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A MODIFICATION OF THE HARTUNG–KNAPP CONFIDENCE INTERVAL ON THE VARIANCE COMPONENT IN TWO–VARIANCE–COMPONENT MODELS

Barbora Arendacká

We consider a construction of approximate confidence intervals on the variance component σ_1^2 in mixed linear models with two variance components with non-zero degrees of freedom for error. An approximate interval that seems to perform well in such a case, except that it is rather conservative for large σ_1^2/σ^2, was considered by Hartung and Knapp in [6]. The expression for its asymptotic coverage when $\sigma_1^2/\sigma^2 \to \infty$ suggests a modification of this interval that preserves some nice properties of the original and that is, in addition, exact when $\sigma_1^2/\sigma^2 \to \infty$. It turns out that this modification is an interval suggested by El-Bassiouni in [5]. We comment on its properties that were not emphasized in the original paper [5], but which support use of the procedure. Also a small simulation study is provided.

Keywords: variance components, approximate confidence intervals, mixed linear model

AMS Subject Classification: 62F25, 62J10

1. INTRODUCTION

We will consider the problem of constructing a confidence interval on the variance component σ_1^2 in a mixed linear model with two variance components, i.e. in a situation when the n-dimensional vector of observations y is supposed to come from $N_n(X\beta, \sigma_1^2 ZZ^T + \sigma^2 I)$ distribution, where X, Z are known matrices and β and $(\sigma_1^2, \sigma^2)^T$ are vectors of unknown parameters, $\sigma_1^2 \geq 0, \sigma^2 > 0$. We suppose that $\mathcal{R}(Z) \nsubseteq \mathcal{R}(X)$, where $\mathcal{R}(A)$ denotes the linear subspace generated by the columns of the matrix A. As the variance component σ_1^2 is not influenced by a translation in mean, the usual step is to exploit the principle of invariance and reduce the problem by constructing a maximal invariant, i.e. transform y into $B^T y \sim N_{n-\mathrm{rank}(X)}(0, \sigma_1^2 B^T ZZ^T B + \sigma^2 I)$, where $BB^T = M = I - X(X^T X)^- X^T, B^T B = I_{n-\mathrm{rank}(X)}$. A minimal sufficient statistic for $(\sigma_1^2, \sigma^2)^T$ in the family of distributions of $B^T y$ is a vector consisting of mutually independent quadratic forms

$$U_i = y^T B F_i B^T y \sim (\lambda_i \sigma_1^2 + \sigma^2)\chi^2_{\nu_i}, \quad i = 1, \ldots, r$$

where $\lambda_1 > \lambda_2 > \cdots > \lambda_r \geq 0$ are the distinct eigenvalues of the matrix $B^T ZZ^T B$, $\nu_i, i = 1, .., r$ are their multiplicities and $F_i, i = 1, \ldots, r$ are the symmetric, idempotent and pairwise orthogonal ($F_i F_j = 0, i \neq j$) matrices belonging to $\lambda_i, i = 1, \ldots, r$ in the spectral decomposition of $B^T ZZ^T B$. In models with $n > \text{rank}([X,\ Z])$, $\lambda_r = 0$ and the nuisance parameter σ^2 can be estimated using solely U_r. That is what we will further suppose. It means that from now on

$$U_r \sim \sigma^2 \chi^2_{\nu_r}.$$

The construction of confidence intervals on σ_1^2 is complicated by the presence of the nuisance parameter σ^2, owing to which the exact solutions are not known. Approximate confidence intervals that can be constructed in a general situation, without any special assumptions on X, Z, except for $n > \text{rank}([X,\ Z])$, include those derived by Park and Burdick (TINGM in [7]), or Hartung and Knapp [6] or Thomas and Hultquist [10]. Based on various simulation studies (see [1, 7, 10]) out of these three the Hartung–Knapp interval seems to perform the best regarding the fact that its coverage was at least as great as the nominal level for a whole range of σ_1^2/σ^2. However, according to the simulations, in models with ν_r not much bigger than $s = \sum_{i=1}^{r-1} \nu_i$ it tends to be conservative with increasing σ_1^2/σ^2. The properties of the Hartung–Knapp interval are discussed in greater detail in Section 2. In Section 3 a modification that preserves some 'nice' properties of the original Hartung–Knapp interval and that is, moreover, exact for $\sigma_1^2/\sigma^2 \to \infty$, is stated, yielding an interval already proposed by El-Bassiouni [5]. This in turn implies some nice properties of El-Bassiouni's procedure, which supports its use. Section 4 is devoted to a small simulation study illustrating properties of the mentioned intervals.

In the following $F_{m,n;\alpha}$ and $\chi^2_{n;\alpha}$ denote the α quantiles of the corresponding F and χ^2 distributions and $s = \sum_{i=1}^{r-1} \nu_i$ (as stated earlier).

2. THE HARTUNG–KNAPP (HK) INTERVAL

Hartung and Knapp [6] considered an $(1-\alpha)100\%$ approximate interval on σ_1^2 based on an exact interval for the ratio of the variance components σ_1^2/σ^2 derived by Wald [13] (see also [8]). Bounds of the HK interval, $[\mathrm{L_{HK}}, \mathrm{U_{HK}}]$, are obtained by multiplying the bounds of the exact interval[1] on σ_1^2/σ^2 by an unbiased estimator of $\sigma^2 : U_r/\nu_r$. More precisely,

$$\mathrm{L_{HK}} = lU_r/\nu_r, \qquad \mathrm{U_{HK}} = uU_r/\nu_r, \tag{1}$$

where l, u are nonnegative solutions (or zeros if nonnegative solutions do not exist) to the following equations:

$$\sum_{i=1}^{r-1} \frac{U_i}{\lambda_i l + 1} = sF_{s,\nu_r;1-\alpha/2} U_r/\nu_r \tag{2}$$

$$\sum_{i=1}^{r-1} \frac{U_i}{\lambda_i u + 1} = sF_{s,\nu_r;\alpha/2} U_r/\nu_r. \tag{3}$$

[1] To be precise, the interval is exact for $\sigma_1^2/\sigma^2 > 0$, the true zero value is covered with probability $1-\alpha/2$ (if the interval is taken equal-tailed). This is caused by putting any negative bounds equal to zero, as the ratio σ_1^2/σ^2 is a non-negative parameter. See also [8].

If there exists a nonnegative solution to (2) or (3), it is unique as the left side of (2), (3) is a strictly decreasing function of l, u respectively, on $(-1/\lambda_1, \infty)$. The solutions can be computed, for example, by the Newton–Raphson method. In case of an unbalanced one-way random effects model another quick approach is to use the bisection method with starting points suggested by Wald [12], see also [6].

Considering that under the limit $\nu_r \to \infty$ the estimator U_r/ν_r converges to σ^2 in probability, it is obvious that the HK interval becomes exact for $\nu_r \to \infty$ (for true $\sigma_1^2 > 0$). Also, the interval contains zero in accordance with the result of the Wald test of nullity of σ_1^2 (on the significance level $\alpha/2$). (Its lower bound is zero if and only if the hypothesis of nullity is not rejected, i. e. if $\nu_r \sum_{i=1}^{r-1} U_i/(sU_r) \le F_{s,\nu_r;1-\alpha/2}$.) This causes the interval to cover the true zero value of σ_1^2 with probability $1-\alpha/2$, which is greater than the nominal confidence level, however, this is a common feature of two-sided confidence intervals on σ_1^2. The preceding two optimal features of the HK interval are not accompanied by being exact when $\sigma_1^2/\sigma^2 \to \infty$.

Let $V_i = U_i/(\lambda_i \sigma_1^2)$, $i = 1, \dots, r-1$. Then as $\sigma_1^2/\sigma^2 \to \infty$, $(V_1, \dots, V_{r-1}, \frac{U_r}{\sigma^2}\frac{\sigma^2}{\sigma_1^2})$ converges in distribution to $(Q_1, \dots, Q_{r-1}, 0)$, where $Q_i \sim \chi^2_{\nu_i}$ and Q_is are mutually independent. The probability of covering the true non-zero value of σ_1^2 by the HK interval is:

$$\mathrm{P}\left(\mathrm{L_{HK}} \le \sigma_1^2 \le \mathrm{U_{HK}}\right) = \mathrm{P}\left(sF_{s,\nu_r;\alpha/2} U_r/\nu_r \le \sum_{i=1}^{r-1} \frac{U_i}{\lambda_i \sigma_1^2 \nu_r/U_r + 1} \le sF_{s,\nu_r;1-\alpha/2} U_r/\nu_r\right)$$
$$= \mathrm{P}\left(sF_{s,\nu_r;\alpha/2} \le \sum_{i=1}^{r-1} \frac{V_i}{1 + \frac{1}{\lambda_i}\frac{U_r}{\nu_r \sigma^2}\frac{\sigma^2}{\sigma_1^2}} \le sF_{s,\nu_r;1-\alpha/2}\right)$$

and under the limit $\sigma_1^2/\sigma^2 \to \infty$ we get

$$\mathrm{P}(\mathrm{L_{HK}} \le \sigma_1^2 \le \mathrm{U_{HK}}) \to \mathrm{P}\left(sF_{s,\nu_r;\alpha/2} \le \sum_{i=1}^{r-1} Q_i \le sF_{s,\nu_r;1-\alpha/2}\right). \tag{4}$$

Denote the limiting probability of coverage $\mathrm{P}_{\alpha,s,\nu_r}$. The differences between $\mathrm{P}_{\alpha,s,\nu_r}$ and the desired confidence level $1-\alpha$ for certain configurations of s, ν_r and 90 %, 95 % and 99 % intervals are stated in Table 1. Naturally, these differences are smaller if s is substantially smaller than ν_r. The tendency of the HK interval to be conservative for large values of σ_1^2/σ^2 was observed also by Hartung and Knapp [6] in their simulation study. Apart from that the HK interval seemed to perform well.

In the next section we present a modification of the HK interval that is exact when $\sigma_1^2/\sigma^2 \to \infty$.

3. A MODIFICATION OF THE HK INTERVAL

The HK interval would be exact for $\sigma_1^2/\sigma^2 \to \infty$, if in (4) we obtained

$$\mathrm{P}\left(\chi^2_{s;\alpha/2} \le \sum_{i=1}^{r-1} Q_i \le \chi^2_{s;1-\alpha/2}\right).$$

Table 1. Differences between the probability of coverage of an $(1-\alpha)\,100\%$ HK interval under the limit $\sigma_1^2/\sigma^2 \to \infty$ and the desired confidence level $1-\alpha$.

	$\mathrm{P}_{\alpha,s,\nu_r}-(1-\alpha)$				
	$s=2$, $\nu_r=1$	$s=2$, $\nu_r=25$	$s=10$, $\nu_r=25$	$s=10$, $\nu_r=50$	$s=30$, $\nu_r=50$
$\alpha=0.1$	0.0474	0.0160	0.0479	0.0294	0.0614
$\alpha=0.05$	0.0244	0.0113	0.0283	0.0187	0.0355
$\alpha=0.01$	0.0050	0.0036	0.0067	0.0051	0.0083

This can be rewritten as

$$\mathrm{P}\left(sF_{s,\nu_r;\alpha/2} \le \frac{sF_{s,\nu_r;\alpha/2}}{\chi^2_{s;\alpha/2}}\sum_{i=1}^{r-1} Q_i \;\&\; \frac{sF_{s,\nu_r;1-\alpha/2}}{\chi^2_{s;1-\alpha/2}}\sum_{i=1}^{r-1} Q_i \le sF_{s,\nu_r;1-\alpha/2}\right). \tag{5}$$

Denote

$$H(t)=\sum_{i=1}^{r-1}\frac{V_i}{t+\frac{1}{\lambda_i}\frac{U_r}{\nu_r\sigma^2}\frac{\sigma^2}{\sigma_1^2}}$$

and $k_{\alpha/2}=\frac{\chi^2_{s;\alpha/2}}{sF_{s,\nu_r;\alpha/2}}$. It is clear that as $\sigma_1^2/\sigma^2\to\infty$, $H(k_{\alpha/2})$ converges in distribution to $\frac{1}{k_{\alpha/2}}\sum_{i=1}^{r-1} Q_i$ and also (e.g. by Cramér–Wold Theorem(see [2], p. 49))

$$(H(k_{\alpha/2}), H(k_{1-\alpha/2})) \xrightarrow{\mathcal{D}} \left(\frac{1}{k_{\alpha/2}}\sum_{i=1}^{r-1} Q_i, \frac{1}{k_{1-\alpha/2}}\sum_{i=1}^{r-1} Q_i\right).$$

Thus

$$\mathrm{P}(sF_{s,\nu_r;\alpha/2} \le H(k_{\alpha/2}) \;\&\; H(k_{1-\alpha/2}) \le sF_{s,\nu_r;1-\alpha/2}) \tag{6}$$

converges to the desired result (5). Working on (6) backwards we obtain that it is equal to

$$\mathrm{P}\left(\mathrm{L_{HK}}\frac{sF_{s,\nu_r;1-\alpha/2}}{\chi^2_{s;1-\alpha/2}} \le \sigma_1^2 \le \mathrm{U_{HK}}\frac{sF_{s,\nu_r;\alpha/2}}{\chi^2_{s;\alpha/2}}\right),$$

which suggests a modification of the HK interval with the lower and upper bounds as follows:

$$\begin{aligned}\mathrm{L}_m &= \mathrm{L_{HK}}sF_{s,\nu_r;1-\alpha/2}/\chi^2_{s;1-\alpha/2},\\ \mathrm{U}_m &= \mathrm{U_{HK}}sF_{s,\nu_r;\alpha/2}/\chi^2_{s;\alpha/2}.\end{aligned} \tag{7}$$

From (7) it is clear that $\mathrm{L}_m, \mathrm{U}_m$ are zeros if and only if $\mathrm{L_{HK}}, \mathrm{U_{HK}}$ are zeros and that for $\nu_r\to\infty$ the HK interval and its modified version coincide. So the modified HK interval has the two optimal properties of the HK interval and moreover, it is exact for $\sigma_1^2/\sigma^2\to\infty$.

The interval given in (7) was originally proposed by El-Bassiouni [5] who at first considered the case with $r = 2$. In such a situation, Boardman [3] commented that the interval (1) should be slightly wider and thus more conservative than the Williams–Tukey interval (see [11, 14]), which had coverage close to the nominal value in his simulation study. El-Bassiouni observed that the Williams–Tukey interval can be written in the form:

$$[c_1 l U_r/\nu_r,\ c_2 u U_r/\nu_r], \tag{8}$$

where $c_1 = sF_{s,\nu_r;1-\alpha/2}/\chi^2_{s;1-\alpha/2}$, $c_2 = sF_{s,\nu_r;\alpha/2}/\chi^2_{s;\alpha/2}$ and l, u, are the bounds of the exact interval on σ_1^2/σ^2. He remarked that c_1, c_2 can be regarded as correction factors adjusting for the fact that the exact bounds of the interval on σ_1^2/σ^2 are multiplied by an estimator of σ^2. Based on the previous, El-Bassiouni's suggestion for an approximate interval in case $r > 2$ was to use expression (8) with l, u as in (2), (3). In the simulation study in [5] only this proposed interval gave coverage not lower than the nominal confidence level across all designs and for all considered values of parameters. In addition to this favourable result we have just shown that this interval possesses some nice properties: namely that it behaves as an exact interval for $\sigma_1^2/\sigma^2 \to \infty$, $\sigma_1^2 = 0$ and $\nu_r \to \infty$. From derivation of (7) it is also clear how the constants c_1, c_2, called correction factors by El-Bassiouni, really improve the behaviour of the interval in comparison to the HK interval.

From El-Bassiouni's derivation of (7) follows that in case $r = 2$, the interval reduces to the Williams–Tukey interval. Thus besides being regarded as a modification of the HK interval, it is also a generalization of the Williams–Tukey interval. In fact, it can be obtained by applying Williams's approach used in [14] (the bounds of the approximate interval are obtained as the intersections of the lower and upper bounds of two exact intervals on σ_1^2 constructed for a known value of σ^2). Although this is not surprising, it was not explicitly stated by El-Bassiouni and we emphasize it here because we think it makes the procedure more attractive (e. g. immediately it is guaranteed that the probability of coverage of the resulting interval is always at least $1 - 2\alpha$, see [14]). Also, it resolves any doubts that might have arisen, whether after the modification as given in (7) it still holds $\mathrm{L}_m \leq \mathrm{U}_m$.

The two exact intervals on σ_1^2 constructed for a known value of σ^2, required by the Williams's approach are:

1. an interval derived from an exact interval on σ_1^2/σ^2, $I_1(\sigma^2)$, whose lower and upper bounds $B_l = l\sigma^2$, $B_u = u\sigma^2$ (l, u defined by (2), (3)) can be obtained by solving the following equations

$$\begin{aligned}\sum_{i=1}^{r-1} \frac{U_i}{\lambda_i B_l + \sigma^2} &= \frac{1}{\sigma^2} sF_{s,\nu_r;1-\alpha/2} U_r/\nu_r \\ \sum_{i=1}^{r-1} \frac{U_i}{\lambda_i B_u + \sigma^2} &= \frac{1}{\sigma^2} sF_{s,\nu_r;\alpha/2} U_r/\nu_r.\end{aligned} \tag{9}$$

2. an interval based on the fact that for a known σ^2, $\sum_{i=1}^{r-1} \frac{U_i}{\lambda_i\sigma_1^2 + \sigma^2} \sim \chi^2_s$. Bounds of this interval, $I_2(\sigma^2)$, can be obtained by solving the following equa-

tions

$$\sum_{i=1}^{r-1} \frac{U_i}{\lambda_i \tilde{B}_l + \sigma^2} = \chi^2_{s;1-\alpha/2}, \qquad \sum_{i=1}^{r-1} \frac{U_i}{\lambda_i \tilde{B}_u + \sigma^2} = \chi^2_{s;\alpha/2}. \tag{10}$$

Comparing (9) with (10) we see, that the lower bounds of I_1 and I_2 intersect at $\sigma^2 = c_1 U_r/\nu_r$ and the upper bounds at $\sigma^2 = c_2 U_r/\nu_r$ with the value of the lower and the upper bounds $\tilde{B}_l = B_l = c_1 l U_r/\nu_r$ and $\tilde{B}_u = B_u = c_2 u U_r/\nu_r$.

Considering all the above stated facts, we will refer to the interval defined in (7) as the El-Bassiouni–Williams–Tukey (EBWT) interval.

Comparison of length. Looking into F tables, from (7) we can see that for all commonly used values of $\alpha (\leq 0.1)$ and $s \geq 3$, the EBWT interval is shorter (for small and moderate values of ν_r) than the HK interval and we may expect it to be overally less conservative. For $s \leq 2$, still $\mathrm{L}_m \geq \mathrm{L}_{\mathrm{HK}}$, however $\mathrm{U}_m \geq \mathrm{U}_{\mathrm{HK}}$, so the EBWT interval can be larger than the HK interval. Actually, in all examples with $s = 2$ considered in our simulation study (see Section 4) the EBWT interval was always larger (or of zero length) than the HK interval. To see how much larger the EBWT interval can be, consider the difference between the lengths of the two intervals: $\mathrm{Length}_{\mathrm{EBWT}} - \mathrm{Length}_{\mathrm{HK}} = (c_2 - 1)\mathrm{U}_{\mathrm{HK}} - (c_1 - 1)\mathrm{L}_{\mathrm{HK}}$. For $s = 2, \nu_r = 30$ the difference can be bounded from above by $(c_2 - 1)\mathrm{Length}_{\mathrm{HK}}$, from which it follows that the EBWT interval can be larger than the HK interval by at most 0.18 %, 0.09 %, 0.02 % of $\mathrm{Length}_{\mathrm{HK}}$ in case of 90%, 95%, 99% confidence intervals, respectively. Similarly, for $s = 1, \nu_r = 50$, $\mathrm{Length}_{\mathrm{EBWT}}$ can be greater than $\mathrm{Length}_{\mathrm{HK}}$ by at most 1.01 % of the length of the HK interval in case of 90 %, 95 % or 99 % intervals. However, for more extreme cases the difference can be larger.

In [5] El-Bassiouni suggested so-called short intervals obtained when in (7), instead of the equal-tailed χ^2 quantiles $\chi^2_{\alpha/2}$, $\chi^2_{1-\alpha/2}$, we use χ^2 quantiles such as in Table 678 in [9], while in F quantiles $\alpha/2$, $1 - \alpha/2$ are applied. Such intervals remain exact for $\sigma_1^2/\sigma^2 \to \infty$ and cover the true zero value with probability $1 - \alpha/2$ (however, for $\nu_r \to \infty$ the constants c_1, c_2 do not converge to 1). We will refer to these intervals as the short EBWT intervals.

4. SIMULATION STUDY

To illustrate the behaviour of the HK, the EBWT and the short EBWT intervals we conducted the following simulation study: in each of 7 concrete examples of our model, as they are stated in Table 2, the vector of observations y was generated 10 000-times for $\sigma_1^2 = 0.001, 0.1, \ldots, 0.9, 0.999$, $\sigma^2 = 1 - \sigma_1^2$ and fixed value of β. The bounds of the HK interval were computed by the Newton–Raphson method with tolerance 10^{-14} (using (2), (3)), the bounds of the EBWT interval and its short version were then obtained using (7) with appropriate quantiles. The choice of the values of the variance components was inspired by their choice in [7]. Designs of Examples 2, 3, 6 were considered also in [1, 6, 7], Examples 4, 5 in [1]. Figure 1 shows the simulated probabilities of coverage of the 95 % HK, EBWT and short EBWT intervals and limits (0.946, 0.954) between which the simulated values should fall

Table 2. Forms of the matrices X, Z (together with values of s, ν_r) in the examples considered in the simulation study. v_k is a $(k \times 1)$ vector of real numbers between 0 and 1, 1_k is a $(k \times 1)$ vector of ones. If not stated otherwise, Z is a block matrix with 1_{n_i} on the diagonal.

Example	X	Z	s	ν_r
1	1_4	$n_i : 1, 1, 2$	2	1
2	$[1_{30}\ v_{30}]$	$n_i : 5, 10, 15$	2	26
3	$[1_{102}\ v_{102}]$	$n_i : 1, 1, 100$	2	98
4	$[1_{30}\ t]$ $t : 30 \times 1,$ $t_i = -3 + 6 * (i-1)/29$	$[\text{diag}(1_6, 1_6, 1_6, 1_6, 1_6)\ w]$ $w : 30 \times 1,$ $w_i = (-2 + 4 * (i-1)/29)^2$	5	23
5	1_{14}	$n_i : 1, 1, 1, 1, 1, 1, 2, 2, 2, 2$	9	4
6	$[1_{59}\ v_{59}]$	$n_i : 1, 1, 4, 5, 6, 6, 8, 8, 10, 10$	9	48
7	1_{12}	$n_i : 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2$	10	1

with probability 0.95 if the true probability of coverage is 0.95 (using the normal approximation to the binomial). Figure 2 presents the average lengths obtained in the different situations. Note that increasing σ_1^2 implies increasing ratio σ_1^2/σ^2(from approximately 0.001 to 999).

Comparing results in Examples $1, 2, 3$ or $5, 6$ one can see the positive effect of higher ν_r in the model on the conservativeness of the HK interval. On the other hand, Examples $1, 4$, and especially 5 and 7 show how much conservative the HK interval can be in case of small ν_r. As expected, in all examples the EBWT interval is less conservative than the HK interval and moreover, its confidence coefficients do not seem to be less than 0.95. Only in Example 7 with $\sigma_1^2 = 0.001$ the simulated probability of coverage is slightly below the lower limit 0.946. The situation is similar for the short EBWT interval, however, unlike the two other ones, it seems to remain a bit conservative for large ν_r and smaller values of σ_1^2/σ^2 as can be seen from results in Example 3.

Comparing the average lengths, the HK interval is shorter compared to the EBWT interval in the first three examples ($s = 2$), while the situation is opposite considering the rest. The biggest differences between the lengths of the HK and EBWT intervals are seen in examples with small ν_r. The short version of the EBWT interval yields the shortest intervals in all examples, the difference is remarkable especially in models with $s = 2$, which agrees with El-Bassiouni's findings in [5].

5. CONCLUSIONS

We have pointed out some favourable properties of the approximate interval on a variance component suggested by El-Bassiouni [5], which supports its use besides the good results it yielded in a simulation study in [5]. The interval can be regarded as a modification of an interval considered e. g. by Hartung and Knapp in [6] or as

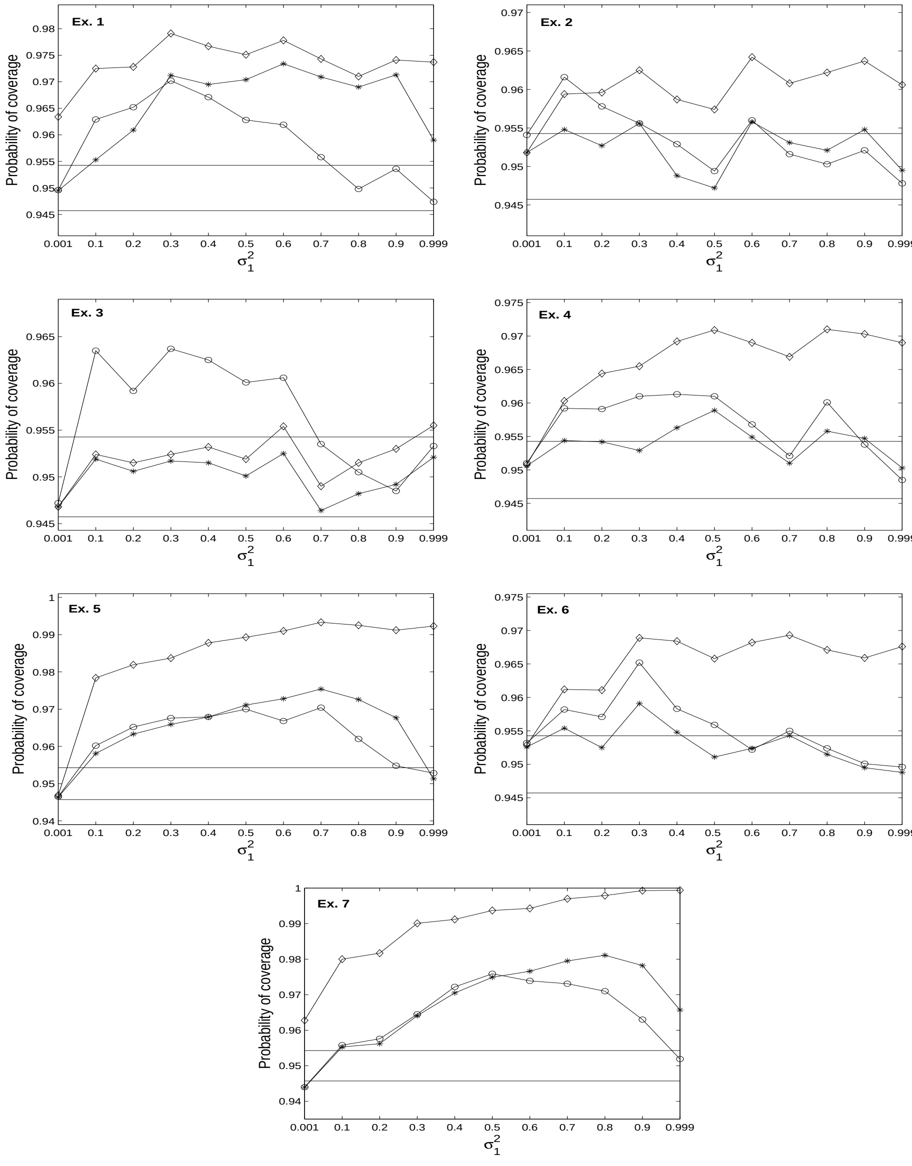

Fig. 1. Simulated probabilities of coverage of 95 % HK(◇), EBWT(*) and short EBWT(o) confidence intervals in examples from Table 2.(10 000 simulations).

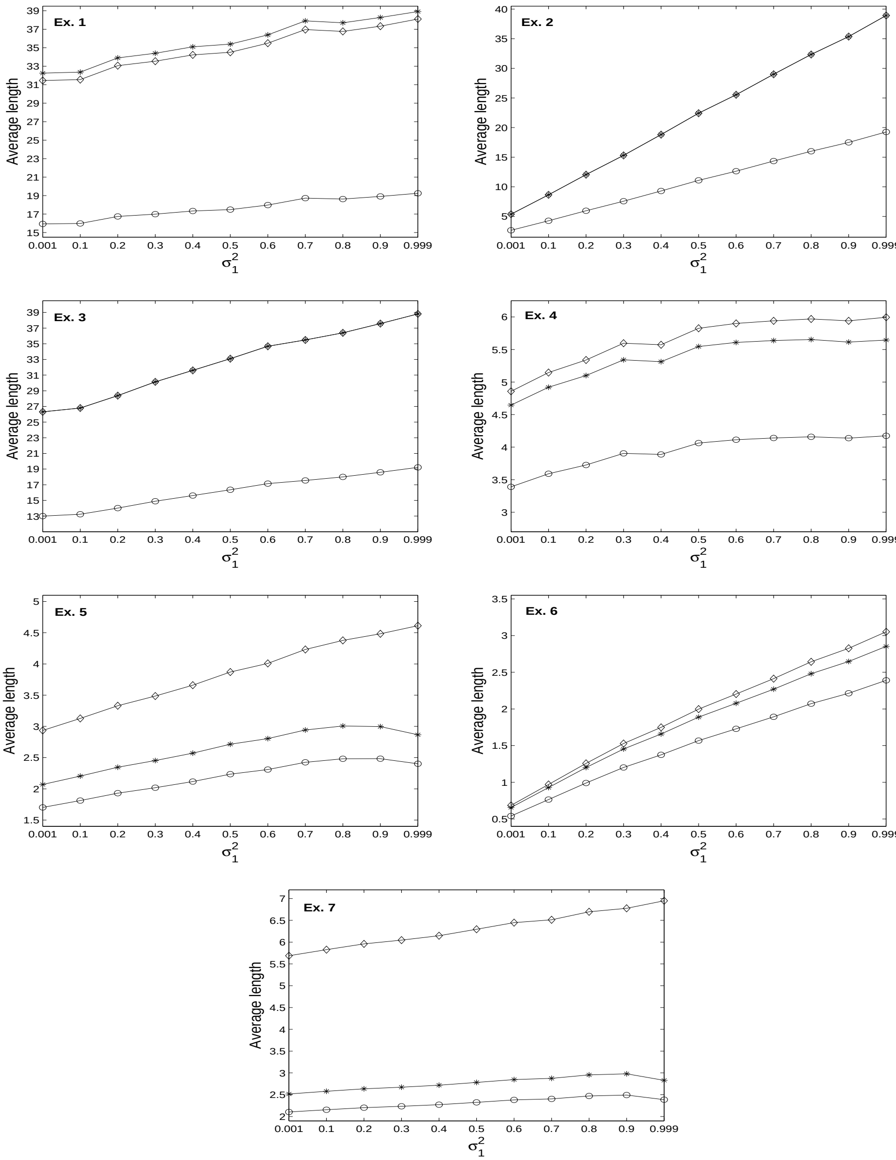

Fig. 2. Average lengths of 95 % HK(◇), EBWT(*) and short EBWT(o) confidence intervals in examples from Table 2 (10 000 simulations).

a generalization of the well-known Willimas–Tukey interval. We have shown how exactly the modification improves the properties of the resulting interval and pointed out that the interval can be derived by directly applying Williams's approach. This is not surprising, however, it was not mentioned explicitly by El-Bassiouni and we think it makes the procedure more attractive. A small simulation study was used to illustrate the behaviour of the approximate interval, referred to as El-Bassiouni–Williams–Tukey with respect to the preceding, its short version (see [5]) and the Hartung–Knapp interval.

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