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SYNCHRONIZATION WITH ERROR BOUND OF NON–IDENTICAL FORCED OSCILLATORS

Jiangen Wang, Jianping Cai, Mihua Ma and Jiuchao Feng

Synchronization with error bound of two non-identical forced oscillators is studied in the paper. By introducing two auxiliary autonomous systems, differential inequality technique and active control technique are used to deal with the synchronization of two non-identical forced oscillators with parameter mismatch in external harmonic excitations. Numerical simulations show the effectiveness of the proposed method.

Keywords: chaotic synchronization with error bound, non-identical forced oscillator, differential inequality, active control

AMS Subject Classification: 74H65, 70K40

1. INTRODUCTION

Chaotic synchronization have been extensively studied since the pioneer work of Pecora and Carroll [10] due to its theoretical challenge and its great potential applications in secure communications, chemical and biomedical sciences, and other fields. A wide variety of approaches have been proposed for the synchronization of chaotic systems such as linear and nonlinear feedback controls, most of which synchronize two identical systems or non-identical autonomous systems [2, 7, 11, 13, 14, 15]. However, more and more non-autonomous systems have been found in engineering and physics [3, 4] therefore much attention should be paid to the synchronization of two non-identical non-autonomous chaotic systems. But it is obvious that the synchronization of two non-identical non-autonomous systems is rather difficult.

Although active control technique sometimes results in relatively complex controllers, it is often used to synchronize two chaotic systems [1, 5, 6, 8, 9, 12]. Synchronization between single and double wells Duffing–Van der Pol oscillators was discussed in [15]. However, the control functions adopted there contain external harmonic excitations, which are inconvenient in practice.

In this paper, synchronization between two non-identical forced oscillators with parameter mismatch in the external harmonic excitations is investigated. Parameter mismatch here implies that amplitude, frequency, and phase can be all or partly different in the external harmonic excitations, which is different from [16], where robust synchronization of chaotic horizontal platform systems with phase difference was studied.

By differential inequality technique, two auxiliary autonomous systems are constructed such that the trajectories of the error system of two non-identical oscillators are bounded in between the trajectories of the two auxiliary autonomous systems, so that the synchronization with a prescribed error bound can be achieved.

2. SYNCHRONIZATION OF NON-IDENTICAL FORCED OSCILLATORS

Consider the master-slave synchronization scheme of two non-identical forced oscillators with parameters mismatch in the external harmonic excitations:

$$\text{Master system:} \quad \ddot{x} + F(x,\dot{x})\dot{x} + G(x) = F_0\cos(\omega_0 t + \varphi_0), \tag{1}$$

$$\text{Slave system:} \quad \ddot{y} + f(y,\dot{y})\dot{y} + g(y) = F_1\cos(\omega_1 t + \varphi_1) + u(t), \tag{2}$$

where $\cdot$ represents the derivative with respect to time t, $f(y,\dot{y})$, $F(x,\dot{x})$, $g(y)$ and $G(x)$ are different functions, $F_0\cos(\omega_0 t + \varphi_0)$ and $F_1\cos(\omega_1 t + \varphi_1)$ are different external harmonic excitations, and $u(t)$ is a controller to be designed below.

Letting $e = y - x$, one obtains the error dynamical system

$$\begin{aligned} &\ddot{e} + f(e+x,\dot{e}+\dot{x})(\dot{e}+\dot{x}) + g(e+x) - F(x,\dot{x})\dot{x} - G(x) \\ &= F_1\cos(\omega_1 t + \varphi_1) - F_0\cos(\omega_0 t + \varphi_0) + u(t). \end{aligned} \tag{3}$$

Definition. (Wu, Cai and Wang [16]) For a given real constant $h > 0$, the synchronization scheme (1)−(2) is called synchronization with error bound h if for any finite initial conditions of systems (1) and (2), there exist $T > 0$ such that $|e(t)| < h$ for all $t > T$.

If h is sufficiently small, then such a synchronization scheme is referred to as nearly complete synchronization.

By active control technique, we choose the controller $u(t)$ as

$$u(t) = -k_1\dot{e} - k_2 e + v(x,\dot{x},e,\dot{e}),$$

where $v(x,\dot{x},e,\dot{e})$ is a nonlinear function, and k_1, k_2 are coupling coefficients, all to be determined later.

Properly choosing controller $u(t)$, the error dynamical system (3) is equivalent to the following equation:

$$\ddot{e} + (a+k_1)\dot{e} + (b+k_2)\,e = F_1\cos(\omega_1 t + \varphi_1) - F_0\cos(\omega_0 t + \varphi_0), \tag{4}$$

where a, b are constants. Obviously, the inequalities $-F_1 - F_0 \le F_1\cos(\omega_1 t + \varphi_1) - F_0\cos(\omega_0 t + \varphi_0) \le F_1 + F_0$ always hold for all t. Taking $M = F_1 + F_0$, two auxiliary autonomous systems can be constructed as follows:

$$\ddot{e_1} + (a+k_1)\dot{e} + (b+k_2)\,e - M = 0, \tag{5}$$

$$\ddot{e_2} + (a+k_1)\dot{e} + (b+k_2)\,e + M = 0. \tag{6}$$

In order to show that the synchronization scheme (1)−(2) can achieve synchronization with error bound h, we first give the following lemma.

Lemma 1. Consider the following two non-autonomous systems:

$$\begin{cases} \ddot{z}_1 = f_1(t, z_1, \dot{z}_1), \\ z_1(t_0) = z_{10}, \quad \dot{z}_1(t_0) = \dot{z}_{10}, \end{cases} \tag{7}$$

and

$$\begin{cases} \ddot{z}_2 = f_2(t, z_2, \dot{z}_2), \\ z_2(t_0) = z_{20}, \quad \dot{z}_2(t_0) = \dot{z}_{20}, \end{cases} \tag{8}$$

where $\cdot$ represents the derivative with respect to time t. If

$$f_1(t, z_1, \dot{z}_1) \leq f_2(t, z_2, \dot{z}_2), \quad \forall\, t > t_0, \tag{9}$$

$$z_{10} \leq z_{20}, \quad \dot{z}_{10} \leq \dot{z}_{20}, \tag{10}$$

then the solutions of systems (7) and (8), $z_1(t)$, $z_2(t)$, satisfy the inequality $z_1(t) \leq z_2(t)$ for all $t \geq t_0$.

Proof. Letting $\varphi(t) = z_2(t) - z_1(t)$, we have $\ddot{\varphi}(t) = \ddot{z}_2(t) - \ddot{z}_1(t) = f_2(t, z_2, \dot{z}_2) - f_1(t, z_1, \dot{z}_1) \geq 0$, for $t \geq t_0$. Hence, $\dot{\varphi}(t)$ is a monotone non-decreasing function for all $t \geq t_0$. By inequalities (10), we get $\dot{\varphi}(t_0) = \dot{z}_{20} - \dot{z}_{10} \geq 0$. Then, the inequalities $\dot{\varphi}(t) \geq \dot{\varphi}(t_0) \geq 0$ hold for all $t \geq t_0$. Similarly, $\varphi(t) \geq \varphi(t_0) \geq 0$ hold for all $t \geq t_0$, i.e. $z_1(t) \leq z_2(t)$ holds for all $t \geq t_0$.

By Lemma 1, if $e(t)$, $e_1(t)$, $e_2(t)$ are respectively the solutions of Eqs. (4), (5) and (6) satisfying the same initial conditions, then the inequalities

$$e_2(t) \leq e(t) \leq e_1(t) \tag{11}$$

hold for all $t > t_0$. After transformation

$$\varepsilon_1 = e_1 - \frac{M}{b + k_2}, \quad \varepsilon_2 = e_2 + \frac{M}{b + k_2}, \tag{12}$$

Eqs. (5) and (6) become

$$\ddot{\varepsilon}_1 + (a + k_1)\dot{\varepsilon}_1 + (b + k_2)\varepsilon_1 = 0, \tag{13}$$

$$\ddot{\varepsilon}_2 + (a + k_1)\dot{\varepsilon}_2 + (b + k_2)\varepsilon_2 = 0. \tag{14}$$

If k_1, k_2 are selected to satisfy $a + k_1 > 0$, $b + k_2 > 0$, then the eigenvalues of the corresponding characteristic equations of Eqs. (5) and (6) are all complex conjugates with negative real parts. Therefore, Eqs. (13) and (14) are asymptotically stable at the origin, that is,

$$\lim_{t \to +\infty} \varepsilon_1(t) = \lim_{t \to +\infty} \varepsilon_2(t) = 0.$$

According to Eq. (12), we obtain

$$\lim_{t \to +\infty} e_1(t) = \frac{M}{b + k_2}, \quad \lim_{t \to +\infty} e_2(t) = -\frac{M}{b + k_2}.$$

Hence, through inequalities (11), we have

$$-\frac{M}{b+k_2} \le \lim_{t\to+\infty} e(t) \le \frac{M}{b+k_2}.$$

For a prescribed error bound $h > 0$, if we choose k_1 and k_2 satisfying

$$k_1 > -a, \quad k_2 > \frac{M}{h} - b, \tag{15}$$

then $\frac{M}{b+k_2} < h$, which implies that there exits $T > 0$ such that

$$|e(t)| < h, \quad \forall\, t > T. \tag{16}$$

Therefore, the scheme (1) – (2) achieves synchronization with error bound h.

If there only exist amplitude and phase mismatches in the external harmonic excitations, i.e., $\omega_1 = \omega_0 = \omega$, then by the differential mean-value theorem, we have

$$-|F_1 - F_0| - F_0|\varphi_1 - \varphi_0| \le F_1\cos(\omega t + \varphi_1) - F_0\cos(\omega t + \varphi_0) \le |F_1 - F_0| + F_0|\varphi_1 - \varphi_0|.$$

As a result, M in (12) becomes $|F_1 - F_0| + F_0|\varphi_1 - \varphi_0|$. Especially, if $\omega_1 = \omega_0 = \omega$ and $\varphi_1 = \varphi_0$, then $M = |F_1 - F_0|$. Similarly, if $\omega_1 = \omega_0 = \omega$ and $F_1 = F_0$, then $M = F_0|\varphi_1 - \varphi_0|$. □

3. EXAMPLES

Two coupled systems consisting of a chaotic oscillator and a periodic oscillator with parameter mismatch in the external harmonic excitations are adopted to verify the effectiveness of the technique developed above.

Example 1. Consider the following master-slave synchronization scheme:

$$\text{Master system:} \qquad \ddot{x} + x - \mu(1 - x^2)\dot{x} = F_0\cos(\omega_0 t + \varphi_0), \tag{17}$$

$$\text{Slave system:} \qquad \ddot{y} + c\dot{y} + dy = F_1\cos(\omega_1 t + \varphi_1) + u(t). \tag{18}$$

For $\mu = 5,\ F_0 = 5,\ \omega_0 = 2.465$ and $\varphi_0 = 0$, the forced Van der Pol oscillator exhibits a chaotic behavior as shown in Figure 1 with initial conditions $x(0) = 2$, $\dot{x}(0) = 2$. The slave system exits a stable limit cycle when $c > 0$ and $u(t) = 0$ (see Ref. [17] for details). For $c = 1$, $d = 6.0$, $F_1 = 5.2$, $\omega_1 = 2.5$, $\varphi_1 = 0.1$ and $u(t) = 0$, the limit cycle of the slave system is shown in Figure 2.

The error system of synchronization scheme (17) – (18) is

$$\begin{aligned} &\ddot{e} + c\dot{e} + de + (d-1)x + (c+\mu)\dot{x} - \mu x^2\dot{x} \\ &\quad = F_1\cos(\omega_1 t + \varphi_1) - F_0\cos(\omega_0 t + \varphi_0) + u(t). \end{aligned} \tag{19}$$

If the controller is selected as

$$u(t) = -k_1\dot{e} - k_2 e + (d-1)x + (c+\mu)\dot{x} - \mu x^2\dot{x}, \tag{20}$$

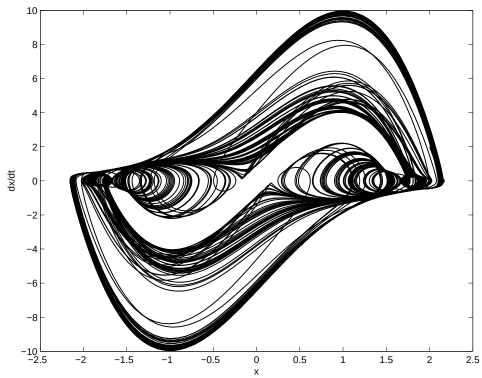

Fig. 1. For $\mu = 5$, $F_0 = 5$, $\omega_0 = 2.465$ and $\varphi_0 = 0$, the forced Van der Pol oscillator (17) exhibits a chaotic behavior with initial conditions $x(0) = 2$, $\dot{x}(0) = 2$.

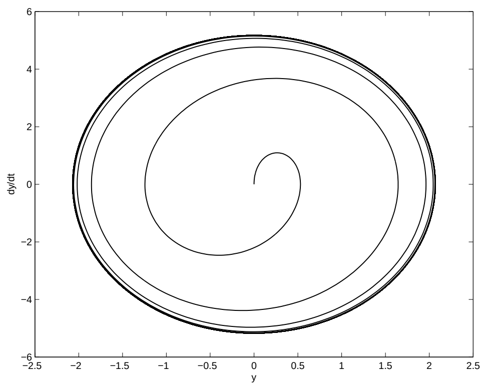

Fig. 2. For $c = 1$, $d = 6.0$, $F_1 = 5.2$, $\omega_1 = 2.5$, $\varphi_1 = 0.1$ and $u(t) = 0$, the slave system (18) has a stable limit cycle.

then the error system (19) becomes

$$\ddot{e} + (k_1 + c)\dot{e} + (k_2 + d)\, e = F_1 \cos(\omega_1 t + \varphi_1) - F_0 \cos(\omega_0 t + \varphi_0).$$

In view of equation (4), we have $a = c$, $b = d$. If k_1, k_2 are chosen to satisfy

$$k_1 > -c, \quad k_2 > \frac{M}{h} - d, \tag{21}$$

then inequality (16) holds. Hence, the master-slave scheme (17)–(18) will achieve synchronization with error bound h.

If the parameter values of the master-slave synchronization scheme(17)–(18) are $\mu = 5$, $F_0 = 5$, $\omega_0 = 2.465$, $\varphi_0 = 0$, and $c = 1$, $d = 6.0$, $F_1 = 5.2$, $\omega_1 = 2.5$, $\varphi_1 = 0.1$ and for a prescribed error bound $h = 0.1$, then we get $M = 10.2$, $k_1 > -1$,

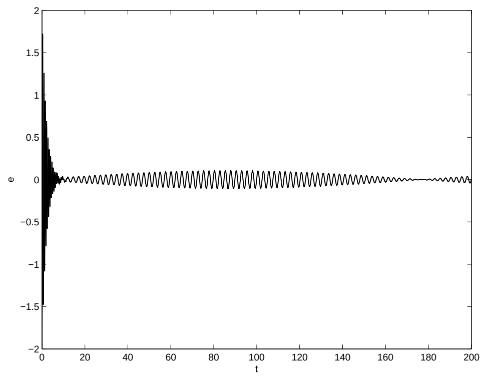

Fig. 3. Time history of the error system (19) with $h = 0.1$ and $u(t) = -97e + 5x + 6\dot{x} - 5x^2\dot{x}$ when amplitude, frequency and phase are mismatched at the same time.

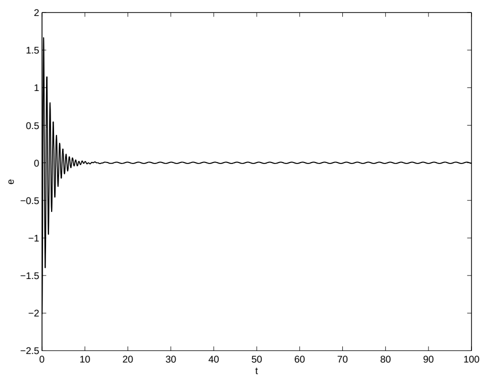

Fig. 4. Time history of the error system (19) with $\omega_1 = \omega_0 = 2.465$, $h = 0.01$ and $u(t) = -65e + 5x + 6\dot{x} - 5x^2\dot{x}$.

$k_2 > 96$. For $k_1 = 0$ and $k_2 = 97$, $u(t) = -97e + 5x + 6\dot{x} - 5x^2\dot{x}$ can be obtained from Eq. (20). The master-slave scheme (17)−(18) can achieve synchronization with error bound $h = 0.1$ as shown in Figure 3 with initial conditions $x(0) = 2$, $\dot{x}(0) = 2$ and $y(0) = 0$, $\dot{y}(0) = 0$, respectively. For a smaller error bound $h = 0.01$, we get $k_1 > -1$, $k_2 > 1014$ from inequalities (21), where the other parameter values are the same as in Figure 3. Obviously, k_2 will become larger if the error bound h becomes smaller. In other words, synchronization scheme (17)−(18) with smaller error bound $h = 0.01$ needs a larger energy input, which means synchronization cost will increase.

If $\omega_1 = \omega_0 = 2.465$, $h = 0.01$ and the other parameter values are the same as in Figure 3, then $M = |F_1 - F_0| + F_0|\varphi_1 - \varphi_0| = 0.7$, $k_1 > -1$, $k_2 > 64$. For $k_1 = 0$, $k_2 = 65$, $u(t) = -65e + 5x + 6\dot{x} - 5x^2\dot{x}$ can be obtained from Eq. (20). Achievement of synchronization scheme (17)−(18) with error bound $h = 0.01$ is

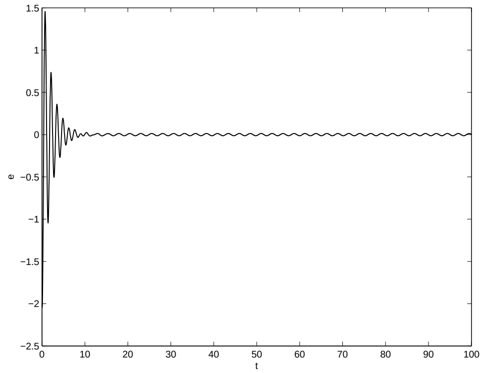

Fig. 5. Time history of the error system (19) with $\omega_1 = \omega_0 = 2.465$, $\varphi_1 = \varphi_0 = 0$, $h = 0.01$ and $u(t) = -15e + 5x + 6\dot{x} - 5x^2\dot{x}$.

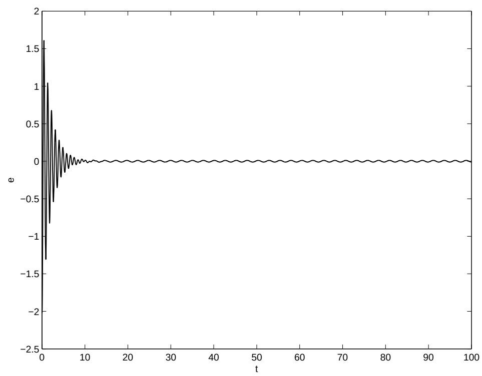

Fig. 6. Time history of the error system (19) with $\omega_1 = \omega_0 = 2.465$, $F_1 = F_0 = 5$, $h = 0.01$, and $u(t) = -45e + 5x + 6\dot{x} - 5x^2\dot{x}$.

shown in Figure 4 with the same initial conditions as in Figure 3. Here, we note that $k_2 = 65 < 1014$, which means synchronization in the case of $\omega_1 = \omega_0$ can achieve more easily than that of $\omega_1 \neq \omega_0$ under the same error bound $h = 0.01$.

Especially, if $\omega_1 = \omega_0 = 2.465$, $\varphi_1 = \varphi_0 = 0$, $h = 0.01$ and the other parameter values are the same as in Figure 3, then $M = |F_1 - F_0| = 0.2$, $k_1 > -1$, $k_2 > 14$. For $k_1 = 0$, $k_2 = 15$, $u(t) = -15e + 5x + 6\dot{x} - 5x^2\dot{x}$ can be obtained from Eq. (20). Achievement of synchronization scheme (17) − (18) with error bound $h = 0.01$ is shown in Figure 5 with the same initial conditions as in Figure 3.

If $\omega_1 = \omega_0 = 2.465$, $F_1 = F_0 = 5$, $h = 0.01$, and the other parameter values are the same as in Figure 3, then $M = F_0|\varphi_1 - \varphi_0| = 0.5$, $k_1 > -1$, $k_2 > 44$. For $k_1 = 0$, $k_2 = 45$, $u(t) = -45e + 5x + 6\dot{x} - 5x^2\dot{x}$ can be obtained from Eq. (16). The master-slave scheme (17) − (18) achieves synchronization with error bound $h = 0.01$ as shown in Figure 6 with the same initial conditions as in Figure 3.

In Example 1, if we put the controller to the periodic system, then the periodic

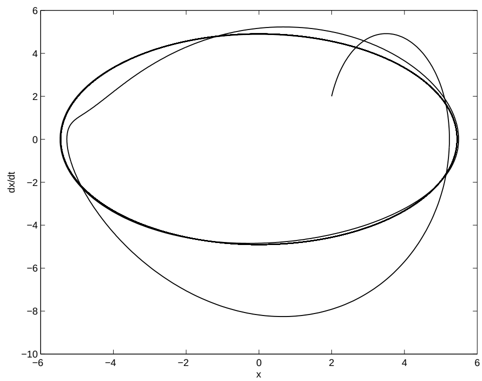

Fig. 7. For $c = 1$, $d = 6.0$, $F_0 = 28.7$, $\omega_0 = 0.9$ and $\varphi = 0.1$, the master system (22) has a stable limit cycle.

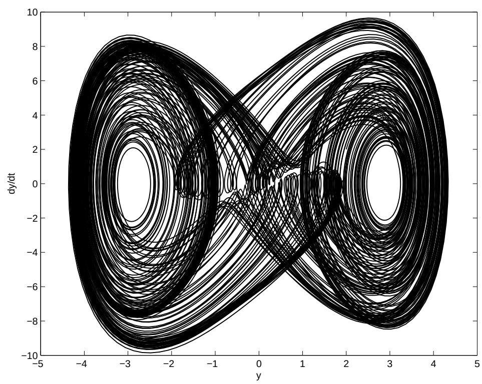

Fig. 8. For $\lambda = 0.2$, $\alpha = 1.0$, $\gamma = 1.0$, $F_1 = 28.5$, $\omega_1 = 0.86$, $\varphi_1 = 0$ and $u(t) = 0$, the forced Duffing oscillator (23) exhibits a chaotic behavior with initial conditions $y(0) = 0$, $\dot{y}(0) = 0$.

system can be controlled to be a chaotic system. In the same way, when we put the controller to the chaotic system, the chaotic system can also be controlled to be a periodic system, as shown in the following example.

Example 2. Consider the following master-slave synchronization scheme:

$$\text{Master system:} \quad \ddot{x} + c\dot{x} + dx = F_0 \cos(\omega_0 t + \varphi_0), \tag{22}$$

$$\text{Slave system:} \quad \ddot{y} + \lambda\dot{y} + \alpha y + \gamma y^3 = F_1 \cos(\omega_1 t + \varphi_1) + u(t). \tag{23}$$

For $c = 1$, $d = 6.0$, $F_0 = 28.7$, $\omega_0 = 0.9$ and $\varphi = 0.1$, the master system has a stable limit cycle, which is shown in Figure 7. For $\lambda = 0.2$, $\alpha = 1.0$, $\gamma = 1.0$, $F_1 = 28.5$, $\omega_1 = 0.86$, $\varphi_1 = 0$ and $u(t) = 0$, the forced Duffing oscillator exhibits a chaotic behavior, as shown in Figure 8 with initial conditions $y(0) = 0$, $\dot{y}(0) = 0$.

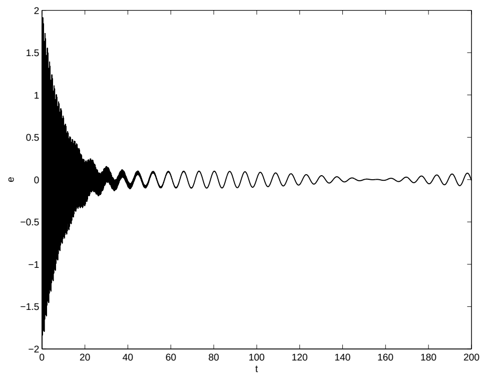

Fig. 9. Time history of the error system (24) with $h = 0.1$ and $u(t) = -572e + e^3 - 5x - 0.8\dot{x} + x^3 + 3x^2e + 3xe^2$ when amplitude, frequency and phase are mismatched at the same time.

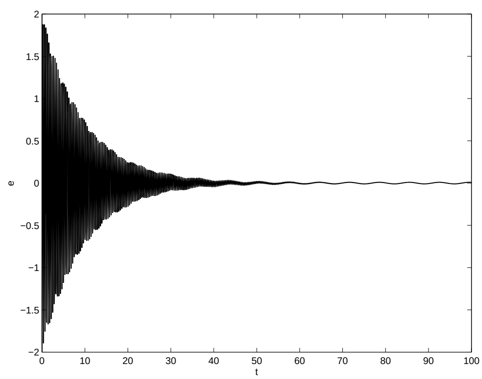

Fig. 10. Time history of the error system (24) with $\omega_1 = \omega_0 = 0.9$, $h = 0.01$, $u(t) = -307e + e^3 - 5x - 0.8\dot{x} + x^3 + 3x^2e + 3xe^2$.

The error system of synchronization scheme (22)–(23) is

$$\begin{aligned} &\ddot{e} + \lambda\dot{e} + \alpha e + \gamma e^3 + (\alpha - d)x + (\lambda - c)\dot{x} + \gamma x^3 + 3x^2\gamma e + 3x\gamma e^2 \\ &\quad = F_1\cos(\omega_1 t + \varphi_1) - F_0\cos(\omega_0 t + \varphi_0) + u(t). \end{aligned} \tag{24}$$

If we select the controller as

$$u(t) = -k_1\dot{e} - k_2 e + \gamma e^3 + (\alpha - d)x + (\lambda - c)\dot{x} + \gamma x^3 + 3x^2\gamma e + 3x\gamma e^2, \tag{25}$$

then the error system (24) is equivalent to the following equation:

$$\ddot{e} + (k_1 + \lambda)\dot{e} + (k_2 + \alpha)e = F_1\cos(\omega_1 t + \varphi_1) - F_0\cos(\omega_0 t + \varphi_0).$$

Similarly, if k_1 and $k2$ are chosen to satisfy

$$k_1 > -\lambda, \quad k_2 > \frac{M}{h} - \alpha, \tag{26}$$

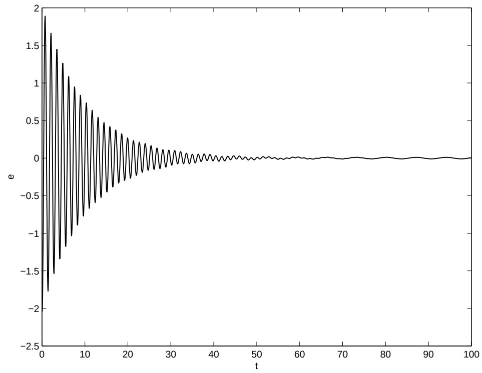

Fig. 11. Time history of the error system (24) with $\omega_1 = \omega_0 = 0.9$, $\varphi_1 = \varphi_0 = 0.1$, $h = 0.01$ and $u(t) = -20e + \dot{e}^3 - 5x - 0.8\dot{x} + x^3 + 3x^2\dot{e} + 3xe^2$.

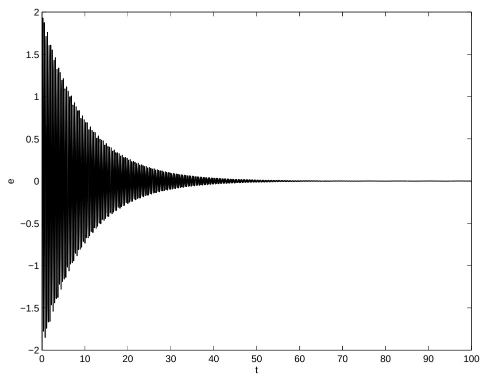

Fig. 12. Time history of the error system (24) with $\omega_1 = \omega_0 = 0.9$, $F_1 = F_0 = 28.7$, $h = 0.01$, and $u(t) = -287e + e^3 - 5x - 0.8\dot{x} + x^3 + 3x^2e + 3xe^2$.

then the master-slave scheme (22) – (23) achieves synchronization with error bound h.

If the parameter values of the master-slave synchronization scheme (22) – (23) are $c = 1$, $d = 6.0$, $F_0 = 28.7$, $\omega_0 = 0.9$ and $\varphi = 0.1$, $\lambda = 0.2$, $\alpha = 1.0$, $\gamma = 1.0$, $F_1 = 28.5$, $\omega_1 = 0.86$, $\varphi_1 = 0$ and for a prescribed error bound $h = 0.1$, then $M = 57.2$, $k_1 > -0.2$, $k_2 > 571$. For $k_1 = 0$, $k_2 = 572$, $u(t) = -572e + e^3 - 5x - 0.8\dot{x}+x^3+3x^2e+3xe^2$ can be got from Eq. (25). The master-slave scheme (22) – (23) can achieve synchronization with error bound $h = 0.1$ as shown in Figure 9 with initial conditions $x(0) = 2$, $\dot{x}(0) = 2$ and $y(0) = 0$, $\dot{y}(0) = 0$, respectively. For a smaller error bound $h = 0.01$, we can get $k_1 > -0.2$, $k_2 > 5719$ from inequalities (26), where the other parameter values are the same as in Figure 9. It can also be seen that when amplitude, frequency and phase are mismatched at the same time, synchronization of two non-identical forced oscillators with error bound needs a larger energy input.

If $\omega_1 = \omega_0 = 0.9$, $h = 0.01$ and the other parameter values are the same as in

Figure 9, then $M = |F_1 - F_0| + F_0|\varphi_1 - \varphi_0| = 3.07$, $k_1 > -0.2$, $k_2 > 306$. For $k_1 = 0$, $k_2 = 307$, $u(t) = -307e + e^3 - 5x - 0.8\dot{x} - 5x - 0.8\dot{x} + x^3 + 3x^2e + 3xe^2$ can be obtained from Eq. (25). Achievement of synchronization scheme (22)–(23) with error bound $h = 0.01$ is shown in Figure 10 with the same initial conditions as in Figure 9. From $k_2 = 307 < 5719$, it can also be seen that the scheme with $\omega_1 = \omega_0$ can achieve synchronization with error bound $h = 0.01$ more easily than that of $\omega_1 \neq \omega_0$.

Especially, if $\omega_1 = \omega_0 = 0.9$, $\varphi_1 = \varphi_0 = 0.1$, $h = 0.01$ and the other parameter values are the same as in Figure 9, then $M = |F_1 - F_0| = 0.2$, $k_1 > -0.2$, $k_2 > 19$. For $k_1 = 0$, $k_2 = 20$, $u(t) = -20e + \dot{e}^3 - 5x - 0.8\dot{x} + x^3 + 3x^2\dot{e} + 3xe^2$ can be obtained from Eq. (25). The master-slave scheme (22)–(23) achieves synchronization with error bound $h = 0.01$ as shown in Figure 11 with the same initial conditions as in Figure 9.

If $\omega_1 = \omega_0 = 0.9$, $F_1 = F_0 = 28.7$, $h = 0.01$, and the other parameter values are the same as in Figure 9, then $M = F_0|\varphi_1 - \varphi_0| = 2.87$, $k_1 > -0.2$, $k_2 > 286$. For $k_1 = 0$, $k_2 = 287$, $u(t) = -287e + e^3 - 5x - 0.8\dot{x} + x^3 + 3x^2e + 3xe^2$ can be obtained from Eq. (25). The master-slave scheme (22)–(23) achieves synchronization with error bound $h = 0.01$ as shown in Figure 12 with the same initial conditions as in Figure 9.

4. CONCLUSIONS

We have presented a synchronization scheme of two non-identical forced oscillators by differential inequality technique and active control technique. Numerical simulations have shown that this method can control a chaotic system to be a periodic system or a periodic system to be a chaotic system. The rest work can be done to reduce the complexity of the controller, for instance through the combination of the differential inequality technique developed in this paper with other control techniques rather than the active control technique.

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