} - \mathbf{f}_0 \sim N_n\left(\mathbf{F}\delta\boldsymbol{\beta} + \frac{1}{2}\boldsymbol{\kappa}(\delta\boldsymbol{\beta}), \sigma^2\mathbf{V}\right), \quad \mathbf{T}\delta\boldsymbol{\beta} + \frac{1}{2}\boldsymbol{\tau}(\delta\boldsymbol{\beta}) = \mathbf{0}. \tag{2}$$

**Lemma 2.1** *If the model (1) is valid, the test of the hypothesis is*

$$(\widehat{\delta\boldsymbol{\beta}})'\mathbf{T}'[\mathbf{T}(\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{T}]^{-1}\mathbf{T}\widehat{\delta\boldsymbol{\beta}} \sim \begin{cases} \sigma^2\chi^2_q(0) & \text{if } H_0 \text{ is true,}\\ \sigma^2\chi^2_q(\delta) & \text{if } H_0 \text{ is not true.}\end{cases}$$

*Here* $\widehat{\delta\boldsymbol{\beta}} = (\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{F}'\mathbf{V}^{-1}(\mathbf{Y} - \mathbf{f}_0)$ *and the parameter of noncentrality* $\delta$ *is*

$$\delta = [E(\widehat{\delta\boldsymbol{\beta}})]'\mathbf{T}'[\mathbf{T}(\mathbf{F}'\mathbf{V}^{-1}\mathbf{T})^{-1}\mathbf{T}']^{-1}\mathbf{T}E(\widehat{\delta\boldsymbol{\beta}})/\sigma^2.$$

**Proof** Cf. [4], chpt. 4. □

**Lemma 2.2** *If the model (2) is valid, then under the null hypothesis* $H_0$ : $\mathbf{T}\delta\boldsymbol{\beta} + \frac{1}{2}\boldsymbol{\tau}(\delta\boldsymbol{\beta}) = \mathbf{0}$, *it is valid*

$$(\widehat{\delta\boldsymbol{\beta}})'\mathbf{T}'[\mathbf{T}(\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{T}]^{-1}\mathbf{T}\widehat{\delta\boldsymbol{\beta}} \sim \sigma^2\chi^2_q(\Delta).$$

*Here*

$$\Delta = \frac{1}{\sigma^2}\Big[-\frac{1}{2}\boldsymbol{\tau}(\mathbf{K}_T\delta\mathbf{u}) + \mathbf{T}(\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{F}'\mathbf{V}^{-1}\frac{1}{2}\boldsymbol{\kappa}(\mathbf{K}_T\delta\mathbf{u})\Big]'$$
$$\times\Big[\mathbf{T}(\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{T}'\Big]^{-1}\Big[-\frac{1}{2}\boldsymbol{\tau}(\mathbf{K}_T\delta\mathbf{u}) + \mathbf{T}(\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{F}'\mathbf{V}^{-1}\frac{1}{2}\boldsymbol{\kappa}(\mathbf{K}_T\delta\mathbf{u})\Big]$$

*and* $\delta\boldsymbol{\beta} = \mathbf{K}_T\delta\mathbf{u}+$ *terms of higher orders. The* $k\times(k-q)$ *matrix* $\mathbf{K}_T$ *is of the rank* $r(\mathbf{K}_T) = k-q$ *and it satisfies the equality* $\mathbf{T}\mathbf{K}_T = \mathbf{0}$.

**Proof** In model (2) the mean value of the estimator

$$\widehat{\delta\boldsymbol{\beta}} = (\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{F}'\mathbf{V}^{-1}(\mathbf{Y}-\mathbf{f}_0)$$

is

$$E(\widehat{\delta\boldsymbol{\beta}}) = (\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{F}'\mathbf{V}^{-1}\Big[\mathbf{F}\delta\boldsymbol{\beta} + \frac{1}{2}\boldsymbol{\kappa}(\delta\boldsymbol{\beta})\Big]$$
$$= \delta\boldsymbol{\beta} + \frac{1}{2}(\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{F}'\mathbf{V}^{-1}\boldsymbol{\kappa}(\delta\boldsymbol{\beta}).$$

Under the null hypothesis $H_0 : \mathbf{T}\delta\boldsymbol{\beta} + \frac{1}{2}\boldsymbol{\tau}(\delta\boldsymbol{\beta}) = \mathbf{0}$ it is valid

$$\delta\boldsymbol{\beta} = \mathbf{K}_T\delta\mathbf{u} - \mathbf{T}^-\frac{1}{2}\boldsymbol{\tau}(\mathbf{K}_T\delta\mathbf{u}) + \text{terms of higher orders.}$$

Thus

$$\mathbf{T}E(\widehat{\delta\boldsymbol{\beta}}) = \mathbf{T}\Big[\mathbf{K}_T\delta\mathbf{u} - \mathbf{T}^-\frac{1}{2}\boldsymbol{\tau}(\mathbf{K}_T\delta\mathbf{u})\Big] + \mathbf{T}\frac{1}{2}(\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{F}'\mathbf{V}^{-1}\boldsymbol{\kappa}(\mathbf{K}_T\delta\mathbf{u}) + \dots$$

In the last term the vector $\delta\boldsymbol{\beta}$ is substituted by $\mathbf{K}_T\delta\mathbf{u}$. Since $\mathbf{T}\mathbf{K}_T = \mathbf{0}$ and $\mathbf{T}\mathbf{T}^- = \mathbf{I}$, the expression $[E(\widehat{\delta\boldsymbol{\beta}})]'\mathbf{T}'\Big[\mathbf{T}(\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{T}'\Big]^{-1}\mathbf{T}E(\widehat{\delta\boldsymbol{\beta}})/\sigma^2 = \Delta$ can be written in the form given in the statement. (Cf. also [1] and [2].) □

**Definition 2.3** The quantity

$$K^{(test)}(\boldsymbol{\beta}_0) = \sup\left\{\frac{2\sqrt{\mathbf{b}'\Big\{\mathbf{T}[\mathbf{F}'(\sigma^2\mathbf{V})^{-1}\mathbf{F}]^{-1}\mathbf{T}'\Big\}^{-1}\mathbf{b}}}{\delta\mathbf{u}'\mathbf{K}_T'\mathbf{F}'(\sigma^2\mathbf{V})^{-1}\mathbf{F}\mathbf{K}_T\delta\mathbf{u}} : \delta\mathbf{u}\in R^{k-q}\right\}$$
$$= \sigma\sup\left\{\frac{2\sqrt{\mathbf{b}'\Big\{\mathbf{T}[\mathbf{F}'\mathbf{V}^{-1}\mathbf{F}]^{-1}\mathbf{T}'\Big\}^{-1}\mathbf{b}}}{\delta\mathbf{u}'\mathbf{K}_T'\mathbf{F}'\mathbf{V}^{-1}\mathbf{F}\mathbf{K}_T\delta\mathbf{u}} : \delta\mathbf{u}\in R^{k-q}\right\} = \sigma\mathbf{K}_0^{(test)}(\boldsymbol{\beta}_0),$$

where

$$\mathbf{b} = -\frac{1}{2}\boldsymbol{\tau}(\mathbf{K}_T\delta\mathbf{u}) + \mathbf{T}(\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{F}'\mathbf{V}^{-1}\frac{1}{2}\boldsymbol{\kappa}(\mathbf{K}_T\delta\mathbf{u}),$$

is a measure of nonlinearity for test.

**Theorem 2.4** *Let $\delta_{max}$ be a solution of the equation*

$$P\{\chi^2_q(\delta_{max}) \geq \chi^2_q(0; 1-\alpha)\} = \alpha + \varepsilon.$$

*Here $\chi^2_q(0; 1-\alpha)$ is $(1-\alpha)$-quantile of the chi-square distribution with $q$ degrees of freeedom. Then*

$$\delta\boldsymbol{\beta} \in \mathcal{L}_T = \left\{\mathbf{K}_T\delta\mathbf{u} : \delta\mathbf{u}'\mathbf{K}'_T(\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{K}_T\delta\mathbf{u} \leq \frac{2\sigma\sqrt{\delta_{max}}}{K_0^{(test)}(\boldsymbol{\beta}_0)}\right\}$$

$$\Rightarrow P_{H_0}\left\{\widehat{\delta\boldsymbol{\beta}}'\mathbf{T}'[\mathbf{T}(\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{T}']^{-1}\mathbf{T}\widehat{\delta\boldsymbol{\beta}} \geq \sigma^2\chi^2_q(0; 1-\alpha)\right\} \leq \alpha + \varepsilon.$$

**Proof** In the model (2) the random variable $\widehat{\delta\boldsymbol{\beta}}'\mathbf{T}'\left[\mathbf{T}(\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{T}'\right]^{-1}\mathbf{T}\widehat{\delta\boldsymbol{\beta}}$ is distributed as $\sigma^2\chi^2_q(\Delta)$, where $\Delta$ is given by Lemma 2.2. With respect to Definition 2.3 we have

$$2\sqrt{\mathbf{b}'\left[\mathbf{T}(\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{T}'\right]^{-1}\mathbf{b}} \leq K_0^{(test)}(\boldsymbol{\beta}_0)\delta\mathbf{u}'\mathbf{K}'_T(\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{K}_T\delta\mathbf{u}.$$

If

$$K_0^{(test)}(\boldsymbol{\beta}_0)\delta\mathbf{u}'\mathbf{K}'_T(\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{K}_T\delta\mathbf{u} \leq 2\sigma\sqrt{\delta_{max}},$$

then

$$2\sqrt{\frac{\mathbf{b}'\left[\mathbf{T}(\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{T}'\right]^{-1}\mathbf{b}}{\sigma^2}} = 2\sqrt{\Delta} \leq 2\sqrt{\delta_{max}}$$

$$\Rightarrow P_{H_0}\left\{\chi^2_q(\Delta) \geq \chi^2_q(0; 1-\alpha)\right\} \leq \alpha + \varepsilon.$$

Thus

$$\delta\mathbf{u}'\mathbf{K}'_T(\mathbf{F}'\mathbf{V}^{-1}\mathbf{F})^{-1}\mathbf{K}_T\delta\mathbf{u} \leq \frac{2\sigma\sqrt{\delta_{max}}}{K_0^{(test)}(\boldsymbol{\beta}_0)} \Rightarrow P_{H_0}\left\{\chi^2_q(\Delta) \geq \chi^2_q(0; 1-\alpha)\right\} \leq \alpha+\varepsilon.$$

□

**Remark 2.5** If $\mathbf{T}\delta\boldsymbol{\beta} + \frac{1}{2}\boldsymbol{\tau}(\delta\boldsymbol{\beta}) \neq \mathbf{0}$, i.e. the null hypothesis is not true, then

$$\delta = \left[\mathbf{T}\delta\boldsymbol{\beta} + \frac{1}{2}\mathbf{T}\mathbf{C}_0^{-1}\mathbf{F}'\mathbf{V}^{-1}\boldsymbol{\kappa}(\delta\boldsymbol{\beta})\right]'(\mathbf{T}\mathbf{C}_0\mathbf{T}')^{-1}\left[\mathbf{T}\delta\boldsymbol{\beta} + \frac{1}{2}\mathbf{T}\mathbf{C}_0^{-1}\mathbf{F}'\mathbf{V}^{-1}\boldsymbol{\kappa}(\delta\boldsymbol{\beta})\right],$$

where $\mathbf{C}_0 = \mathbf{F}'\mathbf{V}^{-1}\mathbf{F}$. Thus at the alternative hypothesis the power function

$$p(\delta\boldsymbol{\beta}) = P_{H_a}\left\{\chi^2_t(\delta) \geq \chi^2_t(0; 1-\alpha)\right\}$$

has a different values at points $\delta\boldsymbol{\beta}$ and $-\delta\boldsymbol{\beta}$, respectively, opposite to the case of the null hypothesis where these values are identical. It makes an investigation of a linearization region for the power function more complicated than it is at the null hypothesis.

**Lemma 2.6** *The* $(1-\alpha)$*-confidence ellipsoid for the parameter* $\boldsymbol{\beta}$ *in the model (1) is*

$$\mathcal{E} = \{\mathbf{u} : (\mathbf{u} - \widehat{\delta\boldsymbol{\beta}})'\mathbf{F}'\mathbf{V}^{-1}\mathbf{F}(\mathbf{u} - \widehat{\delta\boldsymbol{\beta}}) \leq \sigma^2\chi_k^2(0; 1-\alpha)\}.$$

**Proof** Cf. [4], chpt. 4. □

**Remark 2.7** If

$$\sigma^2\chi_q^2(0; 1-\alpha) \ll \frac{2\sigma\sqrt{\delta_{max}}}{K_0^{(test)}(\boldsymbol{\beta}_0)},$$

then the model (2) can be substituted by (1) when the test of hypothesis is performed. Thus the value of $\sigma$ must satisfy the strong inequality $\sigma \ll \sigma_{crit}$, where

$$\sigma_{crit} = \frac{2\sqrt{\delta_{max}}}{\chi_q^2(0; 1-\alpha)K_0^{test}(\boldsymbol{\beta}_0)}.$$

## 3 Numerical example

Let a class of regression function be $\{f(x) = \beta_1 \exp(-\beta_2 x) : \beta_1, \beta_2 \in R^1\}$. The null hypothesis states that all these functions attain the same value equal to 1 at the point $x = 10$ (cf. also [3]).

The measurement is realized at the points $x_i \in \{1, 3, 5, 7, 9\}$. Thus

$$H_0 : \ln\beta_1 - 10\beta_2 = 0, \quad H_a : \ln\beta_1 - 10\beta_2 \neq 0.$$

The regression model is

$$\mathbf{Y} \sim N_5[\mathbf{f}(\boldsymbol{\beta}), \sigma^2\mathbf{I}], \quad \boldsymbol{\beta} \in R^2,$$

where

$$\{\mathbf{f}(\boldsymbol{\beta})\}_i = \beta_1 \exp(-\beta_2 x_i), \quad i = 1, 2, 3, 4, 5,$$

$$t(\boldsymbol{\beta}) = \ln\beta_1 - 10\beta_2 = 0,$$

$$\{\mathbf{F}\}_{i,\cdot} = \left(\exp(-\beta_2^{(0)} x_i), -\beta_1 x_i \exp(-\beta_2^{(0)} x_i)\right), \quad i = 1, \dots, 5,$$

$$\mathbf{F}_i = \begin{pmatrix} 0, & -x_i \exp(-\beta_2^{(0)} x_i) \\ -x_i \exp(-\beta_2^{(0)} x_i), & \beta_1^{(0)} x_i^2 \exp(-\beta_2^{(0)} x_i) \end{pmatrix}, \quad i = 1, \dots, 5,$$

$$\mathbf{T} = \left(\frac{1}{\beta_1^{(0)}}, -10\right), \quad \mathbf{K}_T = \begin{pmatrix} \beta_1^{(0)} \\ 0.1 \end{pmatrix},$$

$$\kappa_i(\mathbf{K}_T\delta u) = (\delta u)^2 \left(-0.2 x_i \beta_1^{(0)} \exp(-\beta_2^{(0)} x_i) + 0.01\beta_1^{(0)} x_i^2 \exp(-\beta_2^{(0)} x_i)\right), \quad i = 1, \dots, 5,$$

$$\mathbf{F}'\mathbf{F} = \begin{pmatrix} \sum_{i=1}^5 \exp(-2\beta_2^{(0)} x_i), & -\sum_{i=1}^5 \beta_1^{(0)} x_i \exp(-2\beta_2^{(0)} x_i) \\ -\sum_{i=1}^5 \beta_1^{(0)} x_i \exp(-2\beta_2^{(0)} x_i), & \sum_{i=1}^5 (\beta_1^{(0)})^2 x_i^2 (\exp(-2\beta_2^{(0)} x_i) \end{pmatrix},$$

$$\mathbf{b} = -\frac{1}{2}\tau(\mathbf{K}_T\delta u) + \mathbf{T}(\mathbf{F}'\mathbf{F})^{-1}\mathbf{F}'\frac{1}{2}\boldsymbol{\kappa}(\mathbf{K}_T\delta u) = \frac{1}{2}(1 + A)(\delta u)^2,$$

where

$$A = \left(\frac{1}{\beta_1^{(0)}}, -10\right) (\mathbf{F}'\mathbf{F})^{-1}\mathbf{F}' \times$$
$$\times \begin{pmatrix} \vdots \\ -0.2x_i\beta_1^{(0)} \exp(-\beta_2^{(0)}x_i) + 0.01\beta_1^{(0)}x_i^2 \exp(-\beta_2^{(0)}x_i) \\ \vdots \end{pmatrix}.$$

Further

$$K^{(test)}(\boldsymbol{\beta}_0) = \sigma \frac{\sqrt{(1+A)\left[(1/\beta_1^{(0)}, -10)(\mathbf{F}'\mathbf{F})^{-1}\begin{pmatrix} 1/\beta_1^{(0)} \\ -10 \end{pmatrix}\right]^{-1}(1+A)}}{(\beta_1^{(0)}, 0.1)\mathbf{F}'\mathbf{F}\begin{pmatrix} \beta_1^{(0)} \\ 0.1 \end{pmatrix}} = \sigma K_0^{(test)},$$

$$K_0^{(test)} = \frac{|1+A|}{(\beta_1^{(0)}, 0.1)\mathbf{F}'\mathbf{F}\begin{pmatrix} \beta_1^{(0)} \\ 0.1 \end{pmatrix}\sqrt{(1/\beta_1^{(0)}, -10)(\mathbf{F}'\mathbf{F})^{-1}\begin{pmatrix} 1/\beta_1^{(0)} \\ -10 \end{pmatrix}}}.$$

$$P\{\chi_1^2(\delta_{max}) \geq \chi_1^2(0; 0.95)\} = 0.05 + 0.05 \Rightarrow \delta_{max} = 0.426, \quad \chi_1^2(0; 0.95) = 3.84,$$

$$\sigma_{crit} = \frac{2\sqrt{0.451}}{3.84K_0^{(test)}(\boldsymbol{\beta}_0)} = \frac{0.349774}{K_0^{(test)}(\boldsymbol{\beta}_0)}.$$

Some numerical values were obtained by the help of [5] and they are given in the following table.

**Table 1**

| $\boldsymbol{\beta}^{(0)}$ | $\binom{0.1}{-0.230}$ | $\binom{0.2}{-0.161}$ | $\binom{0.3}{-0.120}$ | $\binom{0.5}{-0.069}$ |
|---|---|---|---|---|
| $K_0^{(test)}(\boldsymbol{\beta}_0)$ | 0.613 | 0.406 | 0.306 | 0.206 |
| $\sigma_{crit}$ | 0.554 | 0.837 | 1.110 | 1.649 |
| $\boldsymbol{\beta}^{(0)}$ | $\binom{1}{0}$ | $\binom{5}{0.161}$ | $\binom{10}{0.230}$ | $\binom{15}{0.271}$ |
| $K_0^{(test)}(\boldsymbol{\beta}_0)$ | 0.113 | 0.024 | 0.012 | 0.008 |
| $\sigma_{crit}$ | 3.01 | 14.15 2 | 28.31 | 42.47 |

If the value of $\sigma$ in the actual experiment is smaller than $\sigma_{crit}$ from Table 1, then the theory of linear regression model can be used when the test of hypothesis is performed.

It is advisable to notice a strong dependence of the quantities $K_0^{(test)}$ and $\sigma_{crit}$, respectively, on the vector $\boldsymbol{\beta}^{(0)}$.

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## References

[1] Kubáček, L., Kubáčková, L.: *Regression models with a weak nonlinearity.* Technical report Nr. 1998.1, Universität Stuttgart, 1998, 1–67.

[2] Kubáček, L., Kubáčková, L.: Statistics and Metrology. *Vyd. Univ. Palackého, Olomouc*, 2000 (in Czech).

[3] Kubáček, L., Kubáčková, L.: *Statistical problems of a determination of isobestic points.* Folia Fac. Sci. Nat. Univ. Masarykianae Brunensis, Math. **11** (2002), 139–150.

[4] Rao, C. R.: Linear Statistical Inference and its Application. *J. Wiley, New York–London–Sydney*, 1965.

[5] Tesaříková, E., Kubáček, L.: *A test in nonlinear regression models. Demoprogram.* Department of Algebra and Geometry, Faculta of Science, Palacký University, Olomouc, 2004 (in Czech).