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INFINITE-DIMENSIONAL COMPLEX PROJECTIVE SPACES AND COMPLETE INTERSECTIONS

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Abstract. Let V be an infinite-dimensional complex Banach space and $X \subset \mathbf{P}(V)$ a closed analytic subset with finite codimension. We give a condition on X which implies that X is a complete intersection. We conjecture that the result should be true for more general topological vector spaces.

Keywords: infinite-dimensional complex projective space, infinite-dimensional complex manifold, complete intersection, complex Banach space, complex Banach manifold

MSC 2000: 32K05

1. Complete intersections

For any locally convex and Hausdorff complex topological vector space V, let $\mathbf{P}(V)$ be the projective space of all one-dimensional linear subspaces of V. We recall that a holomorphic embedding $j\colon X \to Y$ between infinite-dimensional complex manifolds is said to be locally split if for every $P \in X$ there is an open neighborhood U of $j(P)$ in Y and a complex manifold A such that $U \cong (U \cap j(X)) \times A$; of course, if a good Inverse Function Theorem is true for the local models of X and Y to have local splitness it is sufficient that for every $P \in X$ the linear subspace $(\mathrm{d}j)(P)_*(T_PX)$ is a closed supplemented subspace of the tangent space $T_{j(P)}Y$. It seems much easier to work with complete intersections subvarieties of projective spaces, even the infinite-dimensional ones (see [1] for complex analysis on them). The aim of this note is the proof of the following result.

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Theorem 1. *Let V be an infinite-dimensional complex Banach space and X a closed analytic subset of finite definition of $\mathbf{P}(V)$. Assume the existence of a closed analytic subset Γ of X such that $X \setminus \Gamma$ is smooth and locally split in $\mathbf{P}(V)$ and Γ is contained in $\mathbf{P}(M)$ with M a linear subspace (not necessarily closed) of V with infinite algebraic codimension, and that the following Condition τ holds:*

Condition τ: *There are finitely many continuous homogeneous polynomials h_i, $1 \leqslant i \leqslant x$, on V such that $X = \{Q \in \mathbf{P}(V)\colon\ h_i(Q) = 0$ for all $i\}$ and such that for every $P \in X \setminus \Gamma$ the differentials $\mathrm{d}h_1(P), \dots, \mathrm{d}h_x(P)$ generate a supplementary subspace for the tangent space of X at P inside the tangent space of $\mathbf{P}(V)$ at P.*

Then X is a complete intersection of finitely many hypersurfaces of $\mathbf{P}(V)$.

An immediate corollary of Theorem 1 is that a smooth locally split analytic subset of $\mathbf{P}(V)$ satisfying Condition τ is a complete intersection when V is infinite-dimensional. Obviously, this is no longer true when V is finite-dimensional. Indeed, Condition τ is always satisfied when V is finite-dimensional. Since we conjecture that the result is true for more general infinite-dimensional topological vector spaces (see Remark 2), we consider the projective space $\mathbf{P}(V)$ for more general vector spaces V. To get good properties of the projective space $\mathbf{P}(V)$ we will use the fact that any codimension one closed linear subspace of V has a closed supplement; here we need for instance that V is locally convex. Under this assumption $\mathbf{P}(V)$ is covered by infinitely many charts $\{U_v\}_{v \in V^* \setminus \{0\}}$ such that each U_v is biholomorphic to a codimension one closed linear subspace of V. Indeed, for any continuous linear form $v\colon V \to \mathbb{C}$, $v \neq 0$, the projective space $\mathbf{P}(\mathrm{Ker}(v))$ is a closed codimension one linear projective subspace of $\mathbf{P}(V)$ and we set $U_v := \mathbf{P}(V) \setminus \mathbf{P}(\mathrm{Ker}(v))$. If V is locally convex, then $\mathrm{Ker}(v)$ has a topological supplement, i.e. $V \cong \mathrm{Ker}(v) \oplus \mathbb{C}$. We have $U_v \cong \mathrm{Ker}(v)$: send any $a \in \mathrm{Ker}(v)$ into the element of U_v determined by the equivalence class of $(a, 1) \in \mathrm{Ker}(v) \oplus \mathbb{C}$. Notice that $U_{v'} = U_v$ if and only if $v' = \lambda v$ for some $\lambda \in \mathbb{C} \setminus \{0\}$. We will use these charts to define the locally free rank one sheaves $\mathcal{O}_{\mathbf{P}(V)}(t)$, $t \in \mathbb{Z}$. For an equivalent definition when V is a Banach space, see [2], §7. We set $\mathcal{O}_{\mathbf{P}(V)}(0) := \mathcal{O}_{\mathbf{P}(V)}$. Now we define $\mathcal{O}_{\mathbf{P}(V)}(1)$. We assume that the holomorphic line bundle $\mathcal{O}_{\mathbf{P}(V)}(1)$ is the unique holomorphic line bundle on $\mathbf{P}(V)$ whose restriction to each U_v is trivial and such that for all $v, w \in V^* \setminus \{0\}$ its glueing datum over $U \cap U_w$ is w/u. If $t \geqslant 2$ let $\mathcal{O}_{\mathbf{P}(V)}(t)$ be the tensor power of t copies of the locally free rank one sheaf $\mathcal{O}_{\mathbf{P}(V)}(1)$. If $t < 0$ let $\mathcal{O}_{\mathbf{P}(V)}(t)$ be the dual of the sheaf $\mathcal{O}_{\mathbf{P}(V)}(-t)$. If $t < 0$, then $H^0(\mathbf{P}(V), \mathcal{O}_{\mathbf{P}(V)}(t)) = 0$. If $t \geqslant 0$, then $H^0(\mathbf{P}(V), \mathcal{O}_{\mathbf{P}(V)}(t))$ is isomorphic (as a complex vector space) to the set of all continuous homogeneous degree t polynomials on V.

Lemma 1. *Fix integers $k > 0$, $x > 0$, $y > 0$, a_i, $1 \leqslant i \leqslant x$, and b_j, $1 \leqslant j \leqslant y$, such that $a_1 \leqslant \ldots \leqslant a_x$ and $b_1 \leqslant \ldots \leqslant b_y$. Assume the existence of a* *surjection* $\Phi\colon \bigoplus_{i=1}^{x} \mathcal{O}_{\mathbf{P}^k}(a_i) \to \bigoplus_{j=1}^{y} \mathcal{O}_{\mathbf{P}^k}(b_j)$. *Then* $x \geqslant y$ *and either* $x > k$ *or there is a subset* S *of* $\{1, \ldots, x\}$ *with* $\sharp(S) = y$, *say* $\{i_1, \ldots, i_y\}$ *with* $i_1 < \ldots < i_y$, *such that* $a_{i_j} = b_j$ *for all* j *and* $\Phi'\colon \bigoplus_{j=1}^{y} \mathcal{O}_{\mathbf{P}^k}(a_{i_j}) \to \bigoplus_{j=1}^{y} \mathcal{O}_{\mathbf{P}^k}(b_j)$ *is an isomorphism, where* $\Phi' := \Phi\Big| \bigoplus_{j=1}^{y} \mathcal{O}_{\mathbf{P}^k}(a_{i_j})$ *is the restriction of* Φ.

Proof. The inequality $x \geqslant y$ is obvious. Assume $x \leqslant k$. First assume $y = 1$. The existence of a surjection Φ is equivalent to the existence of x hypersurfaces $Y_i = \{f_i = 0\}$ with f_i a homogeneous form of degree $b_1 - a_i$ such that $Y_1 \cap \ldots \cap Y_x = \emptyset$, with the convention that $Y_i = \mathbf{P}^k$ if $f_i \equiv 0$, $Y_i = \emptyset$ if $b_1 = a_i$ and f_i is a non-zero constant and that $f_i \equiv 0$ if $b_1 < a_i$. Since any k hypersurfaces of the complex projective space $\mathbf{P}^k$ have a common point, we obtain a contradiction, unless $Y_i = \emptyset$ for at least one index, say the index m. In this case $\Phi|\mathcal{O}_{\mathbf{P}^k}(a_{i_m})$ is an isomorphism. Now assume $y \geqslant 2$. Compose Φ with the surjection $\bigoplus_{j=1}^{y} \mathcal{O}_{\mathbf{P}^k}(b_j) \to \mathcal{O}_{\mathbf{P}^k}(b_1)$. By the first part we obtain $a_{i_m} = b_1$ for some m and we may split off a rank one factor in the matrix of homogeneous forms representing Φ. Then we conclude by induction on y. $\square$

Proof of Theorem 1. Since X is finitely defined in a smooth manifold, X_{reg} is a non-empty open dense subset of X and $\mathrm{Sing}(X)$ is a nowhere dense closed analytic subset of X ([2], Ch. V of Part II). By assumption the linear span $\mathbf{P}(M)$ of $\mathrm{Sing}(X)$ has infinite algebraic codimension in $\mathbf{P}(V)$.

First Claim: We claim that the proofs in [3], §1.2, yield that for every $P \in X_{\mathrm{reg}} \setminus \Gamma$ and every integer $z > 0$, X contains a z-dimensional linear subspace A_z such that $P \in A_z$.

Proof of the First Claim: By the Inverse Function Theorem (with respect to a finite-codimensional submanifold) Condition τ implies that for every $P \in X_{\mathrm{reg}}$ the germs at P of $h_1, \ldots, h_x$ generate the ideal sheaf of X in $\mathbf{P}(V)$. First, we will prove the First Claim in the case $z = 1$. Let u_i, $1 \leqslant i \leqslant s$, be finitely many continuous degree $\geqslant 1$ homogeneous polynomials on V. Since V is infinite-dimensional, $\{u_1 = \ldots = u_s = 0\}$ is a non-empty closed analytic subset of $\mathbf{P}(V)$. Hence any two non-empty closed analytic subsets of finite definition of $\mathbf{P}(V)$ have non-empty intersection, i.e. the Connectedness Principle stated in [3], p. 1190, is true in our set-up. Fix $P \in X \setminus \Gamma$. Since X is smooth at P, each h_i has a Taylor expansion at P. Since h_i is a polynomial, this expansion has only finitely many non-zero terms, say $h_i = \sum_{j \leqslant s_i(P)} h_{i,j}$ with each $h_{i,j}$ continuous non-zero homogeneous polynomial on V with respect to the translation of V which sends a representative $\overline{P}$ of P into 0. Since $h_i(P) = 0$, among the $h_{i,j}$ there is no constant polynomial.

Set $C(X,P) := \{Q \in \mathbf{P}(V)\colon h_{i,j}(\bar{Q}) = 0$ for any representative $\bar{Q} \in V$ of Q and all $1 \leqslant i \leqslant x,\ j \leqslant s_i(P)\}$. Thus $C(X,P) \cap X$ is a non-empty finitely determined closed analytic subset of $\mathbf{P}(V)$ containing P. For any $Q \in (C(X,P) \cap X) \setminus \{P\}$ the line $\langle\{P,Q\}\rangle$ is contained in X. Since $C(X,P) \cap X$ is a closed finitely defined analytic subset of $\mathbf{P}(V)$, there is $Q \in (C(X,P) \cap X) \setminus \{P\}$ and hence the First Claim is true if $z = 1$. Now assume $z \geqslant 2$ and that the First Claim is true for the integer $z' := z - 1$. By the inductive assumption there is a $(z-1)$-linear subspace J contained in $(C(X,P) \cap X) \setminus \{P\}$. Take as A_z the linear span of the set $\{P\} \cup J$, i.e. the union of all lines containing P and intersecting J.

Second Claim: For every $P \in X \setminus X \cap \mathbf{P}(M)$ and every integer $z > 0$ the analytic set X contains a z-dimensional linear subspace A_z such that $P \in A_z$ and $A_z \cap \mathbf{P}(M) = \emptyset$. Furthermore, we may find a chain of such linear spaces, i.e. a family of linear spaces $\{A_z\}_{z \geqslant 1}$ such that $P \in A_1$, $\dim(A_z) = z$, $A_z \cap \mathbf{P}(M) = \emptyset$ and $A_z \subset A_{z+1}$ for every $z \geqslant 1$. Furthermore, given any finite-dimensional linear subspace $A \subset X \setminus X \cap \mathbf{P}(M)$ there is such a chain satisfying the additional condition that $A \subset A_z$ for all $z \gg 0$.

Proof of the Second Claim: Since M has infinite codimension in V, there is a linear subspace W of M with infinite (and countable) algebraic dimension (depending only on P and M) such that $P \in \mathbf{P}(W)$ and $W \cap M = \{0\}$ (i.e. $\mathbf{P}(W) \cap \mathbf{P}(M) = \emptyset$). Hence $\mathbf{P}(W) \cap B = \emptyset$. Recall that $X = \{h_1 = \ldots = h_x = 0\}$. Tyurin's algebraic proof recalled in the proof of the First Claim works (in his own set-up) inside $\mathbf{P}(W)$ using the finitely many homogeneous polynomials $h_1|W, \ldots, h_x|W$. We obtain $A_z \subset W$ and hence $\mathbf{P}(A_z) \cap B = \emptyset$.

Third Claim: For every $P \in X \setminus X \cap \mathbf{P}(M)$, every integer $z > 0$ and all lines $L, D \subset X$ such that $P \in L \cap D$ and $L \cup D \subset X \setminus \mathbf{P}(M)$ there are linear spaces $A_z, B_z \subset X \setminus X \cap \mathbf{P}(M)$ such that $L \subset A_z$, $D \subset B_z$, $\dim(A_z) = \dim(B_z) = z$ and $\dim(A_z \cap B_z) = z - 1$.

Proof of the Third Claim: Since M has infinite codimension in V and $L \cup D \subset X \setminus X \cap \mathbf{P}(M)$, there is a linear subspace W of M with infinite (and countable) algebraic dimension (depending only on P, L, D and M) such that $L \cup D \subset \mathbf{P}(W)$ and $W \cap M = \{0\}$, i.e. $\mathbf{P}(W) \cap \mathbf{P}(M) = \emptyset$. Work inside the projective space $\mathbf{P}(W)$ with countable algebraic dimension using the fact that $X \cap \mathbf{P}(W)$ has $h_1|W, \ldots, h_x|W$ as global equations. The case $z = 2$ is [3], Lemma 1.5; the general case is very similar and left to the reader.

Fourth Claim: For any $P, Q \in X \setminus X \cap \mathbf{P}(M)$ there are lines $L, L' \subset X$ such that $P \in L$, $Q \in L'$, $L \cap L' \neq \emptyset$ and $L \cup L' \subset X \setminus X \cap \mathbf{P}(M)$.

Proof of the Fourth Claim: For the existence of L, L', see [3], Lemma 1.4 at p. 1193. For reader's sake we reproduce Tyurin's proof in our set-up. As in the proof of the First Claim set $C(X,P) := \{Q \in \mathbf{P}(V)\colon h_{i,j}(\bar{Q}) = 0$ for any representative $\bar{Q} \in V$ of Q and all $1 \leqslant i \leqslant x$, $j \leqslant s_i(P)\}$. Thus $C(X,P) \cap X$ is a non-empty finitely determined closed analytic subset of $\mathbf{P}(V)$ containing P. For any $Q \in (C(X,P) \cap X) \setminus \{P\}$ the line $\langle\{P,Q\}\rangle$ is contained in X. By the Connectedness Principle we have $C(X,P) \cap C(X,Q) \cap X \setminus \{P,Q\} \neq \emptyset$. For any $A \in C(X,P) \cap C(X,Q) \cap X \setminus \{P,Q\}$ we may take $L := \langle\{P,A\}\rangle$ and $L' := \langle\{Q,A\}\rangle$.

Fifth Claim: For any two lines $L, D \subset X \setminus X \cap \mathbf{P}(M)$ there are planes $A, B \subset X \setminus X \cap \mathbf{P}(M)$ such that $L \subset A$, $D \subset B$ and $A \cap B \neq \emptyset$.

Proof of the Fifth Claim: As in the previous case work on a suitable projective space and then apply [3], Corollary 1.6.

For any $P \in X \setminus \Gamma$ let $c(P)$ be the codimension of X in $\mathbf{P}(V)$ at P. By assumption $0 \leqslant c(P) < +\infty$ and $c(P)$ is an integer valued locally constant function on $X \setminus \Gamma$. We will see in Remark 1 that X is irreducible and hence $X \setminus \Gamma$ is connected. Thus there is an integer $c \geqslant 0$ such that $c(P) = c$ for all $P \in X \setminus \Gamma$. Let $\mathcal{I}$ denote the ideal sheaf of $X \in \mathbf{P}(V)$. The sheaf $\mathcal{I}/\mathcal{I}^2$ is an $\mathcal{O}_X$-module and it is usually called the conormal sheaf of X in $\mathbf{P}(V)$, while its dual is usually called the normal sheaf of X in $\mathbf{P}(V)$. In a neighborhood of each $P \in X \setminus \Gamma$ the conormal sheaf is a holomorphic vector bundle with rank equal to the codimension of X in $\mathbf{P}(V)$; indeed, here we use that the embedding of $X \setminus \Gamma$ in $\mathbf{P}(V)$ is locally split.

For any line $L \subset X \setminus \Gamma$ there are uniquely determined integers $a_{1,L} \geqslant \ldots \geqslant a_{c,L}$ such that $\mathcal{I}/\mathcal{I}^2|L$ has splitting type $a_{1,L} \geqslant \ldots \geqslant a_{c,L}$, i.e. such that $\mathcal{I}/\mathcal{I}^2|L \cong \bigoplus_{i=1}^{c} \mathcal{O}_L(a_{i,L})$.

Sixth Claim: There are integers $a_1 \geqslant \ldots \geqslant a_c$ such that $a_{i,L} = a_i$ for every line $L \subset X \setminus X \cap \mathbf{P}(M)$. Furthermore, $\mathcal{I}/\mathcal{I}^2|A \cong \bigoplus_{i=1}^{c} \mathcal{O}_A(a_i)$ for every finite-dimensional linear subspace $A \subset X \setminus X \cap \mathbf{P}(M)$.

Proof of the Sixth Claim: Fix any chain $\{A_z\}_{z \geqslant 1}$ such that $\dim(A_z) = z$, $A_z \cap \mathbf{P}(M) = \emptyset$ and $A_z \subset A_{z+1}$ for every $z \geqslant 1$ (Second Claim). By Lemma 1 there are c uniquely determined integers $a_1 \geqslant \ldots \geqslant a_c$ such that $\mathcal{I}/\mathcal{I}^2|A_z \cong \bigoplus_{i=1}^{c} \mathcal{O}_{A_z}(a_i)$ for all $z \geqslant 1$. As in Tyurin's proof we see that $a_{i,L} = a_i$ for every line $L \subset X \setminus X \cap \mathbf{P}(M)$; we use the Fourth and the Fifth Claim to get that the splitting type of the restriction of $\mathcal{I}/\mathcal{I}^2$ is the same for all lines contained in the fixed chain and for one line not contained in it. This implies that the integers $a_1 \geqslant \ldots \geqslant a_c$ do not depend on the choice of the chain $\{A_z\}_{z \geqslant 1}$. The last assertion of the Sixth Claim follows from the last assertion of the Second Claim.

Set $d_i := \deg(h_i)$, $1 \leqslant i \leqslant x$. Since the polynomials $h_1, \ldots, h_x$ vanish on X and generate $\mathcal{I}$ at each point of $X \setminus \Gamma$, they induce a surjective map $\Psi\colon \bigoplus_{j=1}^{x} \mathcal{O}_{X \setminus B} \to \mathcal{I}/\mathcal{I}^2|X \setminus \Gamma$; here we use Condition τ. Notice that this implies $x \geqslant c$, because

$\mathcal{I}/\mathcal{I}^2|X\setminus\Gamma$ has rank c. Fix any linear space $A \subset X\setminus X\cap\mathbf{P}(M)$ such that $\dim(A) \geqslant x$ and set $\Phi := \Psi|A$. Apply Lemma 1 to Φ and take c polynomials $f_1, \ldots, f_c$ from the polynomials $h_1, \ldots, h_x$ such that $\deg(f_i) = -a_i$ and the restriction of Φ to the corresponding factors is an isomorphism. Since X has codimension c in $\mathbf{P}(V)$, Theorem 1 will follow from our Last Claim below.

Last Claim: X is the complete intersection of the hypersurfaces $f_1, \ldots, f_c$.

Proof of the Last Claim: Set $\widetilde{X} := \{f_1 = \ldots = f_c = 0\}$. Hence $X \subseteq \widetilde{X}$. Fix $P \in X\setminus\Gamma$. Since $f_1, \ldots, f_c$ generate the finitely generated $\mathcal{O}_{X,P}$-module $(\mathcal{I}/\mathcal{I}^2)_P$, they generate the $\mathcal{O}_X$-module $\mathcal{I}_P$ (Nakayama's lemma); to apply Nakayama's lemma it is essential that $\mathcal{I}_P$ is a finitely generated $\mathcal{O}_{X,P}$-module. Thus we may use $f_1, \ldots, f_c$ instead of $h_1, \ldots, h_x$ for Condition τ. By Condition τ we have $\widetilde{X} \setminus \Gamma = X \setminus \Gamma$. Even more: by Condition τ and the Inverse Function Theorem (with respect to finite codimensional submanifolds) there is an open neighborhood U of $X\setminus\Gamma$ in $\mathbf{P}(V)$ such that $U \cap \widetilde{X} = U \cap X$. Since $\widetilde{X}$ and X are finitely defined in $\mathbf{P}(V)$, to prove the Last Claim it is sufficient to prove that for every irreducible component Z of $\widetilde{X}$ we have $Z \cap (X \setminus \Gamma) \neq \emptyset$; indeed, since $U \cap \widetilde{X} = U \cap X$, if $Z \cap (X \setminus \Gamma) \neq \emptyset$, then $Z \cap X$ contains a non-empty open subset of Z and hence $Z \subseteq X$ by the irreducibility of Z. We have $Z \cap (X \setminus \Gamma) \neq \emptyset$ by the Connectedness Principle proved in the proof of the First Claim. □

Remark 1. The condition "$X\setminus\Gamma$ is smooth and Γ is contained in a linear projective subspace (not necessarily closed) of $\mathbf{P}(V)$ with infinite algebraic codimension" is very strong. It implies that X is irreducible, for the following reason. Assume that X has at least two irreducible components, say Y_1 and Y_2, with $Y_1 \neq Y_2$. Every point of $Y_1 \cap Y_2$ is a singular point of X. Each Y_i is closed and finitely defined in $\mathbf{P}(V)$. $Y_1 \cap Y_2$ is a non-empty finitely defined closed analytic subset of $\mathbf{P}(V)$ by the Connectedness Principle ([3], p. 1190), which was checked in our set-up in the proof of the First Claim in the proof of Theorem 1.

Remark 2. Let V be an infinite-dimensional complex topological vector space. The proof of Theorem 1 shows that in the statement of Theorem 1 instead of assuming that V is a Banach space it is sufficient (except at a critical point: the Last Claim) to assume that V has the following properties:

(a) Every finite-dimensional linear subspace of V is closed and has a closed supplement.

(b) Every finite-codimensional closed linear subspace of V has a closed supplement.

(c) For every closed analytic subset $X \subset \mathbf{P}(V)$ with finite definition there are finitely many continuous homogeneous polynomials $h_1, \ldots, h_x$ such that $X = \{h_1 = \ldots = h_x = 0\}$ and the ideal sheaf of X in $\mathbf{P}(V)$ is generated by $h_1, \ldots, h_x$ at each point of X_{reg}.

Properties (a) and (b) are satisfied if V is locally convex and Hausdorff (Hahn-Banach). Of course, instead of Condition (c) we may just assume that the closed analytic subset of finite definition $X \subset \mathbf{P}(V)$ we want to study satisfies this condition. The critical point in the Last Claim is that for its proof one needs a weak form of the Inverse Function Theorem. The form used in the Last Claim seems to be false outside the Banach setting.

Look again to the proof of Theorem 1. In the First Claim we assumed $P \in X \setminus \Gamma$ and we only needed that B has infinite codimension (as an analytic subset) in X, not that the linear span of B in $\mathbf{P}(V)$ has infinite algebraic codimension. The latter assumption was only needed from the Second Claim on. Hence it may be possible to have partial generalizations of Theorem 1 either assuming less on V or assuming less on Γ or weakening Condition τ.

A k n o w l e d g e m e n t. We want to thank the referee for essential remarks.

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