Luis Alejandro Márquez-Martínez; Claude H. Moog; Martín Velasco-Villa
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THE STRUCTURE OF NONLINEAR TIME DELAY SYSTEMS

LUIS A. MÁRQUEZ–MARTÍNEZ, CLAUDE H. MOOG AND MARTÍN VELASCO–VILLA

Multivariable nonlinear systems with time delays are considered. The delays are supposed to be constant but not commensurate. The goal of this paper is to give a structure algorithm which displays some system invariants for this class of systems.

1. INTRODUCTION

A time delay system is a dynamic system whose evolution in time depends not only on its actual state but also on the past one. This class of systems frequently appears in real processes since there are delays associated to almost every sensor or actuator. Mathematically, a time-delay system is described by means of delay differential equations [3].

The problem of control of time delay systems has been treated in the literature starting with the input-output decoupling problem for a class of linear time delay systems [4, 9]. In particular, the disturbance decoupling problem (DDP) has been treated making use of different approaches (see, e. g., [10] and references therein).

For systems without delays, the study of the structure of linear and nonlinear systems has been useful to solve important control problems, including disturbance decoupling. It has been extensively investigated in the continuous time case as well as in the discrete time case. Some generalizations to linear time delay systems already exist [5]. However, the study of the structure of nonlinear time delay systems is still an open issue.

The objective of this paper is to present a generalization for time delay systems of the well-known Singh's inversion algorithm [8]. It allows to generalize the notion of inverse systems for nonlinear time-delay systems. A sufficient condition that assures the left-invertibility of a system of the considered class is given in terms of the equivalent notion of the rank of a delay system.

A first attempt for the application of this inversion technique to solve control problems as disturbance decoupling in the multiple-input multiple-output case may be found in [6].

This paper is organized as follows. Section 2 presents the special notation used to describe the class of systems under consideration and some preliminary definitions.

The algorithm and a definition of left-invertible systems are described in Section 3. Finally, some concluding remarks are given in Section 4.

2. NOTATIONS AND PRELIMINARY DEFINITIONS

In this section, the class of considered systems will be defined, and the mathematical setting to be used in this paper, which was introduced in [7], will be recalled. This approach is valid for systems with non-commensurable delays. Even if all of the contributions set forth may be extended to this case, for the sake of simplicity it will be considered that all the delays are multiples of an elementary delay h. Furthermore, it will be assumed that the time axis has been scaled to have $h = 1$. Under these assumptions, the considered multiple-input multiple-output (MIMO) nonlinear time-delay systems are described by

$$\Sigma : \begin{cases} \dot{x}(t) &= f(x(t-\tau), \tau \in \mathbb{N}) + \sum_{i=0}^{s} g_i(x(t-\tau), \tau \in \mathbb{N})\, u(t-i) \\ y(t) &= h(x(t-\tau), \tau \in \mathbb{N}) \\ x(t) &= \varphi(t), \quad u(t) = u_0, \quad \forall\, t \in [t_0 - s, t_0] \end{cases} \tag{1}$$

where only a finite number of *constant time delays* occur. The state $x \in \mathbb{R}^n$, the input $u \in \mathbb{R}^m$ and the output $y \in \mathbb{R}^p$. The entries of f and g_i are meromorphic functions of their arguments. The notation $f(x(t-\tau), \tau \in \mathbb{N})$ stands for $f(x(t), x(t-1), \ldots, x(t-s))$, for some $s \in \mathbb{N}$. $\varphi(t)$ is a continuous function of initial conditions.

Let $\mathcal{K}$ be the field of meromorphic functions of a finite number of variables in

$$\left\{x(t-\tau), u^{(k)}(t-\tau),\ \tau, k \in \mathbb{N}\right\}.$$

These variables are independent in the sense that they are not related by any equation except differential/difference equations. Let $\mathcal{E}$ be the formal vector space over $\mathcal{K}$ given by

$$\mathcal{E} = \operatorname{span}_{\mathcal{K}}\{\mathrm{d}\xi \mid \xi \in \mathcal{K}\}.$$

Let $\{\mathrm{d}z_i\}$ be a basis of $\mathcal{E}$. One defines a second vector space as follows:

$$\mathcal{E}^2 := \operatorname{span}_{\mathcal{K}}\{\mathrm{d}z_i(t-k) \wedge \mathrm{d}z_j(t-l)\}, \quad k,\ l \in \mathbb{N}.$$

The wedge product is defined as a linear mapping from $\mathcal{E} \times \mathcal{E}$ to $\mathcal{E}^2$. This mapping is associative, distributive and skew-symmetric:

$$\eta \wedge \omega = -\omega \wedge \eta, \quad \eta, \omega \in \mathcal{E}.$$

Previous equation implies that $\mathrm{d}z_i(t-k) \wedge \mathrm{d}z_i(t-l)$ is zero only for $k = l$, which reflects the independence of the variables defined in $\mathcal{K}$.

The time-shift operator δ is defined by

$$\delta(\xi(t)) = \xi(t-1), \quad \xi(t) \in \mathcal{K}$$

and

$$\delta(\alpha(t)\, \mathrm{d}\xi(t)) = \alpha(t-1)\, \mathrm{d}\xi(t-1), \quad \alpha(t)\, \mathrm{d}\xi(t) \in \mathcal{E}$$

Let $\mathcal{F}[\delta]$ denote the ring of polynomials of the operator δ with coefficients over a field $\mathcal{F}$. Every element of this ring may be written as

$$\alpha(\delta) = \alpha_0(t) + \alpha_1(t)\,\delta + \cdots + \alpha_{r_\alpha}(t)\,\delta^{r_\alpha}, \quad \alpha_i(t) \in \mathcal{F}$$

where r_α is the polynomial degree of $\alpha(\delta)$. Note that if the field is $\mathcal{K}$, this ring does not commute. Addition and product on this ring are defined by

$$\begin{aligned} \alpha(\delta) + \beta(\delta) &= \textstyle\sum_{i=0}^{\max\{r_\alpha, r_\beta\}} (\alpha_i(t) + \beta_i(t))\,\delta^i \\ \alpha(\delta)\,\beta(\delta) &= \textstyle\sum_{i=0}^{r_\alpha + r_\beta} \sum_{j=r_\alpha - i}^{r_\alpha} \alpha_{r_\alpha - j}(t)\,\beta_{i+j-r_\alpha}(t + j - r_\alpha)\,\delta^i. \end{aligned}$$

Lemma 1. For all $a(\delta)$, $b(\delta) \in \mathcal{K}[\delta]$ there exist $\alpha(\delta)$, $\beta(\delta) \in \mathcal{K}[\delta]$ s. t.

$$\alpha(\delta)\,a(\delta) + \beta(\delta)\,b(\delta) = 0.$$

Proof.

$$\begin{aligned} \alpha(\delta)\,a(\delta) &= \sum_{i=0}^{r_\alpha + r_a} \sum_{j=r_\alpha - i}^{r_\alpha} \alpha_{r_\alpha - j}(t)\,a_{i+j-r_\alpha}(t + j - r_\alpha)\,\delta^i \\ &= -\sum_{i=0}^{r_\beta + r_b} \sum_{j=r_\beta - i}^{r_\beta} \beta_{r_\beta - j}(t)\,b_{i+j-r_\beta}(t + j - r_\beta)\,\delta^i \\ &= -\beta(\delta)\,b(\delta). \end{aligned}$$

This implies that $r_\alpha + r_a = r_\beta + r_b$ and

$$\begin{aligned} &\sum_{j=r_\alpha - i}^{r_\alpha} \alpha_{r_\alpha - j}(t) a_{i+j-r_\alpha}(t + j - r_\alpha) \\ = \; &-\sum_{j=r_\beta - i}^{r_\beta} \beta_{r_\beta - j}(t) b_{i+j-r_\beta}(t + j - r_\beta), \quad \forall\, i = 0, \ldots, r_\alpha + r_a \end{aligned} \tag{2}$$

from which we may have up to $(r_\alpha + r_\beta + 2) - (r_\alpha + r_a + 1) = r_\alpha - r_b + 1$ independent solutions. One solution may be obtained from (2) for $r_\alpha > r_b$. □

Corollary 2. For every $a(\delta)$, $b(\delta) \in \mathcal{K}[\delta]$ there exist $\alpha(\delta)$, $\beta(\delta) \in \mathcal{K}[\delta]$ s. t.

$$\alpha(\delta)\,a(\delta)\,b(\delta) = \beta(\delta)\,b(\delta)\,a(\delta).$$

Define $\mathcal{M}$ as the left module over $\mathcal{K}[\delta]$ given by

$$\mathcal{M} = \mathrm{span}_{\mathcal{K}[\delta]}\{\mathrm{d}\xi | \xi \in \mathcal{K}\}.$$

Let $\{\omega_1, \ldots, \omega_r\} \in \mathcal{E}$ be a set of vectors. Then, denote $\mathrm{span}_{\mathcal{K}[\delta]}\{\omega_1, \ldots, \omega_r\}$ as the submodule of $\mathcal{M}$ generated by $\{\omega_1, \ldots, \omega_r\}$.

Under this approach, any element ω of $\mathcal{M}$, also called a 1-form, is said to be exact if there exists a function $\varphi \in \mathcal{K}$ such that $\omega = \mathrm{d}\varphi$.

Note that any 1-form $\omega \in \mathcal{M}$ is also an element of $\mathcal{E}$. Hence, Poincaré's Lemma [1] holds:

Lemma 3. (Poincaré) Consider a 1-form $\omega \in \mathcal{M}$. Then, there exists a function $\xi(t)$ such that (locally) $\omega = \mathrm{d}\xi(t)$ if and only if

$$\mathrm{d}\omega = 0.$$

Under this formalism, systems under consideration may be written as

$$\begin{aligned} \dot{x}(t) &= f(x(\cdot)) + g(x(\cdot), \delta)\, u(t) \\ y(t) &= h(x(\cdot)). \end{aligned} \tag{3}$$

Finally, define the following submodule of $\mathcal{M}$:

$$\mathcal{Y} := \mathrm{span}_{\mathcal{K}[\delta]}\{\mathrm{d}y,\ \mathrm{d}\dot{y}, \ldots\}.$$

3. THE STRUCTURE OF THE SYSTEM

System inversion is an important issue. It is appealing since, whenever an inverse systems exists, it may give a control law that generates any desired trajectory. Having this application in mind, we may accept time advances in the output since, for most applications, the desired trajectory is known in advance. In the case of systems without delays, the use of structural information has proved to be useful to solve this problem. In this section, an extension for nonlinear time–delay systems of Singh's structure algorithm [8] is presented. This allows to extend the notions of rank and invertibility to this class of systems. Section ends with the statement of a sufficient condition for the existence of an inverse system.

3.1. Structure algorithm

Step 0.
Define $\rho_0 = 0$, $\tilde{F}_0 = 0$, and

$$F_0 := y(t) - h(x(t-\tau), \tau \in I\!N) \equiv 0.$$

Step $k+1$.
Assume that in Step k functions $\tilde{F}_k$ and F_k have been defined so that

$$\begin{bmatrix} \tilde{F}_k \\ F_k \end{bmatrix} = \begin{bmatrix} \tilde{a}_k(x, \dot{y}, \ldots, y^{(k)}) \\ a_k(x, \dot{y}, \ldots, y^{(k)}) \end{bmatrix} + \begin{bmatrix} \tilde{b}_k(x, \dot{y}, \ldots, y^{(k-1)}, \delta) \\ 0 \end{bmatrix} u$$

with

$$\mathrm{rank}_{\mathcal{K}[\delta]}\, \tilde{b}_k(\cdot) = \mathrm{rank}_{\mathcal{K}[\delta]}\, \tilde{F}_k = \rho_k.$$

Compute

$$\dot{F}_k = \bar{a}_{k+1}(x, \dot{y}, \ldots, y^{(k+1)}) + \bar{b}_{k+1}(x, \dot{y}, \ldots, y^{(k)}, \delta)\, u$$

and define

$$\rho_{k+1} = \mathrm{rank}_{\mathcal{K}[\delta]} \begin{bmatrix} \tilde{b}_k(x, \dot{y}, \ldots, y^{(k-1)}, \delta) \\ \bar{b}_{k+1}(x, \dot{y}, \ldots, y^{(k)}, \delta) \end{bmatrix}.$$

Permute, if necessary, the components of $\dot{F}_k$ so the first ρ_{k+1} rows of $[\tilde{b}_k^T \; b_{k+1}^T]^T$ become linearly independent over $\mathcal{K}[\delta]$, and define

$$\begin{aligned}\begin{bmatrix} \tilde{F}_{k+1} \\ \hat{F}_{k+1} \end{bmatrix} &:= \begin{bmatrix} I_{\rho_{k+1}} & 0 \\ 0 & I_{p-\rho_{k+1}} \end{bmatrix} \begin{bmatrix} \tilde{F}_k \\ \dot{F}_k \end{bmatrix} \\ &= \begin{bmatrix} \tilde{a}_{k+1}(x, \dot{y}, \ldots, y^{(k+1)}) \\ \hat{a}_{k+1}(x, \dot{y}, \ldots, y^{(k+1)}) \end{bmatrix} + \begin{bmatrix} \tilde{b}_{k+1}(x, \dot{y}, \ldots, y^{(k)}, \delta) \\ \hat{b}_{k+1}(x, \dot{y}, \ldots, y^{(k)}, \delta) \end{bmatrix} u.\end{aligned}$$

Thus, there exist matrices $\tilde{S}_{k+1}(\delta) \in \mathcal{K}^{(p-\rho_{k+1}) \times \rho_{k+1}}[\delta]$ and $\hat{S}_{k+1}(\delta) \in \mathcal{K}_1^{(p-\rho_1)\times(p-\rho_1)}[\delta]$ such that

$$\tilde{S}_{k+1}\tilde{b}_{k+1} + \hat{S}_{k+1}\hat{b}_{k+1} = 0$$

and

$$\mathrm{rank}_{\mathcal{K}[\delta]}\tilde{S}_{k+1}(\delta) = \rho_k.$$

Finally define

$$F_{k+1}(x, \dot{y}) = \tilde{S}_{k+1}\tilde{F}_{k+1} + \hat{S}_{k+1}\hat{F}_{k+1}$$

which yields

$$\begin{bmatrix} \tilde{F}_{k+1} \\ F_{k+1} \end{bmatrix} = \begin{bmatrix} \tilde{a}_{k+1}(x, \dot{y}, \ldots, y^{(k+1)}) \\ a_{k+1}(x, \dot{y}, \ldots, y^{(k+1)}) \end{bmatrix} + \begin{bmatrix} \tilde{b}_{k+1}(x, \dot{y}, \ldots, y^{(k)}, \delta) \\ 0 \end{bmatrix} u. \tag{4}$$

Since ρ_k is a non-decreasing sequence of integers bounded by the number of inputs and outputs, the algorithm converges, at the most, at step n.

The rank ρ of the system is then defined as

$$\rho := \max\{\rho_k, k \geq 1\}.$$

3.2. Invertibility

As stated at the beginning of this section, when considering the problem of output inversion we may accept advances in the output. An inverse system is then defined as

$$\begin{aligned}\dot{\eta}(t) &= F\left(y^{(k)}(t \pm i), \eta(t-i), z(t-i), i \in 0 \cdots m', k \in I\!N\right) \\ u(t) &= H\left(y^{(k)}(t \pm i), \eta(t-i), z(t-i), i \in 0 \cdots m', k \in I\!N\right) \\ z(t+\tau_z) &= K\left(y^{(k)}(t \pm i), \eta(t-i), z(t-i), i \in 0 \cdots m', k \in I\!N\right).\end{aligned} \tag{5}$$

Note that no advances are allowed in the new state. The reason is that, in such case, information about future values of system's state would be needed for proper initialisation.

Definition 1. System (5) is a left inverse system for system (1) if the output $u(t)$ of (5) is equal to the input $u(t)$ of (1) whenever the output $y(t)$ of (1) is chosen as the input $y(t)$ of (5) for a proper initialisation of (5).

Definition 2. System (1) is said to be left–invertible if there exists a left inverse system.

For time-delay systems it is no sufficient to have a rank equal to the number of inputs to be left–invertible, as shown by the following example.

Example 1. Consider system

$$\begin{aligned} \dot{x}_1(t) &= x_2(t)\,u(t-1) \\ \dot{x}_2(t) &= u(t) \\ y(t) &= x_1(t) \end{aligned}$$

for which $\rho = m = 1$. From the structure algorithm we have

$$\dot{y}(t) = x_2(t)\,u(t-1)$$

for which it is easy to see that no inverse of the form (5) may be found.

Previous example shows that additional conditions should be stated to assure the existence of an inverse system.

Assume now that the algorithm converges at the step k. Then, from equation (4) we have

$$\mathrm{d}\tilde{F}_k = \mathrm{d}\Big(\tilde{a}_k(\cdot) + \tilde{b}_k(\cdot,\delta)\,u\Big) \tag{6}$$

with

$$\mathrm{rank}_{\mathcal{K}[\delta]}\{\tilde{b}_k(\cdot,\delta)\} = \rho_k.$$

Rewrite (6) under the form

$$\mathrm{d}\tilde{F}_k = \mathrm{diag}\,\{p_i(\delta)\delta^{\mu_i}\}\,[A(\delta)\,\mathrm{dx(t)} + \mathrm{B}(\delta)\,\mathrm{du(t)}] + \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_{\rho_k} \end{bmatrix} \tag{7}$$

with $p_i(\delta) \in \mathcal{K}[\delta]$, $p_i(0) \neq 0$, $\mu_i \in I\!N$, and $\omega_i \in \mathcal{Y}$, for $i \in 1,\ldots,\rho_k$.

A sufficient condition for the existence of a left inverse for a nonlinear time-delay system is now stated.

Theorem 1. System (1) has a left inverse if

$$\mathrm{rank}_{\mathcal{K}[\delta]}\; B(\delta)|_{\delta=0} = m. \tag{8}$$

P r o o f. Let $\tilde{F}_{k_i}(t)$ represent the ith row of $\tilde{F}_k(t)$. Equation (7) implies

$$\tilde{F}_{k_i}(t) = \tilde{F}_{k_i}\left(y^{(j)}(t-\tau), x(t-\tau'), u(t-\tau'),\, j,\tau,\tau' \in I\!N,\, \tau' \geq \mu_i\right),$$

so

$$\mathrm{col}\{\tilde{F}_{k_i}(t+\mu_i)\} = A(\cdot) + B(0)\,u(t) + B'(\delta)\,u(t)$$

where

$$\begin{aligned} A(\cdot) &= A(x(t-\tau), y^{(j)}(t \pm \tau),\ j\,, \tau \in \mathbb{N}) \\ B(0) &= B(x(t-\tau), y^{(j)}(t \pm \tau),\ j\,, \tau \in \mathbb{N}), \\ B'(\delta) &= B'(x(t-\tau), y^{(j)}(t \pm \tau), \delta,\ j\,, \tau \in \mathbb{N}) \\ &= B(\delta) - B(0). \end{aligned}$$

Define an integer τ_i associated to each column $B_i'(\delta)$ of matrix $B'(\delta)$ as follows:

$$\tau_i = \begin{cases} 0 & \text{if } B_i'(\delta) = 0 \\ \min\{k \in \mathbb{N} \text{ s.t. } B_i'(\delta)\delta^{-k}|_{\delta=0} \neq 0\} & \text{otherwise.} \end{cases}$$

From condition (8), $B(0)$ is an invertible matrix, and an inverse system of the form (5) can be obtained:

$$\begin{aligned} \dot{\eta}(t) &= f(\eta(\cdot)) - g(\eta(\cdot), \delta)(H(\cdot)) \\ u(t) &= H(\cdot) \\ \text{col}\ \{z_i(t+\tau_i)\} &= H_i(\cdot) \end{aligned} \tag{9}$$

where $f(\eta(\cdot))$ and $g(\eta(\cdot), \delta)$ are taken from (3) and

$$\begin{aligned} H(\cdot) &= H(y^{(j)}(t \pm \tau), \eta(t-\tau), z(t-\tau),\ j\,, \tau \in \mathbb{N}) \\ &= -B^{-1}(0)[A(\cdot) + B'(\delta)\,\text{diag}\,\{\delta^{-\tau_i}\}\, z(t).] \end{aligned} \tag{10}$$

□

3.3. Illustrative example

Consider the following system

$$\dot{x}(t) = \begin{bmatrix} x_3(t)\delta & 0 \\ 1 & 0 \\ 0 & 1 - x_1(t)\delta \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

$$\text{y(t)} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}.$$

Structure algorithm

Step 0.

$$F_0 = \begin{bmatrix} y_1(t) - x_1(t) \\ y_2(t) - x_2(t) \end{bmatrix}. \tag{11}$$

Step 1.

$$\dot{F}_0 = \begin{bmatrix} \dot{y}_1(t) \\ \dot{y}_2(t) \end{bmatrix} + \begin{bmatrix} -x_3(t)\delta & 0 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} \tilde{F}_1 \\ \hat{F}_1 \end{bmatrix}$$

$$\rho_1 = \operatorname{rank} \begin{bmatrix} -x_3(t)\delta & 0 \\ -1 & 0 \end{bmatrix} = 1.$$

Folllowing the proof of Lemma 1, we find

$$\begin{aligned} \tilde{S}_1 &= -1 \\ \hat{S}_1 &= x_3(t)\delta \end{aligned}$$

which yields

$$F_1 = -\dot{y}_1(t) + x_3(t)\,\dot{y}_2(t-1), \tag{12}$$

and the Step ends by writing

$$\begin{bmatrix} \tilde{F}_1 \\ F_1 \end{bmatrix} = \begin{bmatrix} \dot{y}_1(t) \\ -\dot{y}_1(t) + x_3(t)\,\dot{y}_2(t-1) \end{bmatrix} + \begin{bmatrix} -x_3(t)\delta & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}.$$

Step 2.

$$\dot{F}_1 = -\ddot{y}_1(t) + x_3(t)\ddot{y}_2(t-1) + \dot{y}_2(t-1)\,(u_2(t) - x_1(t)\,u_2(t-1))$$

$$\begin{aligned} \begin{bmatrix} \tilde{F}_1 \\ \dot{F}_1 \end{bmatrix} &= \begin{bmatrix} \dot{y}_1(t) \\ -\ddot{y}_1(t) + x_3(t)\ddot{y}_2(t-1) \end{bmatrix} \\ &\quad + \begin{bmatrix} -x_3(t)\delta & 0 \\ 0 & \dot{y}_2(t-1)\,(1 - x_1(t)\delta) \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}. \end{aligned}$$

Since $\rho_2 = 2 = m$, the algorithm ends by defining

$$\begin{aligned} \tilde{F}_2 &= \begin{bmatrix} \tilde{F}_1 \\ \dot{F}_1 \end{bmatrix} \\ F_2 &= 0. \end{aligned}$$

Inverse system

$$\begin{aligned} \mathrm{d}\tilde{F}_2 &= \begin{bmatrix} 0 & 0 & -u_1(t-1) \\ \dot{y}_2(t-1)\,u_2(t-1) & 0 & \ddot{y}_2(t-1) \end{bmatrix} \mathrm{d}x \\ &\quad + \begin{bmatrix} -x_3(t)\delta & 0 \\ 0 & \dot{y}_2(t-1)\,(1 - x_1(t)\delta) \end{bmatrix} \mathrm{d}u \\ &\quad + \begin{bmatrix} \mathrm{d}\dot{y}_1(t) \\ -\mathrm{d}\ddot{y}_1(t) + x_3(t)\mathrm{d}\ddot{y}_2(t-1) + (u_2(t) - x_1(t)\,u_2(t-1))\,\mathrm{d}\dot{y}_2(t-1) \end{bmatrix}. \end{aligned}$$

From (11) and (12) one has

$$\begin{aligned} x_1(t) &= y_1(t) \\ x_2(t) &= y_2(t) \\ x_3(t) &= \dot y_1(t)/\dot y_2(t-1) \end{aligned}$$

so $\mathrm{d}x(t) \in \mathcal{Y}$ and $\mathrm{d}\tilde F_2$ may now be written as

$$\mathrm{d}\tilde F_2 = \begin{bmatrix} -\dot y_1(t)/\dot y_2(t-1)\delta & 0 \\ 0 & \dot y_2(t-1) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 - y_1(t)\delta \end{bmatrix} \mathrm{d}u + \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}$$

with

$$\begin{aligned} \omega_1 &= \mathrm{d}\dot y_1(t) - u_1(t-1)\mathrm{d}\big(\dot y_1(t)/\dot y_2(t-1)\big) \\ \omega_2 &= d\big(\ddot y_2(t-1)\,\dot y_1(t)/\dot y_2(t-1) - \ddot y_1(t)\big) \\ &\quad + u_2(t)\mathrm{d}\dot y_2(t-1) - u_2(t-1)\mathrm{d}\big(y_1(t)\,\dot y_2(t-1)\big), \end{aligned}$$

from which $\mu_1 = 1$, $\mu_2 = 0$, and

$$B(0) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad B'(\delta) = \begin{bmatrix} 0 & 0 \\ 0 & -y_1(t)\delta \end{bmatrix}.$$

From previous equation we obtain also $\tau_1 = 0$ and $\tau_2 = 1$.

Since condition (8) is fulfilled, one solution can be found as provided by the proof of Theorem 1:

$$\begin{aligned} \begin{matrix} \tilde F_{2,1}(t+1) \\ \tilde F_{2,1}(t) \end{matrix} &= \begin{bmatrix} \dot y_1(t+1) \\ -\ddot y_1(t) + \ddot y_2(t-1)\,\dot y_1(t)/\dot y_2(t-1) \end{bmatrix} \\ &\quad + \begin{bmatrix} \dot y_1(t+1)/\dot y_2(t) & 0 \\ 0 & \dot y_2(t-1) \end{bmatrix} u(t) + \begin{bmatrix} 0 & 0 \\ 0 & y_1(t)\delta \end{bmatrix} u(t) \end{aligned}$$

so, from equation (10):

$$\begin{aligned} u(t) = -&\begin{bmatrix} \dot y_1(t+1)/\dot y_2(t) & 0 \\ 0 & \dot y_2(t-1) \end{bmatrix}^{-1} \\ &\left[\begin{bmatrix} \dot y_1(t+1) \\ -\ddot y_1(t) + \ddot y_2(t-1)\,\dot y_1(t)/\dot y_2(t-1) \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & -y_1(t) \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} \right]. \end{aligned}$$

An inverse system is given by (9). After a state reduction, one gets

$$\begin{aligned} u_1(t) &= y_2(t) \\ u_2(t) &= \big[\ddot y_1(t) - \ddot y_2(t-1)\,\dot y_1(t)/\dot y_2(t-1) - y_1(t)z(t)\big]/\dot y_2(t-1) \\ z(t+1) &= \big[\ddot y_1(t) - \ddot y_2(t-1)\dot y_1(t)/\dot y_2(t-1) - y_1(t)z(t)\big]/\dot y_2(t-1). \end{aligned}$$

4. CONCLUSIONS

An extension of Singh's inversion algorithm for time-delay systems has been given. Convergence of the algorithm is not sufficient for the existence of an inverse system and suitable additional conditions have been displayed in Theorem 1. These results are innovative also in the special case of linear systems since, to our best knowledge, no inversion algorithm has been explicitly given in the literature. This information can be used for the analysis of control problems such as the disturbance decoupling. Implementation of effective algorithms is still an open problem for future research.

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REFERENCES

[1] G. Conte, C. H. Moog and A. M. Perdon: Nonlinear Control Systems: An Algebraic Setting. (Lecture Notes in Control and Information Sciences 242.) Springer-Verlag, London 1999.

[2] G. Conte and A. M. Perdon: The disturbance decoupling problem for systems over a ring. SIAM J. Control Optim. *33* (1995), 750–764.

[3] J. Hale: Theory of Functional Differential Equations. Springer Verlag, New York 1977.

[4] Z. Iwai, D. E. Seborg, D. G. Fisher and N. Kobayashi: Decoupling of linear time variant systems with time delays in the control variable or state variable. Internat. J. Control *28* (1978), 869–888.

[5] M. Malabre and R. Rabah: Structure at infinity, model matching, and disturbance rejection for linear systems with delays. Kybernetika *29* (1993), 485–498.

[6] L. A. Márquez-Martínez, C. H. Moog and M. Velasco-Villa: The structure of nonlinear time-delay systems. In: 6th IEEE MCCS. Sardinia 1998.

[7] C. H. Moog, R. Castro-Linares, M. Velasco-Villa and L. A. Márquez-Martínez: The disturbance decoupling problem for time-delay systems. IEEE Trans. Automat. Control. To appear.

[8] S. N. Singh: A modified algorithm for invertibility in nonlinear systems. IEEE Trans. Automat. Control *14* (1981), 595–598.

[9] S. G. Tzafestas and P. N. Paraskevopoulos: On the decoupling of multivariable control systems with time delay systems. Internat. J. Control *17* (1973), 2, 405–415.

[10] M. Velasco-Villa and C. H. Moog: Disturbance decoupling problem for time-delay nonlinear systems: dynamic approach. In: IFAC Conference on Control and Systems's Structure. Nantes 1998.

Dr. Luis A. Márquez-Martínez and Dr. Claude H. Moog, IRCCyN, UMR C.N.R.S. 6597, 1 rue de la Noë, BP 92101, 44321 Nantes Cedex 3. France.
e-mails: marquez,moog@ircyn.ec-nantes.fr

Dr. Martín Velasco-Villa, CINVESTAV-IPN, Department of Electrical Engineering, Mechatronics Section, Apdo. Postal 14-740, 07000 Mexico, D. F. Mexico.
e-mail: velasco@mail.cinvestav.mx