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AN ESTIMATOR FOR PARAMETERS OF A NONLINEAR NONNEGATIVE MULTIDIMENSIONAL AR(1) PROCESS

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Abstract. Let $\boldsymbol{e}_t = (e_{t1}, \ldots, e_{tp})'$ be a p-dimensional nonnegative strict white noise with finite second moments. Let $h_{ij}(x)$ be nondecreasing functions from $[0, \infty)$ onto $[0, \infty)$ such that $h_{ij}(x) \leqslant x$ for $i, j = 1, \ldots, p$. Let $\boldsymbol{U} = (u_{ij})$ be a $p \times p$ matrix with nonnegative elements having all its roots inside the unit circle. Define a process $\boldsymbol{X}_t = (X_{t1}, \ldots, X_{tp})'$ by

$$X_{tj} = u_{j1}h_{1j}(X_{t-1,1}) + \ldots + u_{jp}h_{pj}(X_{t-1,p}) + e_{tj}$$

for $j = 1, \ldots, p$. A method for estimating $\boldsymbol{U}$ from a realization $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n$ is proposed. It is proved that the estimators are strongly consistent.

Keywords: autoregressive process, estimating parameters, multidimensional process, nonlinear process, nonnegative process

MSC 2000: 62M10

1. Introduction

Consider a one-dimensional AR(1) process X_t given by $X_t = bX_{t-1} + e_t$ where $b \in [0, 1)$ and e_t is a nonnegative strict white noise such that $Ee_t^2 < \infty$. Define $F(x) = P(e_t < x)$. Bell and Smith [9] proved that

$$b^* = \min\left(\frac{X_2}{X_1}, \ldots, \frac{X_n}{X_{n-1}}\right) \tag{1.1}$$

is a strongly consistent estimator for b if and only if $F(d) - F(c) < 1$ for all $0 < c < d < \infty$. Anděl [3] derived the distribution of b^* under the assumption that F is the distribution function of an exponential distribution. Davis and McCormick [10] obtained the asymptotic distribution of b^* when F varies regularly at 0 and satisfies a

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suitable moment condition. For the case of AR(p) model with $p > 1$ a straightforward generalization of (1.1) does not perform well. Anděl [5] suggested another estimator based on a maximum likelihood argument. His method was modified and generalized by An [1] and by An and Huang [2]. Asymptotic theory is developed in the paper by Feigin and Resnick [11]. The method was applied to nonlinear one-dimensional AR models by Anděl [4] and [6].

2. The model

Non-linear models of time series are important tools in statistical analysis of data. Statistical tests show that many series introduced in textbooks and papers are non-linear (see Tong [13]).

One of the simplest non-linear models is the one-dimensional non-linear AR(1) process X_t defined by

$$X_t = \lambda(X_{t-1}) + e_t \tag{2.1}$$

where λ is a non-linear function and e_t is a white noise. Even in this case it is quite hard task to estimate the function λ from a realization $X_1, \ldots, X_n$ (see Auestad and Tjøstheim [8]). It was proved by Jones [12] under general assumptions about the distribution function of e_t that the process $\{X_t\}$ cannot be stationary because it has "a drift to infinity" with a positive probability if there exist constants $\gamma > 0$ and $q > 1$ such that $|\lambda(x)| \geqslant \gamma |x|^q$ for large $|x|$.

It is a problem how to generalize (2.1) to p-dimensional case. The linear p-dimensional AR(1) process $\boldsymbol{X}_t$ is given by $\boldsymbol{X}_t = \boldsymbol{U}\boldsymbol{X}_{t-1} + \boldsymbol{e}_t$ where $\boldsymbol{U}$ is a $p \times p$ matrix and $\boldsymbol{e}_t$ is a p-dimensional white noise. The model $\boldsymbol{X}_t = \boldsymbol{\lambda}(\boldsymbol{X}_{t-1}) + \boldsymbol{e}_t$ with a non-linear function $\boldsymbol{\lambda}\colon \mathbb{R}_p \to \mathbb{R}_p$ is too general. In this case the function $\boldsymbol{\lambda}$ should be estimated in a whole, not only its parameters. In this paper we introduce a special non-linear model where $\boldsymbol{X}_t$ is created as a linear combination of non-linear functions of components of $\boldsymbol{X}_{t-1}$ plus a white noise (see (2.2) below). For simplicity we assume that the non-linear functions are known and only the parameters of their linear combinations must be estimated.

Remembering that a rapidly growing function λ in the one-dimensional model (2.1) leads to serious problems we assume that our non-linear functions in p-dimensional model do not grow faster than x (assumption A4).

The resulting model (2.2) seems to be quite flexible and its form enables to use procedures for estimating parameters similar to those in linear models.

Now, we introduce some assumptions valid throughout the paper.

A1. Let $\boldsymbol{U} = (u_{ij})$ be a $p \times p$ matrix with nonnegative elements such that all its roots are inside the unit circle.

A2. Let $\boldsymbol{e}_t = (e_{t1}, \ldots, e_{tp})'$ be independent identically distributed random vectors with nonnegative components and with finite second moments.

A3. Let $\boldsymbol{Z}_t = (Z_{t1}, \ldots, Z_{tp})'$ be the p-dimensional AR(1) process defined by $\boldsymbol{Z}_t = \boldsymbol{U}\boldsymbol{Z}_{t-1} + \boldsymbol{e}_t$.

A4. Let $h_{ij}(x)$ be measurable nondecreasing functions mapping $[0,\infty)$ onto $[0,\infty)$ such that $h_{ij}(x) \leqslant x$ for $i, j = 1, \ldots, p$.

A5. Let $P(e_{t1} < c, \ldots, e_{tp} < c) > 0$ for every $c > 0$.

A6. There exists a constant $K_{ij} > 0$ for each pair (i,j), $i, j \in \{1, \ldots, p\}$, such that

$$P\{e_{t1} < c, \ldots, e_{t,j-1} < c, h_{ji}(e_{tj}) > K_{ji}, e_{t,j+1} < c, \ldots, e_{tp} < c\} > 0$$

for every $c > 0$.

Now, introduce a process $\boldsymbol{X}_t = (X_{t1}, \ldots, X_{tp})'$ in the following way. Define $\boldsymbol{X}_t = \boldsymbol{Z}_t$ for $t \leqslant 1$ and

$$X_{ti} = \sum_{j=1}^{p} u_{ij} h_{ji}(X_{t-1,j}) + e_{ti}, \qquad i = 1, \ldots, p, \tag{2.2}$$

for $t \geqslant 2$. Then we say that $\{\boldsymbol{X}_t, t \geqslant 1\}$ is a nonlinear p-dimensional AR(1) process. It follows from our assumptions that all the variables X_{ti} are nonnegative.

3. Auxiliary results

Lemma 3.1. *We have $X_{ti} \leqslant Z_{ti}$ for all t and i.*

Proof. The assertion can easily be proved by complete induction. □

Assume that a realization $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n$ is given. Define

$$u_{ij}^{+} = \min_{2 \leqslant t \leqslant n} \frac{X_{ti}}{h_{ji}(X_{t-1,j})}, \qquad i, j = 1, \ldots, p.$$

Theorem 3.2. *We have $u_{ij}^{+} \to u_{ij}$ a.s. for each $i, j = 1, \ldots, p$ as $n \to \infty$.*

Proof. Let $\boldsymbol{U}^k = \left(u_{ij}^{(k)}\right)$ for $k = 1, 2, \ldots$. Since

$$\boldsymbol{Z}_t = \sum_{k=0}^{\infty} \boldsymbol{U}^k \boldsymbol{e}_{t-k},$$

we can write

$$Z_{ti} = e_{ti} + \sum_{k=1}^{\infty} \sum_{j=1}^{p} u_{ij}^{(k)} e_{t-k,j}, \qquad j = 1, \ldots, p. \tag{3.1}$$

Since the elements u_{ij} are nonnegative, we have also $u_{ij}^{(k)} \geqslant 0$ for all i, j, k. Assumption A1 also implies that $\sum\limits_k \sum\limits_i \sum\limits_j u_{ij}^{(k)} < \infty$ and thus there exists a constant L such that $u_{ij}^{(k)} \leqslant L$ for all i, j, k.

To simplify the notation, we prove the assertion only for $i = j = 1$. The proof for any other pair (i, j) is the same. In our case we have

$$u_{11}^{+} = u_{11} + \min_{2 \leqslant t \leqslant n} \frac{\sum\limits_{j=2}^{p} u_{1j} h_{j1}(X_{t-1,j}) + e_{t1}}{h_{11}(X_{t-1,1})}.$$

Let $\varepsilon > 0$ be a given number. Introduce the events

$$Q_t = \left\{ \omega\colon \ \frac{\sum\limits_{j=2}^{p} u_{1j} Z_{t-1,j} + e_{t1}}{h_{11}(e_{t-1,1})} < \varepsilon \right\}.$$

From the inequalities

$$h_{j1}(X_{t-1,j}) \leqslant X_{t-1,j} \leqslant Z_{t-1,j}, \qquad h_{11}(X_{t-1,1}) \geqslant h_{11}(e_{t-1,1})$$

it follows that $u_{11}^{+} < u_{11} + \varepsilon$ if at least one of the events $Q_2, \ldots, Q_n$ occurs. But $u_{11}^{+} \geqslant u_{11}$ trivially holds. We prove that for every $\varepsilon > 0$ with probability one there exist infinitely many indices $t \geqslant 2$ such that the events Q_t occur. This implies, of course, that $u_{11}^{+} \to u_{11}$ a.s. Using (3.1) we get

$$Q_t = \left\{ \omega\colon \ e_{t1} + \sum_{j=2}^{p} u_{1j} \left[e_{t-1,j} + \sum_{k=1}^{\infty} \sum_{i=1}^{p} u_{ji}^{(k)} e_{t-1-k,i} \right] < \varepsilon h_{11}(e_{t-1,1}) \right\}.$$

Denote $K = K_{11}$. For every integer $M \geqslant 1$ define the events

$$\begin{aligned} Q_{tM1} = \Big\{ \omega\colon \ & h_{11}(e_{t-1,1}) > K, \ e_{t1} < \frac{\varepsilon K}{2Mp^2}, \\ & u_{1j} e_{t-1,j} < \frac{\varepsilon K}{2Mp^2} \quad \text{for} \quad j = 2, \ldots, p, \\ & u_{1j} u_{ji}^{(k)} e_{t-1-k,i} < \frac{\varepsilon K}{2Mp^2} \quad \text{for} \quad j = 2, \ldots, p; \ i = 1, \ldots, p; \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad k = 1, \ldots, M \Big\}, \end{aligned}$$

$$Q_{tM2} = \left\{ \omega\colon \ \sum_{k=M+1}^{\infty} \sum_{j=2}^{p} \sum_{i=1}^{p} u_{1j} u_{ji}^{(k)} e_{t-1-k,i} < \frac{\varepsilon K}{2} \right\}.$$

It is clear that $Q_t \supset Q_{tM1} \cap Q_{tM2}$. Let

$$Q^*_{tM1} = \Big\{\omega\colon\ h_{11}(e_{t-1,1}) > K,\ e_{t1} < \frac{\varepsilon K}{2Mp^2},\\ e_{t-1,j} < \frac{\varepsilon K}{2Mp^2 L} \quad \text{for} \quad j = 2, \ldots, p,\\ e_{t-1-k,i} < \frac{\varepsilon K}{2Mp^2 L^2} \quad \text{for} \quad i = 1, \ldots, p;\ k = 1, \ldots, M\Big\}.$$

Then $Q_{tM1} \supset Q^*_{tM1}$ and our assumptions yield that neither $P(Q_{tM1})$ nor $P(Q^*_{tM1})$ depend on t. Since

$$P(Q^*_{tM1}) = P\left(e_{t1} < \frac{\varepsilon K}{2Mp^2}\right)\\ \times P\left\{h_{11}(e_{t-1,1}) > K,\ e_{t-1,j} < \frac{\varepsilon K}{2Mp^2 L} \quad \text{for} \quad j = 2, \ldots, p\right\}\\ \times \prod_{k=1}^{M} P\left\{e_{t-1-k,i} < \frac{\varepsilon K}{2Mp^2 L^2} \quad \text{for} \quad i = 1, \ldots, p\right\} > 0,$$

we have also $P(Q_{tM1}) > 0$. Further, $P(Q_{tM2}) \to 1$ as $M \to \infty$ and the probability $P(Q_{tM2})$ does not depend on t. Denote $\pi_M = P(Q_{tM1})$. Let w_M be the smallest integer such that $w_M \pi_M \geqslant 1$ holds. Introduce the sets $S_1, S_2, \ldots$ in the following way. Let S_1 contain the elements of w_1 triples

$$(1,2,3),\ (4,5,6),\ \ldots,\ (3w_1 - 2, 3w_1 - 1, 3w_1),$$

let S_2 contain the elements of w_2 four-tuples starting from $3w_1 + 1, 3w_1 + 2, 3w_1 + 3, 3w_1 + 4)$ and so on. The last elements of the triples, four-tuples etc. denote $t_1, t_2, \ldots$. If $t_r \in S_M$ we have $Q_{t_r} \supset Q_{t_r M1} \cap Q_{t_r M2}$. The events $Q_{t_1 M1}, Q_{t_2 M1}, \ldots$ are independent,

$$\sum_{r=1}^{\infty} P(Q_{t_r M1}) \geqslant \sum_{M=1}^{\infty} w_M \pi_M = \infty,$$

$P(Q_{t_r M2}) \to 1$ as $r \to \infty$ and the events $Q_{t_r M1}$ and $Q_{t_r M2}$ are independent for every $r = 1, 2, \ldots$. It follows from the generalized Borel lemma (see Anděl and Dupač [7]) that then with probability one infinitely many events $Q_{t_r M1} \cap Q_{t_r M2}$ occur and thus also infinitely many events Q_t occur with probability one. $\square$

4. Estimating parameters

To simplify the notation, we consider in this section the case $p = 2$ only. The procedure for $p > 2$ is analogous. First of all, we introduce a motivation of our method of estimation.

It is known from simulations that u_{ij}^+ cannot be used as estimator for u_{ij} because the convergence $u_{ij}^+ \to u_{ij}$ is very slow. Assume for a while that e_{t1} and e_{t2} are independent random variables having the exponential distribution $Ex(\lambda_1)$ and $Ex(\lambda_2)$, respectively. Then the conditional likelihood of $\boldsymbol{X}_2, \dots, \boldsymbol{X}_n$, given $\boldsymbol{X}_1$, is

$$\begin{aligned} &\exp\Big\{-\sum_{t=2}^{n}[X_{t1} - u_{11}h_{11}(X_{t-1,1}) - u_{12}h_{21}(X_{t-1,2})]/\lambda_1\Big\} \\ &\times \exp\Big\{-\sum_{t=2}^{n}[X_{t2} - u_{21}h_{12}(X_{t-1,1}) - u_{22}h_{22}(X_{t-1,2})]/\lambda_2\Big\} \end{aligned}$$

under the conditions that

$$X_{t1} - u_{11}h_{11}(X_{t-1,1}) - u_{12}h_{21}(X_{t-1,2}) \geqslant 0, \qquad t = 2, \dots, n,$$

and

$$X_{t2} - u_{21}h_{12}(X_{t-1,1}) - u_{22}h_{22}(X_{t-1,2}) \geqslant 0, \qquad t = 2, \dots, n.$$

If these conditions are not satisfied then the conditional likelihood is zero. The maximum likelihood method leads to the conclusion to estimate u_{11}, u_{12} by the variables which maximize

$$a\sum_{t=2}^{n} h_{11}(X_{t-1,1}) + b\sum_{t=2}^{n} h_{21}(X_{t-1,2}) \tag{4.1}$$

for $a \geqslant 0$, $b \geqslant 0$ under the conditions

$$X_{t1} - ah_{11}(X_{t-1,1}) - bh_{21}(X_{t-1,2}) \geqslant 0, \qquad t = 2, \dots, n, \tag{4.2}$$

and to estimate u_{21}, u_{22} by the variables which maximize

$$c\sum_{t=2}^{n} h_{12}(X_{t-1,1}) + d\sum_{t=2}^{n} h_{22}(X_{t-1,2}) \tag{4.3}$$

for $c \geqslant 0$, $d \geqslant 0$ under the conditions

$$X_{t2} - ch_{12}(X_{t-1,1}) - dh_{22}(X_{t-1,2}) \geqslant 0, \qquad t = 2, \dots, n. \tag{4.4}$$

One can expect that the results will be approximately the same as in the case that we sum up to $t=n+1$ in (4.1) and (4.3). We prove that such estimators are strongly consistent. Note that this assertion will be proved under our general assumptions A1–A6 and that we do not use any specific distribution of $\boldsymbol{e}_t$.

Theorem 4.1. *Let u_{11}^*, u_{12}^* be a solution of the linear program*

$$\max_{a,b}\Big\{a\sum_{t=1}^{n}h_{11}(X_{t1})+b\sum_{t=1}^{n}h_{21}(X_{t2})\Big\} \tag{4.5}$$

for $a\geqslant 0$, $b\geqslant 0$ under the conditions (4.2). *Let u_{21}^*, u_{22}^* be a solution of the linear program*

$$\max_{c,d}\Big\{c\sum_{t=1}^{n}h_{12}(X_{t1})+d\sum_{t=1}^{n}h_{22}(X_{t2})\Big\}$$

for $c\geqslant 0$, $d\geqslant 0$ under the conditions (4.4). *Then $u_{11}^*\to u_{11}$, $u_{12}^*\to u_{12}$, $u_{21}^*\to u_{21}$, $u_{22}^*\to u_{22}$ a.s. as $n\to\infty$.*

Proof. Instead of (4.5) we consider an equivalent formula, namely to solve

$$\max_{a,b}\left\{a\frac{1}{n}\sum_{t=1}^{n}h_{11}(X_{t1})+b\frac{1}{n}\sum_{t=1}^{n}h_{21}(X_{t2})\right\} \tag{4.6}$$

for $a\geqslant 0$, $b\geqslant 0$ under (4.2). Assume first that $u_{11}>0$, $u_{12}>0$. Define

$$\begin{aligned} M_n&=\{(a,b)\colon\ a\geqslant 0, b\geqslant 0, X_{t1}-ah_{11}(X_{t-1,1})-bh_{21}(X_{t-1,2})\geqslant 0,\\ &\qquad t=2,\dots,n\},\\ M&=\{(a,b)\colon\ 0\leqslant a\leqslant u_{11}, 0\leqslant b\leqslant u_{12}\}.\end{aligned}$$

It is clear that $M_2\supset M_3\supset\dots$. We prove that $M_n\to M$ a.s. Theorem 2.2 implies that there exists a sequence of indices t_i such that

$$\frac{X_{t_i1}}{h_{11}(X_{t_i-1,1})}\to u_{11}\qquad \text{a.s.}$$

Since

$$\frac{X_{t1}}{h_{11}(X_{t-1,1})}=u_{11}+\frac{u_{12}h_{21}(X_{t-1,2})+e_{t1}}{h_{11}(X_{t-1,1})}$$

we can see that

$$\frac{h_{21}(X_{t_i-1,2})}{h_{11}(X_{t_i-1,1})}\to 0\qquad \text{a.s.} \tag{4.7}$$

Because
$$\frac{X_{t1}}{h_{21}(X_{t-1,2})} = u_{12} + \frac{u_{11}h_{11}(X_{t-1,1}) + e_{t1}}{h_{21}(X_{t-1,2})}$$
using (4.7) we get
$$\frac{X_{t_i 1}}{h_{21}(X_{t_i-1,2})} \to \infty \qquad \text{a.s.}$$

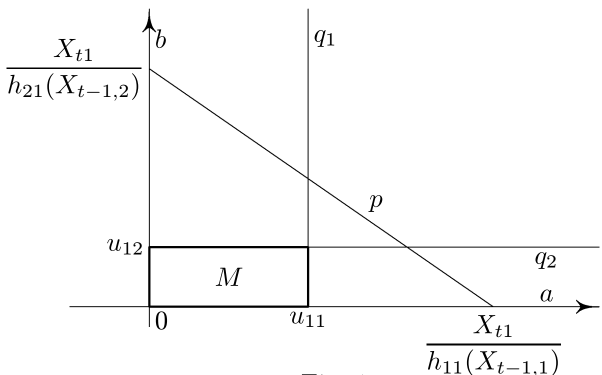

Fig. 1

In this case the straight line p in Fig. 1 approaches the straight line q_1. Quite similarly it can be proved that with probability one there exists a sequence of straight lines p converging to q_2. It is easy to calculate that p intersects q_1 at the point
$$\left(u_{11}, u_{12} + \frac{e_{t1}}{h_{21}(X_{t-1,2})}\right)$$
and thus no straight line p intersects M. Consider the sequence of linear programs (4.6) for $n \to \infty$. We know that
$$h_{11}(X_{t1}) \geqslant h_{11}(e_{t1}), \qquad h_{21}(X_{t2}) \geqslant h_{21}(e_{t2}).$$
Denote
$$H_{11} = Eh_{11}(e_{t1}), \qquad H_{21} = Eh_{21}(e_{t2}).$$
Since $h_{11}(e_{t1})$ as well as $h_{21}(e_{t2})$ are i.i.d. random variables with finite second moments, the law of large numbers yields
$$\frac{1}{n}\sum_{t=1}^{n} h_{11}(e_{t1}) \to H_{11}, \qquad \frac{1}{n}\sum_{t=1}^{n} h_{21}(e_{t2}) \to H_{21} \qquad \text{a.s.}$$
On the other hand,
$$h_{11}(X_{t1}) \leqslant X_{t1} \leqslant Z_{t1}, \qquad h_{21}(X_{t2}) \leqslant X_{t2} \leqslant Z_{t2}.$$

Our assumptions imply that the process $\boldsymbol{Z}_t = (Z_{t1}, Z_{t2})'$ is ergodic. If we denote $L_1 = EZ_{t1}$, $L_2 = EZ_{t2}$, then the ergodic theorem gives

$$\frac{1}{n}\sum_{t=1}^{n} Z_{t1} \to L_1, \qquad \frac{1}{n}\sum_{t=1}^{n} Z_{t2} \to L_2.$$

Thus for every fixed $\varepsilon > 0$ there exists n_0 such that with probability one we have for all $n \geqslant n_0$ that

$$H_{11} - \varepsilon \leqslant \frac{1}{n}\sum_{t=1}^{n} h_{11}(X_{t1}) \leqslant L_1 + \varepsilon, \qquad H_{21} - \varepsilon \leqslant \frac{1}{n}\sum_{t=1}^{n} h_{21}(X_{t2}) \leqslant L_2 + \varepsilon.$$

We choose ε so small that $H_{11} - \varepsilon > 0$, $H_{21} - \varepsilon > 0$. Therefore, the coefficients standing by a and b in the expression (4.6) are positive and bounded for all sufficiently large n and $M_n \to M$. This implies that the sequence of any solutions of the linear programs (4.6) converges to the point (u_{11}, u_{12}) in which (4.6) is maximized on M.

If $u_{11} = 0$ and/or $u_{12} = 0$, the arguments are analogous. The proof for u_{21}^*, u_{22}^* is similar. $\square$

5. A NUMERICAL EXAMPLE

It is known from simulations as well as from theoretical results that the estimators like u_{ij}^* behave quite well in linear autoregressive processes. To illustrate our method in non-linear models we considered the model

$$\begin{aligned} X_{t1} &= u_{11}h_{11}(X_{t-1,1}) + u_{12}h_{21}(X_{t-1,2}) + e_{t1},\\ X_{t2} &= u_{21}h_{12}(X_{t-1,1}) + u_{22}h_{22}(X_{t-1,2}) + e_{t2} \end{aligned}$$

where

$$h_{11}(x) = \begin{cases} x^2 & \text{for } x \in [0,1),\\ \sqrt{x} & \text{for } x \geqslant 1, \end{cases} \qquad h_{21}(x) = \begin{cases} 0 & \text{for } x \in [0,1),\\ 1 & \text{for } x \geqslant 1, \end{cases}$$

$$h_{12}(x) = \begin{cases} x & \text{for } x \in [0,1),\\ 1 & \text{for } x \geqslant 1, \end{cases} \qquad h_{21}(x) = \begin{cases} \dfrac{x}{2} & \text{for } x \in [0,1),\\ 0.5 + \ln x & \text{for } x \geqslant 1 \end{cases}$$

and e_{t1}, e_{t2} are independent random variables with exponential distribution $Ex(1)$. A realization of X_{t1}, X_{t2} of the length 500 was simulated with

$$u_{11} = 0.7, \quad u_{12} = 0.3, \quad u_{21} = 0.1, \quad u_{22} = 0.5.$$

The estimates u_{ij}^* were calculated from the first n variables of this realization ($n = 50, 100, 200, 500$). The results are given in Tab. 1.

Results of a simulation

n	u_{11}^*	u_{12}^*	u_{21}^*	u_{22}^*
50	0.782	0.180	0.000	0.634
100	0.763	0.217	0.146	0.452
200	0.707	0.288	0.134	0.462
500	0.701	0.300	0.100	0.502

Tab. 1

The estimates seem to be satisfactory for $n = 200$ and the agreement with theoretical values is quite good for $n = 500$. Of course, more extensive simulations are needed to see the small sample properties of our estimators in detail.

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