BOOK REVIEWS

H. Grassmann: EXTENSION THEORY. Translated by L. C. Kannenberg, History of Mathematics 19, Amer. Math. Soc., London Math. Soc., 2000, xviii+411 pages, ISBN 0-8218-2031-1, USD 75.–.

This is (the first) English translation of Grassmann's treatise "Die Ausdehnugslehre", published first in 1862. The translation under review is not based on this original edition, but uses "Die Ausdehnugslehre" in the form appearing in the "Hermann Grassmanns gesammelte mathematische und physikalische Werke" published by B. G. Teubner under the editorship of F. Engel and one of Grassmann's sons, Hermann Junior. Here it is necessary to mention that these two editors brought some alteration in the text of the first edition. Their main hope was that this book would become a fundamental monograph especially for references. The translator decided to preserve the rearrangements from Teubner's edition, nevertheless he restored Grassmann's original text to its proper places. This text is marked with {curly brackets}. The parts which substituted this original text in Teubner's edition can be found in Editorial Notes. Following Editorial Notes we find Supplementary Notes the aim of which is to clarify some points in the main body of the book. The book itself consists of two parts, Part 1. The Elementary Conjuctions of Extensive Magnitudes, and Part 2. The Theory of Functions. Both parts may seem at the first glance to be far away from each other. But very quickly we understand Grassmann's deep insight into mathematics. He was able both to develop in detail the necessary technique and simultaneously to present generalizations clarifying several fundamental concepts of mathematics. Though Grassmann's language does not coincide with our contemporary mathematical language, the reading of the text represents no great problem. I can understand that a mathematician permanently using Grassmann's notions and results will not find enough time to read the whole text. In that case I would like to recommend to read at least the original Grassmann's foreword, which I consider to be extremely interesting.

Jiří Vanžura, Brno

J. H. Davis: DIFFERENTIAL EQUATIONS WITH MAPLE. An Interactive Approach. Birkhäuser, Boston, 2001, 432 pages, ISBN 0-8176-4181-5, sFr. 108.–.

The textbook provides a student-friendly introduction to the basics of the theory of differential equations. Theory is presented in a well understandable form, without proofs. The exposition is supported by symbolic computation software MAPLE. This makes it possible to present an interactive and more vivid approach to the subject. The book is divided into 3 parts. The first (MAPLE Use and Programming) gives a description of basic commands and fundamentals of MAPLE programming language. The second part (Differential Equations) starts with examples from various fields in which ordinary differential equations occur. Then, usual basic topics from the theory of ordinary differential equations, like explicitly solvable equations, linear equations of the first or higher orders, Laplace transform methods, systems of equations, stability, periodic solutions etc. are explained. One section is devoted to the introduction to numerical methods. In particular, main attention is paid to the Runge-Kutta methods. Practical importance of ordinary differential equations is illustrated by a section devoted to applications to the circuit theory.

Finally, also partial differential equations are very briefly touched. The third part of the book (MAPLE Application Topics) is devoted to more advanced applications of MAPLE programming language, like plotting, matrix calculus, Laplace transforms or Runge-Kutta designs. Moreover, construction and installation of MAPLE packages are described and illustrated by several examples (root locus, bode plots, LQR utility). The second (theoretical) part of the book contains references to related topics in the third part and vice-versa. Each chapter contains examples providing motivation for practising the theory or MAPLE tools. Enclosed is CD-ROM with MAPLE sources for the large projects of the final part of the book, as well as sample worksheets for smaller examples. Additional related material should be loadable from Birkhäuser's or author's websites.

For understanding, only basic knowledge of mathematical analysis is needed. So, the book can be recommended to students, instructors, engineers or mathematicians who wish to get a quick overview of differential equations and their possible treating by means of the MAPLE software.

Milan Tvrdý, Praha

Jürgen Koslowski, Austin Melton: CATEGORICAL PERSPECTIVES. Birkhäuser, Boston, 2001, 296 pages, ISBN 0-8176-4186-6, DM 196.–.

The book is dedicated to Professor G. E. Strecker on the occasion of his sixtieth birthday. It consists of a preface by Jürgen Koslowski and Austin Melton and of 15 papers of various styles and topics. Among them, there is a paper "10 Rules for Surviving as a Mathematician and Teacher" by G. E. Strecher himself. This paper is written with a gentle sense of humor and every reader will surely enjoy it. Also the papers by Y. T. Rhinghost and A. Zmrzlina are written in a style not quite usual in mathematical literature: as a conversation of a teacher and his (very diligent and very striven) student. Nevertheless, they also discuss themes from category theory, namely the properties of the endofunctors of the category Set of all sets and maps. The other papers are written in the style usual for the papers in good mathematical journals. The introductory paper by Lutz Schröder presents the basic notions and facts of the category theory, so making the book understandable to any reader with some mathematical background. The other papers written by Austin Melton, G. Castellini, H. L. Bentley, G. Preuss, L. Janos, V. A. Lemin, B. S. W. Schroeder, R. El Bashir, H. Herrlich and M. Hušek present results of their research in various fields of the category theory or related problems, e.g. categorial closure operators, reflective and coreflective categories, problems in categories of concrete structures (graphs, ultrametric spaces, extensions of maps in various topological categories) and other questions.

Although the book is far from giving a complete overview of the present stage of the category theory and its possible perspectives, it will be helpful to everybody who wishes to know it.

Věra Trnková, Praha

J. Aguadé, C. Broto, C. Casacuberta: COHOMOLOGICAL METHODS IN HOMOTOPY THEORY. Progress in Mathematics, Birkhäuser, Boston, 2001, ISBN 3-7643-6588-9, hardcover, 424 pages, DM 196,–.

The book contains 25 contributions delivered either at aconference on algebraic topology in Spain (1998) or at a semester organized by Centre de Recerca Matematica (also in 1998). Most of the contributions are related to homotopy theory in a wide sense, some of them deal with various related aspects of group theory, quantum field theory, differential geometry. Although most of the articles are of a research character and bring new results from single subtopics, altogether they give "the state of knowledge in a broad portion of modern homotopy theory" (quoted from Introduction). There are also some articles that have a wider character and bring new look at some known situations (e.g., Tate cohomology, loop spaces and Hopf algebras, and Eilenberg-Moore spaces).

The book can certainly serve to specialists both as a reference book and as a source of open problems and next research.

Miroslav Hušek, Praha

Triebel, Hans: THE STRUCTURE OF FUNCTIONS. Birkhäuser, Basel, 2001, 425 + ix pages, DM 196,–.

The book consists of four chapters: I. Decomposition of functions, II. Sharp inequalities, III. Fractal elliptic operators, and IV. Truncations and semi-linear equations.

In Chapter I the author presents a new approach to the Besov spaces B^s_{pq} and the Triebel-Lizorkin spaces F^s_{pq}. To this end, he introduces quarkonial (subatomic) decompositions of functions and distributions and then defines function spaces mentioned above in terms of the related quarkonial decompositions. Finally, he shows that the spaces defined in this way coincide with the Besov and Triebel-Lizorkin spaces introduced in the classical manner. First the author considers spaces on the whole $\mathbb{R}^n$ and then on domains, on manifolds and fractals. The last two sections of this chapter deal with Taylor expansions of distributions and with traces on sets.

In Chapter II only those spaces $B^s_{pq}(\mathbb{R}^n)$ and $F^s_{pq}(\mathbb{R}^n)$ are considered which are subspaces of $L_1^{\mathrm{loc}}(\mathbb{R}^n)$. Author's intent is to describe singularity behaviour of $f \in A^s_{pq}(\mathbb{R}^n)$, where A^s_{pq} stands either for B^s_{pq} or F^s_{pq}. Of interest are only spaces A^s_{pq} which contain a function whose non-increasing rearrangement $f^* = f^*(t)$ tends to infinity as $t \to 0_+$. Therefore, the case $s > n/p$ is excluded (since then $A^s_{pq}(\mathbb{R}_n)$ is continuously embedded in $L_\infty(\mathbb{R}^n)$). The remaining cases are called sub-critical ($s < n/p$) and critical ($s = n/p$). The author starts with the description of embeddings of spaces $A^s_{pq}(\mathbb{R}^n)$ in $L_1^{\mathrm{loc}}(\mathbb{R}^n)$, $L_\infty(\mathbb{R}^n)$ and other classical targets. If the space $A^s_{pq}(\mathbb{R}^n)$ is embedded in $L_1^{\mathrm{loc}}(\mathbb{R}^n)$ but not in $L_\infty(\mathbb{R}^n)$, following D. Haroske, the author uses the notions of the growth envelope function $\mathcal{E}_G A^s_{pq}$ and the growth envelope $\mathfrak{E}_G(A^s_{pq})$ to treat the subcritical and critical cases. If $s > n/p$, then $A^s_{pq}(\mathbb{R}^n) \subset L_\infty(\mathbb{R}^n)$ and so the functions $f \in A^s_{pq}(\mathbb{R}^n)$ are bounded. However, there is one case which, in the context of continuity, has attracted special attention—namely the case $s = 1 + n/p$. To investigate this super-critical case, the author makes use of the following fact:

$$A^{n/p}_{pq}(\mathbb{R}^n) \subset L_\infty(\mathbb{R}^n) \Longleftrightarrow A^{1+n/p}_{pq}(\mathbb{R}^n) \subset \mathrm{Lip}(\mathbb{R}^n).$$

As a result, the growth envelope $\mathcal{E}_G A^{n/p}_{pq}$ is unbounded if and only if the so called continuity envelope $\mathcal{E}_C A^{1+n/p}_{pq}$ is unbounded. Therefore, the super-critical case $s = 1 + n/p$ is treated by lifting by 1 the inequalities and also the extremal functions responsible for the sharpness in the critical case. The last two sections of this chapter are devoted to Hardy type inequalities (which are obtained as a consequence of sharp imbeddings) and to connections of envelope functions with Green's functions.

In Chapter III the author investigates elliptic operators, mainly the Laplacian in various fractal settings, and demonstrates the relationship between some basic notions of fractal geometry and spectral theory. To this end, the author makes use of the quarkonial decompositions developed in Chapter I. First he treats the case when all the fractals involved are d-sets (Sections 19–21); this setting appears to be natural in the case of B^s_{pq} and F^s_{pq} spaces. The last two sections of Chapter III are devoted to the case when d-sets are replaced by the so-called (d,ψ)-sets. (A typical example of $\psi = \psi(r)$ is the function $|\log r|^b$, $b \in \mathbb{R}$). Then the underlying spaces are properly modified and instead of B^s_{pq} and F^s_{pq} spaces, respectively, the author works with $B^{(s,\psi)}_{pq}$ and $F^{(s,\psi)}_{pq}$ spaces. (The inspiration goes back to a paper by H.-G. Leopold.)

Chapter IV deals with truncations in function spaces $\mathbb{A}^s pq(\mathbb{R}^n)$, where $\mathbb{A}^s_{pq}(\mathbb{R}^n)$ is the real part of the space $A^s_{pq}(\mathbb{R}^n)$. Assuming that $A^s_{pq}(\mathbb{R}^n) \subset L_1^{\mathrm{loc}}(\mathbb{R}^n)$, the author studies the truncation operators T^+ and T given in $\mathbb{A}^s_{pq}(\mathbb{R}^n)$ by

$$f(x) \to T^+ f(x) := f_+(x) = \max\{f(x); 0\}, \quad x \in \mathbb{R}^n,$$

and

$$f(x) \to Tf(x) := |f(x)| = 2T^+ f(x) - f(x), \quad x \in \mathbb{R}^n.$$

In the case that $A^s_{pq}(\mathbb{R}^n) = B^s_{pq}(\mathbb{R}^n)$, the author characterizes those p, q and s for which T^+ (and hence also T) is bounded in $\mathbb{A}^s_{pq}(\mathbb{R}^n)$ (in the case when $A^s_{pq} = F^s_{pq}$, a special case remains open). Beside the boundedness of T^+ and T, the Lipschitz continuity of T^+ and T in $\mathbb{A}^s_{pq}$ is treated. The result is negative—operators T^+ and T are not Lipschitz continuous in the spaces mentioned above. Together with the quarkonial decompositions, the results on truncations in $\mathbb{A}^s_{pq}$ are used to establish a new regularity theory for some semi-linear integral and differential equations of the type

$$u(x) = \int_{\mathbb{R}^n} K(x-y) u_+(y)\,\mathrm{d}y + k(x), \quad x \in \mathbb{R}^n,$$

or

$$(-\Delta + id)u(x) = c|u(x)| + k(x), \quad x \in \mathbb{R}^n,$$

in spaces having the truncation property.

The book is well written and can be recommended to mathematicians interested in function spaces and their application to PDE's.

Bohumír Opic, Praha

C. S. Kubrusly: ELEMENTS OF OPERATOR THEORY. Birkhäuser, Basel, 2001, ISBN 0-8176-4174-2, hardcover, 544 pages, sFr 128.–.

This is a very nice introductory course to functional analysis and operator theory aimed at, roughly, advanced undergraduate or beginning graduate students. Compared to some classics in the area, it is at about the same level as A.E. Taylor's *Introduction to functional analysis,* yet distinctly more elementary than W. Rudin's *Functional analysis* or J. B. Conway's *A course in operator theory.* The six chapters in the book cover, in turn, the necessary prerequisites from set theory (Chapter 1), linear algebra (Chapter 2), and topology (Chapter 3); then Banach spaces (Chapter 4) and Hilbert spaces (Chapter 5) are dealt with, and the exposition culminates by the spectral theorem for (bounded) normal operators (Chapter 6). The presentation is well organized and very pedagogical, with many exercises, examples, illuminating remarks, motivations, etc. To summarize, the book is, in the reviewer's opinion, an excellent resource for anyone—from students to specialists in other areas—seeking a self-contained and well-presented introduction into the field of functional analysis and operator theory.

Miroslav Engliš, Praha

F. G. Timmesfeld: ABSTRACT ROOT SUBGROUPS AND SIMPLE GROUPS OF LIE-TYPE. Monographs in Mathematics, vol. 95. Birkhäuser, Boston, 2001, ISBN 3-7643-6532-3, hardcover, 404 pages, EUR 110.–.

The goal of this book is to give a systematic treatment of groups generated by a conjugacy class of subgroups satisfying certain additional conditions. This theory was originally developed for finite groups in the seventies (M. Aschbacher, B. Fischer, F. G. Timmesfeld) and later extended to infinite groups by the author. The book consists of five chapters. Chapter I deals with the so-called rank one groups (groups generated by two different nilpotent subgroups satisfying a certain condition). In Chapter II, abstract root subgroups are introduced and their theory developed. Chapter III is the main part of the book, dealing with classification of groups generated by abstract root subgroups. Chapter IV is devoted to a revision of the root involution classification. The final Chapter V discusses some applications (quadratic pairs, subgroups generated by long root elements, chamber transitive subgroups of Lie type groups). A large part of the material appears for the first time in a book form. There is a very helpful list of notation, an index and numerous exercises. Apart from requiring reasonable knowledge of abstract group theory and classical groups, the book is quite self-contained and should be accessible for graduate students, while representing a handy and useful resource for anyone actively working in the field.

Miroslav Engliš, Praha

V. P. Pikulin, S. I. Pohozaev: EQUATIONS IN MATHEMATICAL PHYSICS. A Practical Course. Birkhäuser, Basel, 2001, viii+ 207 pages, EUR 98.–.

The main goal of this handbook is to treat and demonstrate the fundamental methods for solving classical problems of mathematical physics.

The three chapters of the book are devoted (as usual) to linear elliptic, hyperbolic and parabolic problems.

Methods of Fourier analysis, conformal mappings and of Green's functions are used and attention is paid also to integral transforms and perturbations.

The book is addressed to readers familiar with the basics of calculus including ordinary differential equations. No problems concerning existence and the spaces to which solutions of the problems belong are discussed in the book.

The book serves students of engineering as well as engineers and scientists. Problems with answers are included and therefore the book can be used for teaching equations of mathematical physics in the classical manner.

Štefan Schwabik, Praha

Anatoly N. Kochubie: PSEUDO-DIFFERENTIAL EQUATIONS AND STOCHASTICS OVER NON-ARCHIMEDEAN FIELDS. Marcel Dekker, New York, 2001, xi+316 pages, ISBN 0-8247-0655-2, \$ 165,–.

Up to isomorphism, there exist only three connected locally compact topological fields: the field of real numbers $\mathbb{R}$, the field of complex numbers $\mathbb{C}$ and the non-commutative field of quaternions $\mathbb{H}$. If we relax the connectedness assumption to non-discreteness, then finite extensions of the fields $\mathbb{Q}_p$ of p-adic numbers and of the fields of Laurent series with coefficients in the finite field $\mathbb{Z}_p = \mathbb{Z}/p\mathbb{Z}$ must be added to the list. These disconnected non-discrete locally compact fields are called local fields. Since the invention of p-adic numbers by K. Hensel more than a hundred years ago, analysis over local fields has been studied intensively within the framework of number theory. The obtained results are in a sense parallel to those known in the classical real or complex analysis, but there are also striking differences, especially in methods and techniques. Recently, in the author's terms, a new "strong impetus to the development of non-Archimedean analysis was given by the hypothesis about the possible p-adic structure of physical space-time at sub-Planck distances ($\leqslant 10^{-33}$ cm)". The aim of the book under review is to present in a systematic manner the current state of the theory of pseudo-differential operators on $\mathbb{C}$-valued functions on local fields and of related topics in the theory of stochastic processes, this choice being motivated by problems arising in mathematical physics.

Let us mention very briefly some of the topics treated in the book. The author starts with Vladimirov's fractional differentiation operator D^α, $\alpha > 0$, and proceeds to more general second order elliptic and hyperbolic operators and their Green functions (Chapter 2). The spectral theory of D^α and of related Schrödinger type operators is thoroughly investigated (Chapter 3). In Chapter 4, a heat equation (1) $\partial_t u + D^\alpha u = f$ on a local field and related parabolic problems are treated. Markov processes whose transition probability is the fundamental solution to (1) are introduced here. Their study continues also in the next chapter, where the reader may find e.g. the corresponding theory of stochastic differential equations. Gaussian measures on infinite-dimensional vector spaces over local fields are among the topics covered by Chapter 6. The last, seventh chapter is devoted to stochastic processes indexed by local fields.

Kochubei's book is basically a monograph, intended for specialists. However, the author tried to make it accessible even for a novice in p-adic analysis and in the first chapter introduces concisely the necessary notions and results concerning local fields. Therefore, the book may be recommended to everybody who wants to get acquainted with this rapidly advancing branch of mathematics.

Jan Seidler, Praha

Antonín Sochor: KLASICKÁ MATEMATICKÁ LOGIKA. Karolinum, Praha, 2001, 402 pages, ISBN 80-246-0218-0.

This book is a new textbook of mathematical logic, written in Czech language. It covers wide range of areas: accessible treatment of fundamentals such as propositional and predicate calculus, more advanced chapters on proof theory and model theory, extensive treatment of arithmetic and Gödel's theorems, and the most challenging chapter on decidability and interpolation.

The whole book is written very carefully, with attention to many details. Some topics are not usually covered in textbooks (or only briefly mentioned), such as logics with multiple sorts of objects, ultraproduct constructions, or Goodstein theorem. Already substantial material covered by the book is further extended by numerous exercises.

This new Czech textbook on mathematical logic will certainly become a valuable resource not only for students but also for researchers in other areas of mathematics.

Jiří Sgall, Praha

Martin Bohner, Allan Peterson: DYNAMIC EQUATIONS ON TIME SCALES. An introduction with applications. Birkhäuser, Basel, 2001, ISBN 0-8176-4225-0, 358 pages, DM 144,–.

The theory of *time scales* (an alternative terminology is a *measure chain*) was introduced by Stefan Hilger in 1988 in his dissertation and the basic ideas of this theory are summarized in his paper published in Result. Math. 18 (1990), 18–56. A time scale $\mathbb{T}$ is any closed subset of real numbers $\mathbb{R}$. For a function $f\colon \mathbb{T} \to \mathbb{R}$, a generalized derivative f^Δ is defined in such a way that it reduces to the usual derivative f' if $\mathbb{T} = \mathbb{R}$ and to the forward difference Δf if

$\mathbb{T} = \mathbb{Z}$-the set of integers. A *dynamic equation* on time scale is the equation for an unknown function which appears in the equation with its derivatives (possibly of higher orders). Hence, dynamic equations are differential equations if $\mathbb{R} = \mathbb{T}$ and difference equations if $\mathbb{T} = \mathbb{Z}$.

The reviewed book is an introduction to the study of dynamic equations on time scales. Many results concerning differential equations carry over quite easily to the corresponding results for difference equations, while other results seem to be completely different in nature from their continuous counterparts. The study of dynamic equations on time scales reveals such discrepancies. Moreover, since there are many other time scales than just the set of real numbers or the set of integer, also much more general results are presented in the book.

Here is the list of chapters of the book: 1. The Time Scales Calculus, 2. First Order Linear Equations, 3. Second Order Linear Equations, 4. Self-Adjoint Equations, 5. Linear Systems and Higher Order Equations, 6. Dynamic Inequalities, 7. Linear Symplectic Dynamic Systems, 8. Extensions. The authors start in Chapter 1 with the fundamental results of time scale calculus giving the basic properties of the generalized derivative and integral. In Chapter 2 the so-called cylindric transformation is introduced and the exponential function on time scales is studied. This generalized exponential function is then used to solve the first order linear dynamic equation. Chapters 3, 4 are devoted to linear second order dynamic equations. First the attention is focused on equations with constant coefficients (both homogeneous and nonhomogeneous) and explicit formulas for their solutions using the generalized trigonometric and hyperbolic equations are presented. Wronski determinants are introduced and Abel's theorem is used to develop a reduction order technique to find a second solution in case one solution is already known. The time scale Laplace transformation is introduced and many of its properties are derived as well. Next, self-adjoint second order dynamic equations on time scales are studied. The classical concepts as Green's function, Riccati equation, Prüfer transformation, oscillation, disconjugacy, eigenvalue problem and many others are extended to time scale dynamic equations.

Chapter 5 is concerned with linear systems of dynamic equations and higher order dynamic equations. Uniqueness and existence theorems are presented, and the matrix exponential on a time scale is introduced. Among others, asymptotic properties of solutions of linear dynamic systems are investigated. Chapter 6 deals with dynamic inequalities on time scales. Analogues of the classical Gronwall's, Hölder's and Jensen's inequalities are presented. In particular, it is also devoted to Opial's and Lyapunov's inequalities. Chapter 7 contains a brief treatment of linear symplectic dynamic systems on time scales. This is a very general class of systems that contains, for example, linear Hamiltonian dynamic systems which in turn contain Sturm-Liouville dynamic equations of higher order and self-adjoint vector dynamic equations. The last chapter deals with various extensions of the results treated in the book.

The book is self-contained and the results are presented in such a way that they are understandable for everybody who passed the basic course of the calculus and linear algebra. More than 200 exercises of various degree of difficulty are presented, some of them are actually research problems in the field of time scale dynamic equations. Compared with another book devoted to dynamic time scale equations, the book of Lakshmikantham, V.; Sivasundaram, S.; Kaymakcalan, B.: *Dynamic Systems on Measure Chains*, Kluwer Academic Publishers, Dordrecht 1996, the reviewed book is aimed more to linear equations and contains many results achieved during the last 5 years.

Ondřej Došlý, Brno

V. V. Beletsky: ESSAYS ON THE MOTION OF CELESTIAL BODIES. Birkhäuser, Basel, 2001, 392 pages, EUR 155,–.

The book is the English translation of the Russian book whose first edition was published in Nauka publishing house in 1972 and which was awarded the 1999 F. A. Zander Prize of the Russian Academy of Sciences. The book contains twelve essays covering interesting problems in mechanics of space flights. Some of the topics discussed are an unperturbed and perturbed motion of a satellite, the problem of two fixed centers, a motion of a satellite under the influence of low thrust ion or plasma engines or radiation pressure, stability and resonance in the solar system, the restricted three body problem, a flight to the Moon, interplanetary flights and so on.

The book is written with high pedagogical proficiency, each mathematical step is clearly explained and has a physical motivation. Let us conclude by quoting from the review of the first edition in Priroda by V. I. Arnold and Ya. B. Zeldovich: "… The general impression that the 'Essays' make is not that of a boring lesson, but rather of a discussion with a brilliant, knowledgeable and wise interlocutor."

Vojtěch Pravda, Praha

N. Balakrishnan, I.A. Ibragimov, V.B. Nevzorov (eds.): ASYMPTOTIC METHODS IN PROBABILITY AND STATISTICS WITH APPLICATIONS. Birkhäuser, Boston, 2001, xxiv+550 pages, ISBN 3-7643-4214-5, DM 220.–.

In order to celebrate the 50th anniversary of establishing the Chair of Probability and Statistics at the St. Petersburg State University, an International Conference was held in 1998. The 38 papers in this volume of the series Statistics for Industry and Technology form a selection of invited talks presented on this occasion.

The title of the book is slightly misleading, as the range of incorporated and solved or discussed problems is much wider than the title suggests. It reflects the tradition of the St. Petersburg School represented e.g. by P.L. Chebyshev, A.M. Lyapunov, A.A. Markov and Yu.V. Linnik, and many lecturers were former alumni of the Chair.

The book is divided into 10 parts the headings of which at best reflect the variability of subjects treated: probability distribution (I) and their characterization (II), probabilities and measures in high-dimensional spaces (III), weak and strong limit theorems (IV), large deviation probabilities (V), empirical processes, order statistics and records (VI), estimation of parameters and hypothesis testing (VII), random walks (VIII), miscellanea (IX), and applications to finance (X).

The papers are mostly theoretical, direct applications are rare. The book is useful by a long list of references turning attention to otherwise unknown papers scattered in Russian journals and in technical reports or proceedings published by various Russian institutions.

Ivan Saxl, Praha

K. H. Borgwardt: OPTIMIERUNG, OPERATIONS RESEARCH, SPIELTHEORIE. Mathematische Grundlagen. Birkhäuser, Basel, 2001, xx + 622 pages, DM 88.–.

The present volume is in fact a collection of four textbooks devoted to linear optimization, nonlinear optimization, integer and combinatorial optimization, and game theory.

The topics presented in the book represent the main mathematical tools for educating students of economy. All the four parts are presented in a thorough and precise form with complete proofs many problems and examples worked out in all the details. Numerical approaches to the solving of the problems are described as well.

The book is designed for teachers and for students and it can be worked out in its respective parts by the student himself. Only some basic knowledge of linear algebra and of the calculus is required.

Štefan Schwabik, Praha

A. O. Petters, H. Levine, J. Wambsganss: SINGULARITY THEORY AND GRAVITATIONAL LENSING. Birkhäuser, Boston, 2001, 603 pages, EUR 98.–.

This monograph studies in detail gravitational lensing—deflection of light in gravitational field. The book is divided into three parts. Introductory Part I briefly summarizes theoretical and experimental history of gravitational lensing. Part II is devoted to astrophysical aspects of gravitational lensing. After summarizing basic physical concepts, the authors discuss how to employ lensing to determine the nature of dark matter, the structure of quasars, Hubble's constant and so on. The most extensive Part III studies mathematical aspects of lensing. The following physical and mathematical problems are examined in the framework of thin-screen, weak-field lensing: the number of images produced by a generic gravitational lens system, lower bounds on the total magnification of a lensed light source and its magnification cross section, generic local and global properties of critical curves and the link between the generic properties of caustics and lens. A rigorous theory of thin-screen, weak-field lensing that applies to any finite number of lens planes is presented. Mathematical topics such as Morse theory and stability and genericity theories of Thom, Whitney, Mather and Arnold are discussed and applied. Several results are published here for the first time.

Alena Pravdová, Praha

N. Karapetiants, S. Samko: EQUATIONS WITH INVOLUTIVE OPERATORS. Birkhäuser, Boston, 2001, ISBN 0-8176-4157-2, hardcover, 456 pages, EUR 120.–.

The book is an enlarged, updated and almost completely rewritten version of the Russian edition (*Equations with involutive operators and their applications,* Izd. Rost. Univ., Rostov-na-Donu, 1988). The principal theme is the Fredholmness (Noetherity) of operator equations where some "nicer" operators are combined with an *involutive* operator (for instance—the equation $(A_1 + QA_2 + \ldots + Q^{n-1}A_n)\varphi = f$, where A_i belong to some class of operators for which Fredholmness is easily decided, and Q satisfies $Q^n = I$). Equations of this type include singular integral equations with Carleman (or generalized Carleman) shifts on curves, convolution type equations, discrete convolutions with oscillating coefficients, multidimensional integral equations with homogeneous kernels and inversion, Hankel-type equations, etc. The exposition is divided into 7 chapters, of which the first contains various preliminaries, Chapters 2–3 the basics of the Fredholm theory and of singular integral equations with Carleman shift, and Chapters 4–7 the main body of the book, namely an abstract approach to the above-mentioned topic as well as its applications to the study of Fredholmness of various concrete equations. Most of the results originate in a series of journal papers by the authors, but the book strives to give ample references to historical motivations, results of other authors, open problems, or related investigations not covered in the book. The exposition is well organized and should be accessible to anyone with basic knowledge of functional analysis and operator theory. The book will be a useful resource to mathematicians working actively in the fields of integral equations or abstract operator theory, as well as to engineers using the techniques from these field in specific applications.

Miroslav Engliš, Praha

Peter Imkeller, Jin-Song von Storch: STOCHASTIC CLIMATE MODELS. Progress in Probability, vol. 49, Birkhäuser, Basel 2001, xxvii+398 pages, ISBN 3-7643-6520-X, EUR 109,–.

The climate system of the Earth is tremendously complex and its various components have very different time scales. In modelling such systems, it is often useful to apply some averaging procedures to obtain equations for slow motions ("the climate"), while the influence of fast motions ("the weather") is incorporated in the form of stochastic terms. Seminal stochastic climate models were proposed by Klaus Hasselmann already in the year 1976, but it seems that recently we have been facing a renewed activity in this direction, partly because new mathematical tools are available. The book under review are proceedings of a conference held in Chorin in summer 1999, whose aim was to bring together mathematicians and climate physicists interested in the stochastic approach and to start a dialogue between these two communities. The main task—to start a dialogue—influenced the style and the contents of the papers included into the proceedings: the physicists provide surveys of various available climate models, whilst the mathematicians present concise overviews of mathematical theories that might be promising in handling such models.

The book is divided into four chapters, with rather loose boundaries, and altogether eighteen papers are included. In Chapter 1, *The hierarchy of climate models*, the reader may find three papers (by D. Olbers, K. Fraedrich and J.-S. von Storch) introducing several types of climate models, and R. Temam's papers on some PDEs arising in general circulation models. Chapter 2, entitled *The emergence of randomness: Chaos, averaging, limit theorems*, opens with a paper by L. Arnold, which contains a reinterpretation of Hasselmann's program in the rigorous language of modern probability theory. Further, M. Denker and M. Kesseböhmer's paper on large deviations and multifractal formalism for expanding dynamical systems, Y. Kifer's paper on mathematical theory of averaging (both in deterministic and stochastic cases), and an article about averaging from a computational point of view by C. Rödenbeck *et al.*, are included in this chapter. The third chapter *Tools and methods: SDE, dynamical systems, SPDE, multiscale techniques* also contains four papers. J. Zabczyk is the author of a mini course on stochastic partial differential equations. P. Müller sketches a statistical mechanics approach to stochastic climate models. P. Imkeller reviews in an informal manner rigorous results concerning energy balance models, concentrating on phenomena like stability, bifurcations and stochastic resonance. A paper by J. Duan *et al.* differs in its style from the rest of the book, since it is devoted to a proof of a particular result on a stochastic quasigeostrophic equation. In the last chapter *Reduced stochastic models and particular techniques* a rather wide spectrum of problems is addressed. J. Egger discusses the Charney-De Vore model of the atmospheric circulation at midlatitudes. The paper by J. A. Freund *et al.* is about the close relation between stochastic resonance and the noise-induced synchronization. Two papers by P. Imkeller, A. H. Monahan and L. Pandolfo are devoted to the phenomenon of localization of planetary (Rossby) waves, the first of them developing the physical background, the companion paper being more mathematical. Rossby waves in a stochastically fluctuating medium are also the topic of an article by P. Sardeshmukh *et al.* Finally, W. A. Woyczyński contributed with a short survey of some mathematical results concerning passive tracer transport in stochastic flows.

Mathematical treatment of stochastic climate models seems to be a very challenging topic and the Proceedings under review are an excellent starting point for everybody who wants to work in this interesting field.

Bohdan Maslowski, Praha

A. B. Cruzeiro, J.-C. Zambrini (eds.): STOCHASTIC ANALYSIS AND MATHEMATICAL PHYSICS. Progress in Probability, vol. 50, Birkhäuser, Boston, 2001, 158 pages, ISBN 0-8176-4246-3, DM 210,–.

In the Preface to the book under review it is stated that the volume is an outgrowth of a meeting organized by the Group of Mathematical Physics in Lisbon. The topics of the papers included in the book seem to be tied together rather loosely: Two papers (by H. Airault and P. Malliavin, and by R. Léandre) are devoted to analysis on manifolds. L. Coutin and L. Decreusefond treat stochastic Volterra equations with singular kernels such that applications to the fractional Brownian motion are covered. Markov and martingale uniqueness of Nelson diffusions on infinite dimensional spaces is studied by L. Wu, while in the paper by M. Oberguggenberger and F. Russo stochastic nonlinear wave equations are dealt with. A. S. Üstünel's contribution contains results on measure-preserving shifts on the Wiener space. R. Rebolledo in his paper provides a brief introduction to quantum dynamical semigroups and quantum Markov flows. Two papers (P. Lescot's on generalized Mehler semigroups and C. Léonard's on entropic projections) are only short surveys of their authors' recent results to be published in detail elsewhere.

The high quality of the papers notwithstanding, it is not easy to imagine who should spend more than a hundred Euros to get one or two papers she or he is interested in-papers that might well appear in any relevant mathematical journal.

Jan Seidler, Praha

W. D. Wallis: MAGIC GRAPHS. Birkhäuser, Boston, Basel, Berlin, xiv+146 pages, EUR 49.50.–.

The fascinating notion of the magic square has become inspiration, in the graph theory, for various defined magic labelings of (finite undirected) graphs without loops and multiple edges, which the author lists in Section 4 of (introductory) Chapter I. The author recalls the magic labeling defined in the early sixties by J. Sedlček. However, the book is built mainly on another definition introduced several years later by A. Kotzig and A. Rosa. Kotzig and Rosa labeled vertices and edges by mutually different positive integers from one through the sum of the number of vertices and edges in such a way that the sum of labelings of a vertex u, a vertex v and the edge uv be constant for any two vertices u, v forming an edge. The author calls such a labeling an edge-magic total labeling (EMTL for short); Chapter II, the most extensive, is devoted to its study. Chapter III studies the vertex-magic total labeling (VMTL for short) of a graph: this is such a labeling of vertices and edges of a graph by positive integers from one through the sum of the number of vertices and edges in which the sum of labelings of a vertex and the labelings of all edges incident with it is constant.

A graph for which an EMTL or VMTL exists is called by the author edge-magic or vertex-magic, respectively. Theorems stating that graphs of some kind are/are not edge magic or vertex magic belong to the main results proved in Chapter II and III, respectively. Let us note here at least some of them. The cycle C_n is both edge-magic and vertex-magic for every $n \geqslant 3$. The complete graph K_n is edge-magic iff $n < 7$ and $n \neq 4$, and is vertex-magic for every $n \neq 2$. The complete bipartite graph $K_{m,n}$ is edge-magic for all $m, n \geqslant 1$ and is vertex-magic iff $|m - n| \leqslant 1$. As concerns trees: every catterpillar is edge-magic (it is an open problem whether every tree is edge-magic). A path with at least three vertices is vertex-magic. A tree with more end vertices than twice its inner vertices is not vertex-magic.

The topic of a short Chapter IV is the totally magic labeling of a graph, that is, such a labeling of vertices and edges by integers from one through the sum of the number of vertices and edges which is simultaneously EMTL and VMTL (it is not required that the constant for EMTL and that for VMTL be equal). As is apparent from this chapter, graphs which admit such a labeling are considerably rare. The last sections of Chapters II through IV are devoted to a certain weakening of magic labelings, which are named edge-magic injections, vertex-magic injections and total-magic injections. Starting with Chapter I (in which, among other, also some applications of magic labeling are presented), the exposition is accompanied by exercises (there is a special section devoted to the solution of some of them) and proposed problems (comments to them also form a separate section).

The study of graph labelings is a topical—and I believe also attractive—branch of the graph theory. Even if the fundaments of the theory presented in the book were constituted roughly three decades ago, their development during the last several years is remarkable. The experts will certainly appreciate the number of new results and, of course, also of open problems introduced in the book, and last but not least, also the well thought-out organization of the monograph.

Ladislav Nebeský, Praha

A. Dijksma, M. A. Kaashoek, A. C. M. Ran (eds.): RECENT ADVANCES IN OPERATOR THEORY. The Israel Gohberg Aniversary Volume, Operator Theory, Advances and Applications, vol. 124, Birkhäuser, Basel, 2001, 616 pages, DM 260,–.

The volume contains the proceedings of the International Workshop on Operator Theory and its Applications (IWOTA) held in Groningen, the Netherlands in June/July 1998. The book is devoted to Israel Gohberg, one of the founders of the series of IWOTA workshops and an outstanding leader in operator theory, on the occasion of his 70th birthday.

The papers included in the volume reflect the wide range of topics discussed at the workshop. They deal with interpolation, completion and extension problems, dilation theory, indefinite metric spaces, spectral problems of (partial) differential equations, operator polynomials and analytic operator functions, numerical ranges of operators, reproducing kernels, scattering theory and problems from harmonic analysis. The book contains also a review of Israel Gohberg contributions to Mathematics and a complete list of his publications.

Vladimír Müller, Praha

H. Amann, J. Escher: ANALYSIS III. Birkhäuser, Basel, 2001, xii + 480 pages, softcover, EUR 29.–.

This is the concluding third volume of a treatise of the advanced calculus. The work as a whole forms a modern, profound and concise background for studying advanced mathematics.

The following topics are dealt with:*Elements of measure theory, Theory of* (Lebesgue) *integration* (including Lebesgue spaces, convolutions, Fourier transform), *Manifolds and differential forms* (including multilinear algebra, vector fields, Riemann metric, vector analysis, etc.) and *Integration on manifolds* (based on integration of differential forms with all the necessary integral theorems).

This volume and also the whole work of Amann and Escher is a well structured work presenting the calculus for the student, the teacher and it contains many other topics which can be used for seminars. It is really not an easy task to write a good and modern textbook on analysis nowadays. The authors managed this job in an excellent way in my opinion. The book has to be recommended to teachers for using or at least for inspiration.

Štefan Schwabik, Praha