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Book reviews

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BOOK REVIEWS

José M. Gracia-Bondía, Joseph C. Várilly, Héctor Figueroa: ELEMENTS OF NON-COMMUTATIVE GEOMETRY. Birkhäuser Advanced Texts, ISBN 0-8176-4124-6, Birkhäuser, Basel, 2000, 704 pages, CHF 118.–.

Modern geometry is based on a duality between commutative associative algebras and spaces. This duality is given by the functor $X \mapsto C(X)$ which assigns to each space X (topological, algebraic, smooth, etc.) its algebra $C(X)$ of functions (continuous, regular, smooth, etc.).

From this point of view, every commutative associative algebra satisfying suitable assumptions represents a space. The idea of noncommutative geometry is to extend this duality and interpret *noncommutative* associative algebras (again satisfying suitable assumptions) as *noncommutative* spaces and then invoke geometric intuition to obtain new results and insights. Since the appearance of the seminal book "Géometrie non commutative" (1990) by A. Connes, this approach has found stunning applications in various fields of mathematics and turned out to be especially well-suited for quantum physics.

The present book is a systematic course in noncommutative differential geometry and operator theory, with applications to quantum physics. Its topics cover C^*-algebras, vector bundles and C^*-modules, K-theory, Fredholm operators, Clifford algebras, Spin representations, noncommutative integration and differential calculus, spectral triples and Connes' spin manifold theorem. As applications, noncommutative tori, quantum theory and Kreimer-Connes-Moscovici algebras are discussed.

The book will be helpful to all mathematicians and mathematical physicists who wish to learn about noncommutative geometry and its ramifications. It is, however, not a textbook and reading it assumes some preliminary knowledge of functional analysis, algebra and geometry.

Martin Markl, Praha

Anna B. Romanowska, Jonathan D. H. Smith: MODES. World Scientific, New Jersey, 2002, GBP 60.–.

This is a comprehensive book on modal theory. The word mode is relatively new and means an idempotent and entropic algebra. More explicitly, a τ- algebra (A, Ω) is idempotent if for every basic operation ω and every x in A,

$$x \ldots x\omega = x;$$

it is entropic if each basic operation ω is a homomorphism $\omega : (A^{\omega\tau}, \Omega) \to (A, \Omega)$.

This notion generalizes those in well known areas such as algebras of convex sets, affine spaces as well as of some classes of ordered sets. Their applications reach from computer science to economics, physics and biology.

The book is intended for both non-specialists and beginning graduate students. The first four chapters present a general introduction to universal algebra. The chapters are Algebras, Categories of algebras, Varieties, prevarieties and quasivarieties, and Constructing algebras and quasivarieties.

The introduction to modal theory itself is in the fifth chapter. Here, basic properties and basic examples of modes are given.

The next three chapters develop further the modal theory by discussing properties of special modes, in particular those of Mal'cev modes and binary modes. In the nineth chapter, interesting applications of modal theory in statistical mechanics are shown. The authors call this approach hierarchical statistical mechanics.

In the concluding tenth chapter, the most recent results and some open problems are presented.

The book which has 625 pages, explains a very broad spectrum of notions and results in modern algebra. Each chapter, except the last, contains exercises as well as comments on the literature. It certainly fills in a gap in the spectrum of mathematical literature.

Miroslav Fiedler, Praha

Jørgen Hoffmann-Jørgensen, Michael B. Marcus, Jon A. Wellner (eds.): HIGH DIMENSIONAL PROBABILITY III. Progress in Probability 55, Birkhäuser, Basel, 2003, viii+346 pages, ISBN 3-7643-2187-3, EUR 128.–.

This proceedings volume contains twenty articles from the conference High Dimensional Probability, held in Sandjberg (Denmark) in June 2002. This conference was the thirteenth in a series that started in 1973, initially under the name Probability in Banach spaces. During these thirty years the methods developed to study Gaussian processes and limit theorems in Banach spaces have proved themselves useful in many other branches of probability theory, applications to empirical processes being particularly noteworthy. The book under review provides a sample of recent developments in this direction.

The papers are divided into seven sections: I. Measures on general spaces and inequalities, II. Gaussian processes, III. Limit theorems, IV. Local times, V. Large and small deviations, VI. Density estimation, VII. Statistics via empirical process theory. To indicate the type of results the reader may expect to find in the book, we give the contents of the first two sections. J. Hoffmann-Jørgensen, motivated by the empirical processes theory, extends the classical inequalities for independent random variables to (non-measurable, in general) random elements in linear spaces. J. Kawabe provides a Prokhorov-type theorem on weak compactness of a set of vector measures. A. V. Uglanov shows that a linear operator from a Hilbert space into a Hausdorff locally convex space is absolutely summing if and only if it factors through a Hilbert space as a composition of a Hilbert-Schmidt operator and a bounded one, and then discusses probabilistic consequences of this result. P. Deheuvels and G. Martynov derive explicit Karhunen-Loève expansions for processes $t^\theta W(t^\varrho)$ and $t^\theta B(t^\varrho)$, where W, B are a Brownian motion and a Brownian bridge, respectively. Finally, X. Fernique considers the following problem: let γ be a Gaussian measure on a Fréchet space and H its reproducing kernel Hilbert space. Let μ be a probability measure absolutely continuous with respect to γ. The author proved in his preceeding paper that there exist a Gaussian random variable G with the law γ and an H-valued random variable Z such that μ is the law of $G+Z$. Now he relates the integrability of $\|Z\|_H$ to that of the Radon-Nikodym derivative $\mathrm{d}\mu/\mathrm{d}\gamma$.

All contributions are full-length research papers with complete proofs.

Ivo Vrkoč, Praha