

# Applications of Mathematics

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## Book Reviews

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## BOOK REVIEWS

*Marc Yor*: SOME ASPECTS OF BROWNIAN MOTION. Part I: Some special functionals, Part II: Some recent martingale problems. Lectures in Mathematics, Birkhäuser Verlag, Basel 1992, 1997; vii+136, xii+144 pages, ISBN 3-7643-2807-X, 3-7643-5717-7; price sFr 38,-, sFr 32,-.

An in-depth investigation of the Wiener process (i.e., the Brownian motion) has been one of the central topics in the modern probability theory for at least fifty years. Nevertheless, research in this field is still active and many fine and deep results have been obtained only recently, as M. Yor's book under review can witness. These lecture notes are drawn from a course delivered by the author in the winter term 1991–92 at the ETH in Zürich. Although the second volume appeared five years later than the first, they both form a unified treatise, in which all the topics dealt with are intertwined in an amazing way. The book consist of eighteen chapters; let us list the titles of them to give some idea of its rich contents: The Gaussian space of Brownian motion, The laws of some quadratic functionals of Brownian motion, Squares of Bessel processes and Ray-Knight theorems for Brownian local times, An explanation and some extensions of the Ciesielski-Taylor identities, On the winding number of planar Brownian motion, On some exponential functionals of Brownian motion and the problem of Asian options, Some asymptotic laws for multidimensional Brownian motion, Some extensions of Paul Lévy's arc sine law for Brownian motion, Further results about reflecting Brownian motion perturbed by its local time at 0, On principal values of Brownian and Bessel local times, Probabilistic representations of the Riemann zeta function, Some examples and applications of enlargements of filtrations, Martingale inequalities at any time, On the martingales which vanish on the set of Brownian zeroes, On Azéma's martingales and the chaos representation property, The filtration of truncated Brownian motion, The Brownian filtration, Tsirel'son's example, and Walsh's Brownian motions, Complements relative to Part I.

Professor Yor is one of the leading experts in the field, many results presented in the book are due to him or even appear here for the first time. *Some aspects of the Brownian motion* is an interesting text providing a possibility to get acquainted with new achievements in the study of Brownian motion. The reader, however, is presupposed to have a working knowledge of stochastic calculus and excursion theory.

*Bohdan Maslowski*

*Lars Hörmander*: NOTIONS OF CONVEXITY. Birkhäuser, Basel-Berlin-Boston, 1994, str. 424, cena sFr 88,-.

Cílem monografie je výklad široké koncepce konvexity nacházející aplikace v rozmanitých partiích matematiky.

Tradiční pojetí konvexity je podáno v kapitole I (věnované konvexním funkcím jedné proměnné) a v kapitole II (zabývající se konvexitou v konečněrozměrném vektorovém prostoru). Vzhledem k významu subharmonicity pro teorii funkcí více komplexních proměnných je další (třetí) kapitola věnována subharmonickým funkcím v  $\mathbb{R}^n$ . Role lineárních funkcí objevujících se v definici konvexních funkcí na  $\mathbb{R}$  z kap. I je převzata harmonickými funkcemi (čímž konvexita přejde v subharmonicitu). Výklad teorie plurisubharmonických funkcí je podán

v kap. IV. Třídy funkcí, jež se chovají podobně jako konvexní funkce vzhledem k invarianci vůči podgrupám lineární grupy, jsou zkoumány v kap. V; jsou užitečné v souvislosti s konvexitou vzhledem k diferenciálním operátorům, jež je předmětem studia kap. VI. Závěrečná kap. VII je věnována podmínkám řešitelnosti mikrodiferenciálních rovnic. Nároky na čtenáře narůstají postupně; pro počáteční kapitoly stačí základní kalkulus a lineární algebra. Míra a integrál jsou požadovány v kap. III, kde se též potřebuje teorie distribucí. Výklad tedy přináší užitek jak studentům, tak specialistům, kteří naleznou originální materiál v závěrečných kapitolách. Monografie si bezpochyby získá široký okruh zájemců podobně jako autorova známá předchozí díla „The Analysis of Linear Partial Differential Operators“ (Springer 1983–1985), „An Introduction to Complex Analysis in Several Variables“ (North Holland 1990), na která jsou ostatně četné odkazy v textu.

*Josef Král*

*V.I. Lebedev*: AN INTRODUCTION TO FUNCTIONAL ANALYSIS AND COMPUTATIONAL MATHEMATICS. Birkhäuser, Boston, 1997, xi+255 pages, ISBN 3-7643-3888-1, price DM 118,-.

Although the title of the book is quite general, it contains necessary elements of functional analysis enabling to solve several problems of numerical and applied mathematics. The book consists of three chapters:

1. Functional spaces and problems in the theory of approximation.
2. Linear operators and functionals.
3. Iteration methods for the solution of operator equations.

The first chapter introduces basic definitions and results in metric, Banach and Hilbert spaces, the contraction mapping principle, Chebyshev polynomials, Lagrange interpolation polynomials, etc.

In the second chapter, the author examines properties of spaces of linear operators and their spectrum. He proves important theorems of functional analysis, like uniform boundedness principle, the Hahn-Banach theorem and the Riesz theorem. Further, he presents Sobolev spaces and embedding theorems. These results are applied to prove the existence and uniqueness of a weak solution of a second order elliptic problem with Dirichlet boundary conditions. Basic properties of its Galerkin and Ritz approximations are derived as well.

Finally, the last chapter deals with a general theory of iteration methods and their convergence rate. Next particular methods (steepest descent, the Chebyshev two-step method, the Newton method etc.) are investigated. Moreover, several standard approaches to accelerate the convergence are studied.

The reader is supposed to have a basic knowledge in numerical mathematics, theory of functions of real variables, initial and boundary value problems for differential equations, and methods of linear algebra.

*Michal Křížek*