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Book Reviews

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# BOOK REVIEWS

Hamhalter, J.:
QUANTUM MEASURE THEORY.
Fundamental Theories of Physics 134.
Kluwer Academic Publishers, Dordrecht 2003, viii + 410 pp.
ISBN 1-4020-1714-6/hbk

Theory of quantum structures was introduced in the beginning of thirties by Birkhoff and von Neumann as mathematical foundations of quantum physics. It was recognized that events connected with any measurement process of quantum observables do not fulfil the axioms of probability theory and statistics. During the last 70 years, many approaches to axiomatize quantum mechanics to were developed. Besides algebraic the most important base concerns with the realm of the Hilbert space.

Today there are couple of monographs on quantum structures: V. S. Vadaradarajan (1968), G. Kalmbach (1983, 1986), S. Gudder (1979, 1988), P. Pták and S. Pulmannová (1991), A. Dvurečenskij (1993), A. Dvurečenskij and S. Pulmannová (2000), and the present book enriches the literature on this topic.

The present monograph extends the latest and very important results concentrated around the measure theory on quantum structures. The book is focusing to the extension of Gleason's theorem for von Neumann algebras, and therefore, it is very close to direct applications to mathematical physics.

The monograph consists of 11 chapters, bibliography, and index.

**Chapter 1** presents shortly the book.

**Chapter 2** gives elements of $C^*$-algebras, von Neumann algebras, Jordan and ordered structures, and operator algebras.

The corner-stone of the book as well as of the theory under review is the Gleason theorem, studied in **Chapter 3** (A. M. Gleason, 1957). It says that every $\sigma$-additive state $\mu$ on the projection lattice $P(H)$ of a separable Hilbert space $H$ is in a one-to-one correspondence with a von Neumann operator $T$ such that $\mu(P) = \mathrm{tr}(TP)$, $P \in P(H)$. It is fascinative that the heart of the proof is lying in the three-dimensional Hilbert space $\mathbb{R}^3$. The original proof was very complicated, and only in the middle of eighties R. Cooke, M. Keane, and W. Moran (1985) gave a more elementary proof, and the book presents it. If we extend the notion of state to a charge, in every $P(H)$, $\dim H < \infty$, there is a unbounded charge. The result of S. Dorofeev and A. Sherstnev gave unexpected results that in infinite-dimensional Hilbert space, every $\sigma$-additive charge is bounded.

In 1987, J. Hamhalter and P. Pták proved an interesting results that a real separable inner product space $S$ is complete if and only if the system $F(S)$ of all closed subspace $M$ of $S$ such that $M^{\perp\perp} = M$ admits a $\sigma$-additive state. This was a first measure-theoretic completeness criterion. This important results initiated the study of completeness criteria in the last 15 years, see Dvurečenskij (1993).

**Chapter 4** is devoted to completeness criteria. These criteria often use algebraic criteria. The author in his book is concentrating mainly to state-criteria which were proved by him and also by other authors.

**Chapter 5** is devoted to the generalized Gleason theorem. It deals with answer to the problem posed by G. W. Mackey who asked: "Does any bounded finitely additive measure on the projection lattice $P(M)$ of a von Neumann algebra $M$ extend to a bounded linear functional on $M$?" This problem was solved for many years. In this chapter, the author presents the Generalized Gleason Theorem which says that every bounded Banach-valued measure on $P(M)$ of a von Neumann algebra $M$ without direct summand of type $I_2$ extends uniquely to a bounded linear operator $T$ from $M$ to the Banach space. This is a result of L. J. Bunce and J. D. M. Wright (1994).

**Chapter 6** is studying basic principles of quantum measure theory. It shows when a measure on $P(H)$ is bounded, as well it gives some generalizations of known result of the classical measure theory like Vitali-Hahn-Saks Theorem, Egorov theorem, Yosida-Hewitt decomposition, Lyapunov theorem.

**Chapter 7** gives some applications of Gleason's theorem. The first deals with the Gleason theorem for multimeasures (decoherence functionals, history approach), dynamical aspects, and problem of hidden variables.

**Chapter 8** presents some problems of orthomorphisms of projections. This is a generalized version of the Wigner theorem. In particular, it describes the structure of completely additive orthomorphisms between Hilbert space projection lattices and it gives the well-know Wigner Unitary-Antiunitary Theorem as a byproduct.

**Chapter 9** describes the restriction and extension properties of states on $C^*$-algebras. The author derives some results on determinacy of the system of orthogonal pure states by biorthogonal systems of elements in the $C^*$-algebra. Then he is concentrated to the extension of measures.

**Chapter 10** is concentrated on Jauch-Piron states. Such states have the kernel which is closed under forming finite supremas of projections. The aim is to show how topological and algebraical aspects of the projection lattice are closely related, and it gives a new look to axioms in the operator-algebraic approach to quantum mechanics.

The last **Chapter 11** is focused to problems of independence of quantum systems like von Neumann and $C^*$-algebras. The independence is a well-known notion in classical probability theory, and in this chapter some applications like $C^*$-independence and $W^*$-independence are studied.

Finally, References contain 334 items.

The monograph under review collects many important and highly nontrivial results and efforts of many authors. It is important to note that the material is based also on the research done by J. Hamhalter, and the author belongs to influencing researchers in this field. The style is very fresh, the author is keeping the regard of the reader permanently on his trip trough the book, and I recommend to book for students and experts interested in operator algebra, noncommutative measure theory and mathematical foundations o quantum physics. The monograph is welcome in the quantum structures realm.

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