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Book Reviews

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BOOK REVIEWS

HANDBOOK OF MEASURE THEORY, VOL. I, II.
Edited by E. Pap.
Elsevier Science B.V., Dordrecht 2002, xi + 786 pp. (I), xi + 789–1607 pp. (II).
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The book is available from the Amsterdam address,
Elsevier Science B.V.,
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1000 AE Amsterdam,
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Elsevier Science Inc.,
P.O. Box 945,
Madison Square Station,
New York, NY 10159-0945,
USA.

The Handbook is a collection of the work of 43 contributors, outstanding specialists in different branches of measure theory, whose names, as well as the editor name, guarantee a high quality of the content.

The main goal of this Handbook is "to survey measure theory with its many different branches and its relations with other areas of mathematics". Besides of aggregating many classical branches of measure theory, this Handbook covers new fields and approaches, and shows many applications. Reach sources of new results are traditional conferences in Germany and Italy, and the working group GEM (Generalized Measures), some parts of the book are related to the results obtained there.

The content of the book yields many close relations to other branches of mathematics, e.g. real analysis, probability theory, statistics, ergodic theory, geometry, differential equations, optimization, variational analysis, decision making, etc..

Some of the chapters give an overview of the related topics with precise definitions and statements of the theorems without proofs. This makes of them an outstanding introduction to the problematic. For the proofs and further information, an interested reader is referred to a huge list of related papers and monographs.

The Handbook consists of nine parts published in two volumes.

Volume I
(consists of Parts 1–4)

Part 1: Classical Measure Theory, contains Chapters 1–7.

Chapter 1, History of measure theory, starts with a short outlook of the history of measure theory, which arose from practical needs to measure lengths, areas and volumes; the non-commensurability of some of them lead to the concept of real numbers (written by D. Paunić).

Then, to make it easier to follow the whole book, in *Chapter 2*, Some elements of the classical measure theory, some basic notions and results from the classical measure theory are given in a very condensed form (E. Pap).

In *Chapter 3*, Paradoxes in measure theory, a review of known paradoxes is given, including Hausdorff, von Neumann and Banach-Tarski paradox and some more recent developments are presented (M. Lackowich).

Chapter 4, Convergence theorems for set functions, starts with the Nikodým boundedness and convergence theorems and related results, as Brooks-Jewett, Vitali-Hahn-Saks, Orlicz-Pettis, Hewitt-Yosida theorem. Extensions of the classical results to more general set functions and measures on special kinds of partially ordered structures are presented (P. de Lucia and E. Pap).

Chapter 5, Differentiation, brings some aspects of the theory of differentiation, focusing mainly on the interaction between differentiation properties and covering properties. Examples of different derivation bases in $\mathbb{R}$ and $\mathbb{R}^n$ are described (B. S. Thomson).

In *Chapter 6*, Radon-Nikodým theorems, versions of Radon-Nikodým theorems for σ-additive, finitely additive, Banach-valued σ-additive and Banach-valued finitely additive measures and some further results are given (D. Candeloro and A. Volčič).

In *Chapter 7*, One-dimensional diffusions and their convergence in distribution, normalized Brownian motion and its properties are described. Then more general diffusion processes are examined. Some convergence theorems for diffusions are proved. Diffusions as limits of stretched Brownian motions and stretched random walks are considered (J. K. Brooks).

Part 2: Vector Measures, contains Chapters 8–10.

In *Chapter 8*, Vector integration in Banach spaces and application to stochastic integration, three stages of integrability are considered: the classical integrability, the Bochner integrability and integrability with respect to a vector measure with finite semivariation. The most important application is the stochastic integral in Banach spaces with respect to summable processes (N. Dinculeanu).

In *Chapter 9*, The Riesz theorem, the Riesz theorem and its applications are studied. The Riesz theorem together with Hahn-Banach theorem is shown to be a very powerful tool in all abstract analysis producing the proofs of such important results as e.g. Choquet's representation theorem and the Stone-Wierestrass theorem. Besides of the classical Riesz theorem, the Riesz theorem for operators and for vector valued continuous function spaces is presented (J. Diestel and J. Swart).

In *Chapter 10*, Stochastic processes and stochastic integration in Banach spaces, the stochastic integral in Banach spaces is developed. Convergence theorems are applied to establish Itô's formula. Then regularity problems are treated. It is shown that regularity is equivalent to the Doob-Meyer decomposition of the process. Some necessary backgrounds on the quasimartingale theory are given in the Appendix (J. K. Brooks).

Part 3: Integration Theory, contains Chapters 11–14.

Chapter 11, Daniell integral and related topics, brings a description of the different formulations of the Daniell scheme of extending certain linear functionals on a vector space. Appropriate abstract forms of classical results, as the Radon-Nikodým theorem and Fubini theorem, are given and a way of further generalizations is noted (M. Díaz Carrillo).

In *Chapter 12*, Pettis integral, Pettis integral for Banach space valued functions is presented. Some generalizations of classical notions and results are given, as Radon-Nikodým property, conditional expectation, differentiation, Fubini theorem. Spaces of Pettis integrable functions are studied (K. Musial).

In *Chapter 13*, The Henstock-Kurzweil integral, the Henstock-Kurzweil integral as a generalization of the Riemann integral and its relation to the problem of primitives is considered. The case of the Henstock-Kurzweil integral on the real line, multidimensional Riemann-type integrals, Henstock-Kurzweil integral for vector valued functions and that on general spaces are studied (B. Bongiorno).

In *Chapter 14*, Set-valued integration and set-valued probability theory, main results from the theory of measurable multifunctions are presented with a closer look at the integration, conditional expectation and convergence theorems as strong laws of large numbers and martingale convergence theorems (C. Hess).

Part 4: Topological Aspects of Measure Theory, contains Chapters 15–18.

The purpose of *Chapter 15*, Density topologies, is to present basic properties of the density topology with the emphasis on the one-dimensional case. A brief presentation of density topologies on the plane is given. Then so-called Ψ-density topologies on the real line are studied. A local property of measurable sets discovered by Lusin is presented (W. Wilczyński).

In *Chapter 16*, FN-topologies and group-valued measures, basic result about the Fréchet-Nikodým topologies (FN-topologies) on Boolean rings and the applications in theory of group- or vector-valued measures are given. Exhaustivity, order continuity and extensions of FN-topologies and measures are treated. Vitali-Hahn-Saks and Nikodým theorems are proved (H. Weber).

In *Chapter 17*, On products of topological measure spaces, the main problem is whether the product of a family of topological measure spaces is a topological measure space. In particular, the Haar measure on a compact group is studied. Then liftings for a product measure are discussed (S. Grekas).

Chapter 18, Perfect measures and related topics, the classical results are summarized and the progress involving perfect measures since the late seventies is outlined (D. Ramachandran).

Volume II

(consists of Parts 5–9)

Part 5: Order and Measure Theory, contains Chapters 19–23.

In *Chapter 19*, Riesz spaces and ideals of measurable functions, measurable functions, their ideals, various kinds of convergence, Dedekind completeness and supports, and duals of ideals of measurable functions are studied (M. Väth).

Chapter 20, Measures on quantum structures, is devoted to measures on different types of algebraic structures, which arisen in mathematical foundations of quantum mechanics. Generalizations of Gleason's theorem and some measure-theoretic completeness criteria of inner product spaces are given. States on commutative BCK-algebras and on pseudo-MV-algebas (i.e. on non-commutative extensions of MV-algebras) are studied (A. Dvurečenskij).

Chapter 21, Probability on MV-algebras, starts with the backgrounds of MV-algebras. Representations of MV-algebras by intervals in lattice ordered groups and the Loomis-Sikorski theorem for MV-algebras is presented. Some theorems of probability theory are extended to the case of MV-algebras, e.g. central limit theorem, laws of large numbers, some of them using an additional operation of product on MV-algebras, e.g. joint observables, conditional expecation, individual ergodic theorem (D. Mundici and B. Riečan).

Chapter 22, Measures on clans and on MV-algebras, extends the results of the preceding chapter. Uniform MV-algebras and measures on MV-algebras with values in a locally convex spaces are studied. The main topics of interest are representations of measures, decompositions of measures, extensions of measures and Vitali-Hahn Saks and Nikodým theorems (G. Barbieri and H. Weber).

Chapter 23, Triangular norm-based measures, presents the necessary preliminaries about triangular norms and fuzzy sets. Next, T-tribes are introduced and properties of T-measures and their integral representations and decompositions are studied. Liapunoff type theorems concerning the compactness and convexity of the range of T-measures are proved (D. Butnariu and E. P. Klement).

Part 6: Geometric Measure Theory, consists of Chapters 24 and 25.

Chapter 24, Geometric measure theory: Selected concepts, results and problems, brings parts of the structure theory for integral dimensional sets. Then working with general Radon measures on $\mathbb{R}^n$, the relation between their m-densities and m-rectifiability is investigated. Preiss's theorem which yields a positive solution of one of the central problems in geometric measure theory, is presented without proof. Some applications in the modern calculus of variations is given (M. Chlebík).

Chapter 25, Fractal measures, gives a brief account of Hausdorff measures and their variants and extensions as the natural tools for the study of fractals. The second part of the chapter considers measures with a fractal structure and their analysis (K. J. Falconer).

Part 7: Relation to Transformation and Duality, contains Chapters 26–30.

In *Chapter 26*, Positive and complex Radon measures in locally compact Hausdorff spaces, several notions of regularity for measures defined on certain δ-rings or σ-rings of subsets of a locally compact Hausdorff space and the existence and uniqueness of their regular extensions is studied. The results are used to characterizations of the complex Radon measures. Isomorphisms among various spaces of real and complex measures are proved. Some generalizations and applications of obtained results are given (T. V. Panchapagesan).

Chapter 27, Measures on algebriac-topological structures is devoted to countably additive measures invariant under groups of transformations. General necessary and sufficient conditions for the existence of finite and σ-finite invariant measures on arbitrary G-spaces are discussed. Then invariant measures defined for all subsets of a given set X are investigated and related to the problem of the existence of a σ-additive σ-finite measure on the power set of X vanishing on all singleton sets. Existence of Borel measures on Polish spaces is discussed and Borel actions of Polish groups are considered with some connections to descriptive dynamics. Extensions of invariant measures are treated and a survey of related results is given (P. Zakrzewski).

Chapter 28, Liftings, studies the existence of liftings and densities in dependence of the strict localizability of measure spaces. Then liftings for functions are studied and some constructions of them are given. Some conditions of the existence of liftings on topological measure spaces and on topological groups are discussed. Liftings on products and projective limits of probability spaces and various Fubini products with applications to stochastic processes are considered. Some results on liftings for abstract valued functions are given and some applications of lifting theory are mentioned (W. Strauss, N. D. Macheras and K. Musial).

Chapter 29, Ergodic theory, introduces a list of a number of the most important examples of measure-preserving systems. Then an overview of the principal convergence theorems of ergodic theory with particular emphasis on von Neumann's mean ergodic theorem and Birkhoff's pointwise ergodic theorem. Ergodicity of measure-preserving systems is considered. Poincaré's recurrence theorem is proved, and some related results are formulated. Several kinds of mixing are investigated. Some further ergodic theorems, e.g. the theorem by Campbell and Petersen are given. Information function and entropy are investigated and entropy of some special systems is given. Some constructions in ergodic theory are described (F. Blume).

Chapter 30, Generalized derivatives, deals with the theories on partial differential equations. Three types of generalized derivatives are exposed: that of Sobolev type, distributional one, and the one appearing in the Mikusiński operational calculus. Characteristic applications to partial differential equations are demonstrated (E. Pap and A. Takači).

Part 8: Relation to the Foundations of Mathematics, contains Chapters 31 and 32.

Chapter 31, Real valued measurability, some set-theoretical aspects, deals with the axioms of set theory and their relations to measure theory. The well known results by Ulam are formulated and a proof of Solovay's theorem on consistency is given. Connections of the existence of a total extension of Lebesgue measure and the existence of a measurable cardinal are discussed. Relations to cardinal monotony and the existence of nonregular ultrafilters are described. A discussion of the problem of metrizability of normal Moore spaces, known as Normal Moore Space Conjecture, and its solution is given (A. Jovanović).

Chapter 32, Nonstandard analysis and measure theory, deals with the extension of the real number system, Transfer Principle, and the infinitesimal calculus. Some aspects of classical measure theory are treated in the nonstandard approach, e.g., Poisson processes, Brownian motion, the martingale convergence theorem, the law of large numbers (P. A. Loeb).

Part 9: Non-Additive Measures, consists of Chapters 33–37.

Chapter 33, Monotone set function-based integrals, extends the notion of the Lebesgue integral based on a σ-additive measure, to integrals with respect to fuzzy measures, i.e. monotone set functions vanishing at the empty set (with some continuity properties in the infinite case). A class of general fuzzy integrals is introduced with respect to a general fuzzy measure. A fuzzy integral is associated to any couple of fitting operations $\oplus$ and $\odot$. As special cases, the Choquet and the Sugeno integrals are discussed and compared. An appropriate choice of the operations $\oplus$ and $\odot$ is discussed. Several examples of operations $\oplus$ and $\odot$ with the corresponding fuzzy integrals are presented (P. Benvenuti, R. Mesiar and D. Vivona).

Chapter 34, Set functions over finite sets: Transformations and integrals, starts with introducing some real-valued set functions: games, capacities or fuzzy measures, unanimity games and pseudo-boolean functions. The Choquet integral for positive integrands is introduced, and two extensions to real-valued integrands are considered: the so-called Choquet integral, and the Šipoš integral. The k-additive fuzzy measures are studied. The case of ordinal structures is considered, where instead of Choquet and Šipoš integrals, the Sugeno integral is to be used. It is shown how constructions similar to the cardinal case can be done in this context (M. Grabisch).

In *Chapter 35*, Pseudo-additive measures and their applications, some results on special non-additive measures — pseudo-additive (decomposable) measures and the corresponding integrals are presented. For the range of set functions instead of the field of real numbers a semiring on a real interval is taken. The so-called pseudo-analysis is developed in an analogous way as classical analysis, introducing $\oplus$-measure, pseudo-integral, pseudo-Laplace transform etc.. The advantage of the pseudo-analysis is that it covers, with unified methods, problems from many different fields: system theory, optimization, control theory, differential equations, difference equations etc.. Generalizations related to the case when the operations $\oplus$ and $\odot$ are non-commutative and non-associative, and when the distributivity law is relaxed are discussed (E. Pap).

Chapter 36, Qualitative possibility functions and integrals, gives an introduction to the comparative counterpart to set-functions. Families of confidence relations on finite sets are surveyed, such as comparative probabilities and possibility orderings. Such orderings play an important role in artificial intelligence, in theory revision and non-monotonic reasoning. The ordinal version of conditioning is introduced. Qualitative counterparts to probabilistic independence on finite ordinal scales are introduced and compared with the possibility theory. Qualitative counterparts to integrals are described, focusing on possibility integrals (D. Dubois and H. Prade).

The aim of *Chapter 37*, Measures of information, is to present results in a "mixed" theory of information, so-called inset theory, where the measures of information may depend on both the probabilities and the events. Thus the probabilistic and non-probabilistic concepts are combined. Branching inset information measures are introduced and recursivity and generalized additivity are considered. Subadditive information measures and their properties are studied. Belief structures and information measures in a theory of evidence are surveyed. Characterizations of measures of fuzziness, fuzzy entropies and weighted entropies are given (W. Sander).

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