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SOME PROPERTIES OF RELATIVELY STRONG PSEUDOCOMPACTNESS

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Abstract. In this paper, we study some properties of relatively strong pseudocompactness and mainly show that if a Tychonoff space X and a subspace Y satisfy that $Y \subset \overline{\operatorname{Int} Y}$ and Y is strongly pseudocompact and metacompact in X, then Y is compact in X. We also give an example to demonstrate that the condition $Y \subset \overline{\operatorname{Int} Y}$ can not be omitted.

Keywords: relative topological properties, pseudocompact spaces, compact space $MSC\ 2010:\ 54\text{D}20,\ 54\text{D}30$

1. INTRODUCTION

A. V. Arhangel'skii and H. M. M. Genedi [3] introduced the theory of relative topological properties in 1989. Many results on "absolute" topological properties can be interpreted as theorems on relative topological properties, which is a guideline of studying relative topology. In [2], it was shown that if Y is strongly pseudocompact and paracompact in X, then Y is compact in X. We know that pseudocompact metacompact spaces are compact [6], so it is natural to consider the following question:

Question. Let X be a Tychonoff space and Y a subspace of X such that Y is strongly pseudocompact and metacompact (strongly metacompact) in X. Is then Y compact in X?

In this paper, we provide an example to answer negatively the above question and also obtain the following theorem:

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Theorem. Let Y be a subspace of a Tychonoff space X satisfying $Y \subset \overline{\operatorname{Int} Y}$, where $\overline{\operatorname{Int} Y}$ denotes the closure of $\operatorname{Int} Y$ in X. If Y is metacompact and strongly pseudocompact in X, then Y is compact in X.

Throughout this paper, we assume that all spaces are T_1 . Unless otherwise stated, when we say that a set U is open (closed), we mean it is open (closed) in X even if U is a subset of a subspace Y of X, \overline{U} denotes the closure of U in X and Int U denotes the interior of U in X. For any set $A \subset X$ and collection \mathscr{U} of subsets of X, $\operatorname{St}(A, \mathscr{U})$ denotes the set $\bigcup \{U \in \mathscr{U} : U \cap A \neq \emptyset\}$. A collection \mathscr{U} of subsets of Xis said to be *point finite* (locally finite) on a subset A [5] of X if for each $x \in A$ the collection $\{U \in \mathscr{U} : x \in U\}$ is finite (there is a neighborhood V of x in X such that $\{U \in \mathscr{U} : U \cap V \neq \emptyset\}$ is finite). Let \mathscr{U} and \mathscr{V} be be two collections of subsets of X. \mathscr{V} is said to be a partial refinement of \mathscr{U} if for each $V \in \mathscr{V}$ there is a $U \in \mathscr{U}$ such that $V \subset U$. If in the above definition \mathscr{V} and \mathscr{U} are two covers of X, we say that \mathscr{V} is a refinement [4] of \mathscr{U} .

Other undefined notions and terminologies are as in [4].

2. Main results

Let Y be a subspace of a space X. Y is said to be strongly pseudocompact in X [2] (see also [1]) if every family $\mathscr{U} = \{U_{\alpha}: \alpha \in \Lambda\}$ of open subsets of X which is locally finite on Y and satisfies $U_{\alpha} \cap Y \neq \emptyset$ for every $\alpha \in \Lambda$, is finite. The following theorem characterizes strong pseudocompactness of Y in X in terms of collections similar to collections with the finite intersection property.

Theorem 2.1. Let Y be a subspace of X. Then the following conditions are equivalent:

- (1) Y is strongly pseudocompact in X.
- (2) For every decreasing sequence $W_1 \supset W_2 \supset \ldots$ of open subsets of X which satisfies $W_i \cap Y \neq \emptyset$ for $i = 1, 2, \ldots, \left(\bigcap_{i=1}^{\infty} \overline{W_i}\right) \cap Y \neq \emptyset$.
- (3) If $\{V_i\}_{i=1}^{\infty}$ is a countable collection of open subsets of X such that $V_{i_1} \cap V_{i_2} \cap \ldots \cap V_{i_k} \cap Y \neq \emptyset$ for every finite set $\{i_1, i_2, \ldots, i_k\} \subset \{1, 2, \ldots\}$, then $\left(\bigcap_{i=1}^{\infty} \overline{V_i}\right) \cap Y \neq \emptyset$.

Proof. First we will show that $(1) \Rightarrow (2)$. Suppose that $\{W_i\}_{i=1}^{\infty}$ is a decreasing sequence of open subsets of X such that $W_i \cap Y \neq \emptyset$ for i = 1, 2, ... By the definition of strong pseudocompactness of Y in X, $\{W_i\}_{i=1}^{\infty}$ is not locally finite at some point y_0 of Y. So every neighborhood U of y_0 in X meets infinitely many W_i 's, hence $y_0 \in \bigcap_{i=1}^{\infty} \overline{W_i}$.

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To prove that $(2) \Rightarrow (3)$ it suffices to consider the decreasing sequence $V_1, V_1 \cap V_2, \ldots$ It is easy to see that the sequence satisfies condition (2), so $\left(\bigcap_{i=1}^{\infty} \overline{V_i}\right) \cap Y \neq \emptyset$.

Finally, we shall show that $(3) \Rightarrow (1)$. Suppose that $\{U_i\}_{i=1}^{\infty}$ is a collection of open subsets of X locally finite on Y and such that $U_i \cap Y \neq \emptyset$ for each *i*. Let $V_k = \bigcup_{i=k}^{\infty} U_i$. Then the collection $\{V_k\}_{k=1}^{\infty}$ satisfies condition (3), which implies that $\left(\bigcap_{k=1}^{\infty} \overline{V_k}\right) \cap Y \neq \emptyset$. So there is a $y_0 \in Y$ such that $y_0 \in \overline{V_k}$ for $k = 1, 2, \ldots$. Since $\{U_i\}_{i=1}^{\infty}$ is locally finite on Y, there exists a neighborhood U_{y_0} of y_0 in X and a finite subcollection $\{U_k\}_{k=1}^{\infty}$ other than $U_{i_1}, U_{i_2}, \ldots, U_{i_m}\}$ of $\{U_k\}_{k=1}^{\infty}$ such that U_{y_0} does not intersect any member of $\{U_k\}_{k=1}^{\infty}$ other than $U_{i_1}, U_{i_2}, \ldots, U_{i_m}$. Put $k_0 = \max\{i_1, i_2, \ldots, i_m\}$. Then $U_{y_0} \cap V_{k_0+1} = \emptyset$ and so $y_0 \notin \overline{V_{k_0+1}}$, a contradiction. Therefore, Y is strongly pseudocompact in X.

Corollary 2.1. Suppose that Y is a subspace of a Tychonoff space X such that Y is strongly pseudocompact in X. If $\{D_n\}_{n=1}^{\infty}$ is a sequence of open subsets of X such that $\overline{D_n \cap Y} \supset Y$ for n = 1, 2, ..., then $\bigcap_{n=1}^{\infty} (D_n \cap Y) \supset Y$. In particular, if $\overline{D_n \cap Y} = Y$ for n = 1, 2, ..., then $\bigcap_{n=1}^{\infty} (D_n \cap Y) = Y$.

Proof. Let $\{D_n\}_{n=1}^{\infty}$ be a sequence of open subsets of X such that $\overline{D_n \cap Y} \supset Y$ for n = 1, 2, ... Then for each $y \in Y$, if U is a neighborhood of y in X, we have $U \cap D_n \cap Y \neq \emptyset$ for n = 1, 2, ... Let $V_1 = U$. So there exists a $y_1 \in Y$ such that $y_1 \in U \cap D_1 \cap V_1$. By the regularity of X, there is an open subset V_2 such that $y_1 \in V_2 \subset \overline{V_2} \subset U \cap D_1 \cap V_1$. Clearly $U \cap V_2$ is a neighborhood of y_1 , so $U \cap V_2 \cap D_2 \cap Y \neq \emptyset$. Similarly there is a $y_2 \in Y$ and an open subset V_3 such that $y_2 \in V_3 \subset \overline{V_3} \subset U \cap D_2 \cap V_2$. Continuing the process, we can obtain a sequence $\{V_n\}_{n=1}^{\infty}$ of open subsets of X such that $V_1 \supset V_2 \supset \ldots$ and $V_n \cap Y \neq \emptyset$ for $n = 1, 2, \ldots$ By Theorem 2.1, $\left(\bigcap_{n=1}^{\infty} \overline{V_n}\right) \cap Y \neq \emptyset$ and so $U \cap \left(\bigcap_{n=1}^{\infty} D_n\right) \cap Y \neq \emptyset$. Since U is any neighborhood of y in $X, y \in \left(\bigcap_{n=1}^{\infty} D_n\right) \cap Y = \bigcap_{n=1}^{\infty} (D_n \cap Y)$. Hence $\overline{\bigcap_{n=1}^{\infty} (D_n \cap Y)} \supset Y$. If $\overline{D_n \cap Y} = Y$, it is easy to prove that $\overline{\bigcap_{n=1}^{\infty} (D_n \cap Y)} = Y$. \Box

Let Y be a subspace of a space X. Y is said to be *compact in* X [3] (see also [1]), if for every open cover of X there is a finite subfamily \mathscr{H} such that $Y \subset \bigcup \mathscr{H}$. Y is said to be *metacompact in* X [5] provided every open cover \mathscr{U} of X has an open partial refinement \mathscr{V} point finite on Y. If in the above definition \mathscr{V} covers X, then we say that Y is strongly metacompact in X [5]. The following theorem characterizes the relative version of the result that a pseudocompact metacompact space is compact. **Theorem 2.2.** Let Y be a subspace of a Tychonoff space X satisfying $Y \subset \overline{\operatorname{Int} Y}$. If Y is metacompact and strongly pseudocompact in X, then Y is compact in X.

To prove Theorem 2.2, we need the following two lemmas.

Lemma 2.1. Let Y be a subspace of a Tychonoff space X and let G be an open subset of X such that $G \cap Y \neq \emptyset$. If Y is strongly pseudocompact in X, then for every sequence $\{F_n\}_{n=1}^{\infty}$ of open subsets of X such that $\overline{F_n \cap G \cap Y} \supset G \cap Y$ for n = 1, 2, ..., we have $\bigcap_{n=1}^{\infty} (F_n \cap G \cap Y) \supset G \cap Y$. In particular, if $G \subset Y$ and $\overline{F_n \cap G} \supset G$ for n = 1, 2, ..., then $\overline{\bigcap_{n=1}^{\infty} (F_n \cap G)} \supset G$.

Proof. Let G be an open subset of X such that $G \cap Y \neq \emptyset$ and let $\{F_n\}_{n=1}^{\infty}$ be a sequence of open subsets of X such that $\overline{F_n \cap G \cap Y} \supset G \cap Y$ for n = 1, 2, ...Then $\{(G \cap F_n) \cup (X - \overline{G \cap Y})\}_{n=1}^{\infty}$ is a collection of open subsets of X. It is easy to show that $\overline{[(G \cap F_n) \cup (X - \overline{G \cap Y})] \cap Y} \supset Y$ for n = 1, 2, ... By Corollary 2.1, we have $\left[\bigcap_{n=1}^{\infty} (G \cap F_n \cap Y)\right] \cup [(X - \overline{G \cap Y}) \cap Y] \supset Y$.

Pick $y \in \widehat{G} \cap Y$, and let U be a neighborhood of y in X. Without loss of generality, we may assume that $U \subset G$, then $\left(U \cap \left[\bigcap_{n=1}^{\infty} (G \cap F_n \cap Y)\right]\right) \cup [U \cap Y \cap (X - \overline{G \cap Y})] \neq \emptyset$, i.e., $U \cap \left[\bigcap_{n=1}^{\infty} (G \cap F_n \cap Y)\right] \neq \emptyset$, so $y \in \overline{\bigcap_{n=1}^{\infty} (G \cap F_n \cap Y)}$. It follows that $\overline{\bigcap_{n=1}^{\infty} (F_n \cap G \cap Y)} \supset G \cap Y$.

Lemma 2.2. Let Y be a subspace of a Tychonoff space X satisfying $Y \subset \overline{\operatorname{Int} Y}$ and let \mathscr{U} be a collection of open subsets of X covering Y and point finite on Y. If Y is strongly pseudocompact in X, then there exists a subset A of Y such that $\overline{A} \supset Y$ and \mathscr{U} is locally finite on A.

Proof. Let \mathscr{U} be a collection of open subsets of X such that \mathscr{U} is point finite on Y and $Y \subset \bigcup \mathscr{U}$. Put $A = \{x \in Y : \mathscr{U} \text{ is locally finite at } x\}$. We shall show that $\overline{A} \supset Y$. Pick $y \in Y$, and let V be an open neighborhood of y in X. It suffices to prove that $V \cap A \neq \emptyset$. Let $X_n = \{x \in Y : x \text{ is in at most } n \text{ elements of } \mathscr{U}\}$. Then $\overline{X_n} \cap (Y - X_n) = \emptyset$ for each n. In fact, for each $z \in Y - X_n$, z is in at least n+1 elements of \mathscr{U} . Without loss of generality, we may assume that $U_1, U_2, \ldots, U_{n+1}$ are distinct elements of \mathscr{U} such that $z \in \bigcap_{i=1}^{n+1} U_i$. Then $\bigcap_{i=1}^{n+1} U_i$ is a neighborhood of zwhich does not meet X_n , so $z \notin \overline{X_n}$. It follows that $\overline{X_n} \cap (Y - X_n) = \emptyset$. Hence $Y \cap \overline{X_n} = [(Y - X_n) \cup X_n] \cap \overline{X_n} = X_n$. For $n = 1, 2, \ldots$, let $E_n = V \cap X_n$. **Claim.** Int $(E_n) \neq \emptyset$ for some n.

Suppose that for each n, $\operatorname{Int}(E_n) = \emptyset$. Let $F_n = V - \overline{X_n}$. Then F_n is an open subset of X and $\overline{F_n \cap V \cap \operatorname{Int} Y} \supset V \cap \operatorname{Int} Y$ for $n = 1, 2, \ldots$. In fact, by $Y \cap \overline{X_n} = X_n, \overline{F_n \cap V \cap \operatorname{Int} Y} \supset \overline{(V \cap \operatorname{Int} Y) - E_n}$. Since $\operatorname{Int}(E_n) = \emptyset$ for each n, $\overline{(V \cap \operatorname{Int} Y) - E_n} \supset V \cap \operatorname{Int} Y$ for each n. Using the condition $Y \subset \operatorname{Int} Y$ and Lemma 2.1, we have

$$\left(\overline{\bigcap_{n=1}^{\infty} F_n}\right) \cap V \cap \operatorname{Int} Y = \overline{\bigcap_{n=1}^{\infty} (F_n \cap V \cap \operatorname{Int} Y)} \supset V \cap \operatorname{Int} Y \neq \emptyset.$$

But $\bigcap_{n=1}^{\infty} F_n = V - \bigcup_{n=1}^{\infty} \overline{X_n} \subset V - Y$ and so $\left(\overline{\bigcap_{n=1}^{\infty} F_n} \right) \cap V \cap \operatorname{Int} Y = \emptyset$, a contradiction.

Let k be the least element such that $\operatorname{Int}(E_k) \neq \emptyset$. Then for each $x \in \operatorname{Int}(E_k) = V \cap \operatorname{Int}(X_k)$, $x \notin \operatorname{Int}(E_{k-1}) = V \cap \operatorname{Int}(X_{k-1})$ and so there exists a neighborhood U of x such that $U \subset E_k$ and $U \cap (X \setminus X_{k-1}) \neq \emptyset$. Pick $z \in U \cap (X \setminus X_{k-1})$, then $z \in \operatorname{Int}(E_k)$ and z is in k elements of \mathscr{U} . Assume that U_1, U_2, \ldots, U_k are distinct elements of \mathscr{U} such that $z \in \bigcap_{i=1}^k U_i$. Let $W = \operatorname{Int}(E_k) \cap \bigcap_{i=1}^k U_i$. By the definition of E_k , W can not intersect any element of \mathscr{U} other than U_1, U_2, \ldots, U_k , hence $z \in A$. Since $E_n = V \cap X_n$ for each $n, z \in \operatorname{Int}(E_k) \subset V$. It follows that $z \in V \cap A$. This concludes the proof.

Proof of Theorem 2.2. Let \mathscr{U} be an open cover of X. By the regularity of Xand the metacompactness of Y in X, there is a collection \mathscr{W} of open subsets of Xcovering Y such that \mathscr{W} is point finite on Y and $\{\overline{W}: W \in \mathscr{W}\}$ refines \mathscr{U} . By Lemma 2.2, there is a subset A of Y such that \mathscr{W} is locally finite on A and $\overline{A} \supset Y$. For each $y \in Y$, fix an open subset U_y of X such that U_y meets only finitely many members of \mathscr{W} . Let $U = \bigcup_{y \in Y} U_y$ and let V_1 be a nonempty open subset of U which meets only finitely many members of \mathscr{W} and $V_1 \cap Y \neq \emptyset$. Inductively pick for each $i \in \mathbb{N}$, if possible, an open set $V_i \subset U - \bigcup_{k=1}^{i-1} \operatorname{St}(V_k, \mathscr{W})$ such that V_i meets only finitely many members of \mathscr{W} and $V_i \cap Y \neq \emptyset$.

Claim. The induction stops at some *i*.

Suppose that the induction proceeds infinitely, we can obtain an infinite sequence $\{V_i\}_{i=1}^{\infty}$ of nonempty open subsets of U such that V_i meets only finitely many members of \mathscr{W} and $V_i \cap Y \neq \emptyset$ for each i, also $V_{i+1} \subset U - \bigcup_{k=1}^{i} \operatorname{St}(V_k, \mathscr{W})$. Let $U_n = \bigcup_{k=n}^{\infty} V_k$. Then $\{U_n\}_{n=1}^{\infty}$ is a decreasing sequence of open subsets of X such that $U_n \cap Y \neq \emptyset$ for $n = 1, 2, \ldots$ By Theorem 2.1, $\left(\bigcap_{n=1}^{\infty} \overline{U_n}\right) \cap Y \neq \emptyset$ and so there exists a $y \in Y$ such that $y \in \bigcap_{n=1}^{\infty} \overline{U_n}$. Pick $W \in \mathscr{W}$ such that $y \in W$. Then for each natural number n, $W \cap U_n \neq \emptyset$, which implies there exist distinct natural numbers l, k such that k < l and $W \cap V_k \neq \emptyset$, $W \cap V_l \neq \emptyset$. So $\operatorname{St}(V_k, \mathscr{W}) \cap V_l \supset W \cap V_l \neq \emptyset$, which contradicts $V_l \subset U - \bigcup_{i=1}^{l-1} \operatorname{St}(V_i, \mathscr{W})$. Hence, the sequence $\{V_i\}_{i=1}^{\infty}$ is finite.

Let the above sequence be $\{V_1, V_2, \ldots, V_m\}$. We claim that $A \subset \bigcup_{i=1}^m \operatorname{St}(V_i, \mathscr{W})$. Otherwise, if there is a $y_0 \in A$ such that $y_0 \notin \bigcup_{i=1}^m \operatorname{St}(V_i, \mathscr{W})$, then there exists an open neighborhood V of y_0 such that $V \cap \bigcup_{i=1}^m \operatorname{St}(V_i, \mathscr{W}) = \emptyset$. Put $V_{m+1} = V \cap U$. It is easy to see that $V_{m+1} \subset U - \bigcup_{i=1}^m \operatorname{St}(V_i, \mathscr{W})$ and $V_{m+1} \cap Y \neq \emptyset$, which is a contradiction.

If $\mathscr{H} = \{H \in \mathscr{W} : H \cap V_i \neq \emptyset \text{ for some } i \leq m\}$, then \mathscr{H} is a finite subcollection of \mathscr{W} such that $Y \subset \overline{A} \subset \bigcup \mathscr{H} = \bigcup_{H \in \mathscr{H}} \overline{H}$. Since $\{\overline{W} : W \in \mathscr{W}\}$ refines \mathscr{U}, Y is compact in X.

The following example demonstrates that in Theorem 2.2 the condition $Y \subset \overline{\operatorname{Int} Y}$ is necessary.

Example 2.1. There exists a Tychonoff space X and its subspace Y such that Y is strongly pseudocompact and metacompact (strongly metacompact) in X, but Y is not compact in X.

Proof. Let $X = [0, \omega_1) \times [0, \omega_1) - \{(0, 0)\}$. For each $\alpha \in (0, \omega_1)$, let $H_\alpha = [0, \omega_1) \times \{\alpha\}$ and $G_\alpha = \{\alpha\} \times [0, \omega_1)$. Define a topology on X as follows: for $\alpha \in (0, \omega_1)$, a neighborhood of $(0, \alpha)$ contains $(0, \alpha)$ and all but finitely many points of H_α . The neighborhood of $(\alpha, 0)$ contains $(\alpha, 0)$ and all but finitely many points of G_α . All other points of X are isolated. Let $Y = [(0, \omega_1) \times \{0\}] \cup [\{0\} \times (0, \omega_1)]$ with the subspace topology of X.

Claim 1. X is a Tychonoff space.

Clearly X is T_1 and X has a base consisting of open-and-closed sets.

Claim 2. Y is metacompact in X.

In fact, X is a metacompact space because any open cover of X has a natural open refinement \mathscr{V} such that each point x of X is in at most 2 members of \mathscr{U} . Thus Y is metacompact (strongly metacompact) in X.

Claim 3. Y is not compact in X.

Since the sets $A = \{(0, \alpha): 0 < \alpha < \omega_1\}$ and $B = \{(\alpha, 0): 0 < \alpha < \omega\}$ are two disjoint closed sets of X which can not be separated in X, Y is not normal in X (Y is said to be *normal in* X [3] if for any two disjoint closed subsets A and B of X there exist two disjoint open subsets U and V in X such that $A \cap Y \subset U$ and $B \cap Y \subset V$). By Theorem 5.1 of [1] (see also [3]), Y is not compact in X.

Claim 4. Y is strongly pseudocompact in X.

Let $\{V_i\}_{i=1}^{\infty}$ be a collection of nonempty open subsets of X such that $V_i \cap Y \neq \emptyset$ for each *i*. It suffices to prove that $\{V_i\}_{i=1}^{\infty}$ is not locally finite at some point of Y. Assume that $\{V_i\}_{i=1}^{\infty}$ is locally finite at each point of Y. Let $V_1^{(1)} = V_1$. Pick $y_1 \in V_1^{(1)} \cap Y$, then there are infinitely many members of $\{V_i\}_{i=2}^{\infty}$ which can not contain y_1 . Otherwise, $\{V_i\}_{i=1}^{\infty}$ is not locally finite at y_1 . Without loss generality, pick $V_1^{(2)}, V_2^{(2)}, \ldots$ such that $y_1 \notin V_k^{(2)}$ for $k = 1, 2, \ldots$, where $V_k^{(2)} \in \{V_i\}_{i=2}^{\infty}$ for each k, and let B_{y_1} be a basic neighborhood of y_1 in X such that $B_{y_1} \subset V_1^{(1)}$. Pick $y_2 \in V_1^{(2)} \cap Y$, similarly there are infinitely many members of $\{V_i^{(2)}\}_{i=2}^{\infty}$ which can not contain y_2 . Otherwise, $\{V_i\}_{i=1}^{\infty}$ is not locally finite at y_2 . We pick $V_1^{(3)}, V_2^{(3)}, \ldots$ such that $y_2 \notin V_k^{(3)}$ for $k = 1, 2, \ldots$, where $V_k^{(3)} \in \{V_i^{(2)}\}_{i=2}^{\infty}$ for each k, and let B_{y_2} be a basic neighborhood of y_2 in X such that $B_{y_2} \subset V_1^{(2)}$. Continuing the process, so we can obtain an infinite sequence of distinct points of Y: y_1, y_2, \ldots such that $B_{y_n} \subset V_1^{(n)}$, where B_{y_n} is a basic neighborhood of y_n in X and $V_1^{(n)} \in \{V_i\}_{i=1}^{\infty}$ for $n = 1, 2, \ldots$.

We shall show that there is a $y_0 \in Y$ such that $\{B_{y_n}\}_{n=1}^{\infty}$ is not locally finite at y_0 , and so $\{V_i\}_{i=1}^{\infty}$ is not locally finite at y_0 which contradicts our assumption. Clearly, there are infinitely many elements of $\{y_n\}_{n=1}^{\infty}$ which are contained in $(0, \omega_1) \times \{0\}$ or $\{0\} \times (0, \omega_1)$. Without loss of generality, we suppose that there are infinitely many elements of $\{y_n\}_{n=1}^{\infty}$ which are contained in $(0, \omega_1) \times \{0\}$ and denote these elements by $y_{n_1} = (\alpha_{n_1}, 0), y_{n_2} = (\alpha_{n_2}, 0), \ldots$ Let

$$B_{y_{n_j}} = \{\alpha_{n_j}\} \times [0, \omega_1) \setminus \{(\alpha_{n_j}, \beta_1^{\alpha_{n_j}}), (\alpha_{n_j}, \beta_2^{\alpha_{n_j}}), \dots, (\alpha_{n_j}, \beta_{i_j}^{\alpha_{n_j}})\},\$$

where $\beta_k^{\alpha_{n_j}} \in (0, \omega_1)$ for $1 \leq k \leq i_j$ and $i_j \in \mathbb{N}$ for $j = 1, 2, \dots$ Put

$$\beta = \sup\{\beta_1^{\alpha_{n_1}}, \dots, \beta_{i_1}^{\alpha_{n_1}}, \beta_1^{\alpha_{n_2}}, \dots, \beta_{i_2}^{\alpha_{n_2}}, \dots\}$$

Pick $y_0 = (0, \beta + 1)$, then any neighborhood of y_0 meets infinitely many members of $\{B_{y_{n_i}}\}_{i=1}^{\infty}$. It follows that $\{V_i\}_{i=1}^{\infty}$ is not locally finite at y_0 .

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