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# ALMOST SUFFICIENT AND NECESSARY CONDITIONS FOR PERMANENCE AND EXTINCTION OF NONAUTONOMOUS DISCRETE LOGISTIC SYSTEMS WITH TIME-VARYING DELAYS AND FEEDBACK CONTROL*

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*Abstract.* A class of nonautonomous discrete logistic single-species systems with time-varying pure-delays and feedback control is studied. By introducing a new research method, almost sufficient and necessary conditions for the permanence and extinction of species are obtained. Particularly, when the system degenerates into a periodic system, sufficient and necessary conditions on the permanence and extinction of species are obtained. Moreover, a very important fact is found in our results, that is, the feedback control and delays are harmless for the permanence and extinction of species for discrete single-species systems. This shows that in a discrete single-species system introducing the feedback control to factitiously control the permanence and extinction of species is useless.

*Keywords*: discrete system, permanence, extinction, feedback control, time-varying delay

*MSC 2010*: 92D25, 39A30

## 1. Introduction

In recent years, the dynamical behavior, such as the local and global stability, persistence, permanence, extinction and existence of positive periodic solutions, etc., for the discrete-time population models have been extensively studied. Many important results have been established in many articles (see [2]–[6], [8], [10]–[19], [24], [25]–[34], [36]–[39] and references cited therein). Especially, we see that the discrete-time population models with feedback controls are investigated in [7], [9], [20]–[22], [23].

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For the population models with feedback controls, as we well know, an important subject is to study the effects of the feedback controls on the dynamical behavior of the models. In [7], we see that the following nonautonomous discrete-time single species model with delay and feedback control is proposed:

$$\begin{aligned} N(n+1) &= N(n)\exp\Big[r(n)\Big(1-\frac{N(n-m)}{k(n)}-c(n)\mu(n)\Big)\Big], \\ \mu(n+1) &= (1-a(n))\mu(n)+b(n)N(n-m). \end{aligned} \tag{1}$$

The author established a sufficient condition for the permanence of system (1) (see Theorem 2.1 in [7]). From this result we see directly that the feedback controls have negative influence on the permanence of the system. In [20], the authors discussed system (1) with periodic coefficients and established a sufficient condition for the existence of positive periodic solutions by applying the continuation theorem of coincidence degree theory. However, from the main result (see Theorem 2.1 in [20]) we can easily see that the feedback controls cannot influence the existence of a positive periodic solution. Thus, an important and interesting open problem is proposed here, that is, whether or not in system (1) the feedback controls influence the permanence and extinction of species. One of the main purposes of this paper is to study the effects of the feedback control and delays on the permanence and extinction of system (1). We will give an explicit answer to this problem.

In this paper, we investigate the following nonautonomous discrete single-species system with time-varying pure-delays and feedback control which is more general than system (1)

$$\begin{aligned} x(n+1) &= x(n)\exp\Big\{r(n)-\sum_{j=1}^{m}a_j(n)x(n-\tau_j(n))-c(n)u(n-\delta(n))\Big\}, \\ u(n+1) &= u(n)\gamma(n)+a(n)x(n-\sigma(n)), \end{aligned} \tag{2}$$

where $x(n)$ is the density of the species at time $n$ and $u(n)$ is the control variable at time $n$. For system (2) we will establish almost sufficient and necessary conditions for the permanence and extinction of the species by introducing a new research method. Particularly, when the system degenerates into a periodic one, sufficient and necessary conditions for the permanence and extinction of species are obtained. In addition, we will discuss the effects of the feedback control and delays on the permanence and extinction. We will see that under some quite weak assumptions the feedback control and delays in system (2) do not influence the permanence and extinction of species $x$.

## 2. Preliminaries

Let $\mathbb{N}$ denote the set of all nonnegative integers. For any bounded sequence $\zeta(n)$ we denote $\zeta^u = \sup_{n\in\mathbb{N}}\{\zeta(n)\}$ and $\zeta^l = \inf_{n\in\mathbb{N}}\{\zeta(n)\}$. In this paper, we first introduce the following assumptions for system (2).

($H_1$) $r(n)$ is a bounded sequence defined on $\mathbb{N}$, $a_i$ ($i = 1, 2, \ldots, m$), $c(n)$, $a(n)$, and $\gamma(n)$ are nonnegative bounded sequences defined on $\mathbb{N}$, and $\tau_i(i)$ ($i = 1, 2, \ldots, m$), $\delta(n)$, and $\sigma(n)$ are nonnegative bounded integer sequences defined on $\mathbb{N}$.

($H_2$) There exists an integer $\omega > 0$ such that

$$\liminf_{n\to\infty} \sum_{k=n}^{n+\omega} r(k) > 0.$$

($H_3$) There exists an integer $\omega > 0$ such that

$$\limsup_{n\to\infty} \sum_{k=n}^{n+\omega} r(k) \leqslant 0.$$

($H_4$) There exists an integer $\lambda > 0$ such that

$$\liminf_{n\to\infty} \sum_{k=n}^{n+\lambda} \sum_{j=1}^{m} a_j(k) > 0.$$

($H_5$) There exists an integer $\sigma > 0$ such that

$$\limsup_{n\to\infty} \prod_{k=n}^{n+\sigma} \gamma(k) < 1.$$

Let $\tau = \max\{\tau_i(n), \delta(n), \sigma(n) \colon n \in \mathbb{N}, i = 1, 2, \ldots, m\}$. Based on the biological background of system (2), in this paper we only consider the solution of system (2) with the initial conditions

$$x(\theta) = \varphi(\theta) \geqslant 0, \;\; u(\theta) = \psi(\theta) \geqslant 0$$
$$\text{for all } \theta = -\tau, -\tau+1, \ldots, -1, \;\; \varphi(0) > 0, \;\; \psi(0) > 0.$$

We first consider the nonautonomous difference inequality system

$$y(n+1) \leqslant y(n)\exp\{\alpha(n) - \beta(n)y(n)\}, \quad n \in \mathbb{N}, \tag{3}$$

where $\alpha(n)$ and $\beta(n)$ are bounded sequences defined on $\mathbb{N}$ and $\beta(n) \geqslant 0$ for all $n \in \mathbb{N}$. We have the following result.

**Lemma 1.** *Assume that there exists an integer $\lambda > 0$ such that*

$$\liminf_{n\to\infty} \sum_{k=n}^{n+\lambda} \beta(k) > 0. \tag{4}$$

*Then there exists a constant $M > 0$ such that for any nonnegative solution $y(n)$ of system (3) with an initial value $y(n_0) = y_0 \geqslant 0$, where $n_0 \in \mathbb{N}$ is an integer,*

$$\limsup_{n\to\infty} y(n) < M.$$

Proof. By (4), there exist a constant $\delta > 0$ and an integer $n_1 \geqslant n_0$ such that

$$\sum_{k=n}^{n+\lambda} \beta(k) > \delta \quad \text{for all } n \geqslant n_1. \tag{5}$$

Now we present two cases to prove the conclusion of Lemma 1.

*Case 1.* Assume that there exists an integer $n_2 \geqslant n_1$ such that $y(n_2) < y(n_2 + \lambda + 1)$. Then from the first equation of system (2) we have

$$y(n_2+\lambda+1) \leqslant y(n_2)\exp\Bigl\{\sum_{k=n_2}^{n_2+\lambda} \alpha(k) - \sum_{k=n_2}^{n_2+\lambda} \beta(k)y(k)\Bigr\}.$$

This implies

$$\sum_{k=n_2}^{n_2+\lambda} \alpha(k) > \sum_{k=n_2}^{n_2+\lambda} \beta(k)y(k). \tag{6}$$

In view of (5), there exists $i_0$ $(0 \leqslant i_0 \leqslant \lambda)$ such that $\beta(n_2+i_0) > \delta/(1+\lambda)$. Hence, combining it with (6), we have

$$(1+\lambda)\alpha^u > \beta(n_2+i_0)y(n_2+i_0) > \frac{\delta}{1+\lambda} y(n_2+i_0),$$

which implies

$$y(n_2+i_0) < \frac{(1+\lambda)^2\alpha^u}{\delta}.$$

Consequently, for every $i = 1, 2, \dots, \lambda + 1$ we further obtain

$$\begin{aligned} y(n_2 + i_0 + i) &= y(n_2 + i_0) \exp\Big\{ \sum_{k=n_2+i_0}^{n_2+i_0+i-1} \alpha(k) - \sum_{k=n_2+i_0}^{n_2+i_0+i-1} \beta(k) y(k) \Big\} \\ &\leqslant y(n_2 + i_0) \exp\Big\{ \sum_{k=n_2+i_0}^{n_2+i_0+i-1} \alpha(k) \Big\} \\ &\leqslant y(n_2 + i_0) \exp\{\alpha^u i\} < \frac{(1+\lambda)^2 \alpha^u}{\delta} \exp\{(\lambda+1)\alpha^u\}. \end{aligned}$$

Let

$$M_1 = \frac{(1+\lambda)^2 \alpha^u}{\delta} \exp\{(\lambda+1)\alpha^u\}.$$

We claim that

$$y(n) < M_1 \quad \text{for all } n \geqslant n_2 + i_0 + \lambda + 1. \tag{7}$$

In fact, if (7) is not true, then there exists an integer $n_3 > n_2 + i_0 + \lambda + 1$ such that $y(n_3) \geqslant M_1$ and $y(n) < M_1$ for all $n_2 + i_0 \leqslant n < n_3$, which implies $y(n_3 - \lambda - 1) < M_1$. Hence, $y(n_3 - \lambda - 1) < y(n_3)$. In view of the discussion above, we immediately have $y(n_3) < M_1$, which is a contradiction. Therefore, (7) is true.

*Case 2.* Assume that $y(n) \geqslant y(n + \lambda + 1)$ holds for all $n \geqslant n_1$. Then for each $i = 0, 1, 2, \dots, \lambda$ we have

$$y(n_1 + i + k(\lambda + 1)) \geqslant y(n_1 + i + (k+1)(\lambda + 1)) \quad \text{for all } k \in \mathbb{N}.$$

Hence, the sequence $y(n_0 + i + k(\lambda + 1))$ is decreasing in $k \in \mathbb{N}$. For any integer $n \geqslant n_1$ we have that there exist integers $i_n$ and $k_n$ such that $n = n_1 + i_n + k_n(\lambda + 1)$, where $0 \leqslant i_n \leqslant \lambda$ and $k_n \in \mathbb{N}$. Hence, we have $y(n) = y(n_1 + i_n + k_n(\lambda + 1))$ and $y(n) \leqslant y(n_1 + i_n)$. Choosing the constant $M_2 = \max\{y(n_1), y(n_1 + 1), \dots, y(n_1 + \lambda)\}$, we obtain $y(n) \leqslant M_2$ for all $n \geqslant n_1$.

Combining Cases 1 and 2, further choosing a constant $M > \max\{M_1, M_2\}$, we finally obtain that the conclusion of Lemma 1 is true. □

Next, we consider the nonautonomous linear difference equation

$$v(n+1) = \gamma(n) v(n) + \omega(n), \tag{8}$$

where $\gamma(n)$ and $\omega(n)$ are nonnegative bounded sequences defined on $\mathbb{N}$. We have the following results.

**Lemma 2.** *Assume that there exists an integer* $\lambda > 0$ *such that*

$$\limsup_{n\to\infty} \prod_{k=n}^{n+\lambda} \gamma(k) < 1. \tag{9}$$

*Then there exists a constant* $M > 0$ *such that for any nonnegative solution* $v(n)$ *of system* (8) *with an initial value* $v(n_0) = v_0 \geqslant 0$, *where* $n_0 \in \mathbb{N}$ *is an integer,*

$$\limsup_{n\to\infty} v(n) < M.$$

Proof. By (9), there exist a constant $\varepsilon_0 \in (0,1)$ and an integer $N_0 \geqslant n_0$ such that

$$\prod_{k=n}^{n+\lambda} \gamma(k) < 1 - \varepsilon_0 \quad \text{for all } n \geqslant N_0.$$

We first prove that there exist positive constants $M_0 > 1$ and $\delta_0 \in (0,1)$ such that for any integers $n_1$, $n_2$ with $n_2 > n_1 \geqslant 0$

$$\prod_{k=n_1}^{n_2-1} \gamma(k) \leqslant M_0 \delta_0^{n_2-n_1}. \tag{10}$$

Choose a constant $\Gamma > 0$ such that $\gamma(n) \leqslant \Gamma$ for all $n \in \mathbb{N}$. For any integers $n_1$, $n_2$ with $n_2 > n_1 \geqslant 0$, if $n_2 \leqslant N_0$, then we have

$$\begin{aligned} \prod_{k=n_1}^{n_2-1} \gamma(k) \leqslant \Gamma^{n_2-n_1} &= \Big(\frac{\Gamma}{1-\varepsilon_0}\Big)^{n_2-n_1} (1-\varepsilon_0)^{n_2-n_1} \\ &\leqslant \Big(\frac{\Gamma}{1-\varepsilon_0}\Big)^{N_0} (1-\varepsilon_0)^{n_2-n_1} \\ &\leqslant \Big(\frac{\Gamma}{1-\varepsilon_0}\Big)^{N_0+\lambda+1} ((1-\varepsilon_0)^{1/(1+\lambda)})^{n_2-n_1}. \end{aligned} \tag{11}$$

If $n_2 > N_0$ and $n_1 \leqslant N_0$, let $n_2 = N_0 + s(\lambda+1) + \varrho + 1$, where $0 \leqslant \varrho \leqslant \lambda$ and $s \in \mathbb{N}$. Then we obtain

$$\begin{aligned} \prod_{k=n_1}^{n_2-1} \gamma(k) &= \prod_{k=n_1}^{N_0-1} \gamma(k) \prod_{k=N_0}^{N_0+s(\lambda+1)-1} \gamma(k) \prod_{k=N_0+s(\lambda+1)}^{N_0+s(\lambda+1)+\varrho} \gamma(k) \\ &\leqslant \Gamma^{N_0} (1-\varepsilon_0)^s \Gamma^{\lambda+1} \\ &\leqslant \Gamma^{N_0+\lambda+1} (1-\varepsilon_0)^{(n_2-\varrho-N_0)/(\lambda+1)} \\ &= \Gamma^{N_0+\lambda+1} [(1-\varepsilon_0)^{1/(1+\lambda)}]^{n_2-n_1} [(1-\varepsilon_0)^{1/(1+\lambda)}]^{n_1-N_0-\varrho} \\ &\leqslant \Gamma^{N_0+\lambda+1} [(1-\varepsilon_0)^{1/(1+\lambda)}]^{n_2-n_1} [(1-\varepsilon_0)^{1/(1+\lambda)}]^{-N_0-\lambda-1} \\ &\leqslant \Big(\frac{\Gamma}{1-\varepsilon_0}\Big)^{N_0+\lambda+1} ((1-\varepsilon_0)^{1/(1+\lambda)})^{n_2-n_1}. \end{aligned} \tag{12}$$

If $n_3 \geqslant N_0$, let $n_4 = N_3 + s(\lambda+1) + \varrho + 1$, where $0 \leqslant \varrho \leqslant \lambda$ and $s \in \mathbb{N}$. Then we get

$$
\begin{aligned}
(13) \qquad \prod_{k=n_1}^{n_2-1} \gamma(k) &= \prod_{k=n_1}^{N_0+s(\lambda+1)-1} \gamma(k) \prod_{k=N_0+s(\lambda+1)}^{N_0+s(\lambda+1)+\varrho} \gamma(k) \\
&\leqslant (1-\varepsilon_0)^s \Gamma^{\lambda+1} \\
&\leqslant \Gamma^{\lambda+1}(1-\varepsilon_0)^{(n_2-\varrho-N_0)/(\lambda+1)} \\
&= \Gamma^{\lambda+1}[(1-\varepsilon_0)^{1/(1+\lambda)}]^{n_2-n_1}[(1-\varepsilon_0)^{1/(1+\lambda)}]^{n_1-N_0-\varrho} \\
&\leqslant \Gamma^{N_0+\lambda+1}[(1-\varepsilon_0)^{1/(1+\lambda)}]^{n_2-n_1}[(1-\varepsilon_0)^{1/(1+\lambda)}]^{-N_0-\lambda-1} \\
&\leqslant \Big(\frac{\Gamma}{1-\varepsilon_0}\Big)^{N_0+\lambda+1}((1-\varepsilon_0)^{1/(1+\lambda)})^{n_2-n_1}.
\end{aligned}
$$

Choosing constants

$$
M_0 = \Big(\frac{\Gamma}{1-\varepsilon_0}\Big)^{N_0+\lambda+1} \quad \text{and} \quad \delta_0 = (1-\varepsilon_0)^{1/(1+\lambda)},
$$

then from (11)–(13) we obtain that (10) holds.

In the following, for the sake of convenience we set $\prod_{k=n}^{n-1} \gamma(k) = 1$ for any $n \in \mathbb{N}$. By the variation-of-constants formula for the difference equation (see [1]), we find that any solution $v(n)$ of equation (8) with an initial value $v(n_0) = v_0$, where $n_0 \in \mathbb{N}$, can be expressed by the formula

$$
(14) \qquad v(n) = v_0 \prod_{k=n_0}^{n-1} \gamma(k) + \sum_{i=n_0}^{n-1} \Big[\prod_{k=i+1}^{n-1} \gamma(k)\Big]\omega(i) \quad \text{for all } n > n_0.
$$

From (10) and (14) we obtain

$$
\begin{aligned}
(15) \qquad v(n) &\leqslant v_0 \prod_{k=n_0}^{n-1} \gamma(k) + \omega^u \sum_{i=n_0}^{n-1} \Big[\prod_{k=i+1}^{n-1} \gamma(k)\Big] \\
&\leqslant v_0 M_0 \delta_0^{n-n_0} + \omega^u \sum_{i=n_0}^{n-1} [M_0 \delta_0^{n-i-1}] \\
&\leqslant v_0 M_0 \delta_0^{n-n_0} + \omega^u M_0 \frac{1-\delta_0^{n-n_0}}{1-\delta_0}.
\end{aligned}
$$

From $\delta_0 < 1$ we have $\delta_0^k \to 0$ as $k \to \infty$. Hence, from (15) we finally have

$$
\limsup_{n\to\infty} v(n) \leqslant \frac{M_0 \omega^u}{1-\delta_0}.
$$

If we choose a constant $M > M_0\omega^u/(1-\delta_0)$, then the conclusion of Lemma 2 holds. This completes the proof of Lemma 2. $\square$

**Lemma 3.** *Assume that the conditions of Lemma* 2 *hold. Then for any constants* $\varepsilon > 0$ *and* $M_1 > 0$ *there exist positive constants* $\hat{\delta} = \hat{\delta}(\varepsilon)$ *and* $\hat{n} = \hat{n}(\varepsilon, M_1)$ *such that for any* $\hat{n}_0 \in \mathbb{N}$ *and* $0 \leqslant v_0 \leqslant M_1$, *when* $\omega(n) < \hat{\delta}$ *for all* $n \geqslant \hat{n}_0$, *we have*

$$v(n, \hat{n}_0, v_0) < \varepsilon \quad \textit{for all } n \geqslant \hat{n}_0 + \hat{n},$$

*where* $v(n, \hat{n}_0, v_0)$ *is the solution of equation* (8) *with the initial condition* $v(\hat{n}_0, \hat{n}_0, v_0) = v_0$.

P r o o f. By using the variation-of-constants formula for the difference equation, we have

$$v(n, \hat{n}_0, v_0) = v_0 \prod_{k=\hat{n}_0}^{n-1} \gamma(k) + \sum_{i=\hat{n}_0}^{n-1} \Bigl[ \prod_{k=i+1}^{n-1} \gamma(k) \Bigr] \omega(i). \tag{16}$$

For any constants $\varepsilon > 0$ and $M_1 > 0$, if $0 \leqslant v_0 \leqslant M_1$ and $\omega(n) \leqslant \hat{\delta}$ for all $n \geqslant \hat{n}_0$, then from (10) and (16) we have

$$\begin{aligned} v(n, \hat{n}_0, v_0) &\leqslant M_1 \prod_{k=\hat{n}_0}^{n-1} \gamma(k) + \hat{\delta} \sum_{i=\hat{n}_0}^{n-1} \Bigl[ \prod_{k=i+1}^{n-1} \gamma(k) \Bigr] \\ &\leqslant M_1 M_0 \delta_0^{n-\hat{n}_0} + \hat{\delta} M_0 \sum_{i=\hat{n}_0}^{n-1} \delta_0^{n-i-1} \\ &\leqslant M_1 M_0 \delta_0^{n-\hat{n}_0} + \frac{\hat{\delta} M_0}{1-\delta_0}. \end{aligned} \tag{17}$$

Choosing

$$\hat{n} = \frac{\ln \varepsilon - \ln(2M_0 M_1)}{\ln \delta_0} + \hat{n}_0 + 1 \quad \text{and} \quad \hat{\delta} = \frac{\varepsilon(1-\delta_0)}{2M_0},$$

then from (17) we finally obtain $v(n, \hat{n}_0, v_0) < \varepsilon$ for all $n \geqslant \hat{n}_0 + \hat{n}$. This completes the proof of Lemma 3. □

## 3. Main results

**Theorem 1.** *Assume that* ($H_1$), ($H_2$), ($H_4$) *and* ($H_5$) *hold. Then species* $x$ *in system* (2) *is permanent.*

P r o o f. Let $(x(n), u(n))$ be any positive solution of system (2). Then from system (2) we directly obtain

$$x(n+1) \leqslant x(n) \exp\{r(n)\} \quad \text{for all } n \in \mathbb{N}.$$

Hence,

$$x(n-\tau) \leqslant x(n)\exp\{-r^u\tau\} \quad \text{for all } n \geqslant \tau.$$

Further, from system (2) it immediately follows that

$$x(n+1) \leqslant x(n)\exp\biggl\{r(n) - \exp\{-r^u\tau\}\sum_{i=1}^{m} a_i(n)x(n)\biggr\} \quad \text{for all } n \geqslant \tau.$$

From Lemma 1 it follows that there exists a constant $\bar{x} > 0$ such that

$$\limsup_{n\to\infty} x(n) < \bar{x}. \tag{18}$$

Hence, there exists a large enough $N_1 \geqslant \tau$ such that $x(n) < \bar{x}$ for all $n \geqslant N_1$. From the second equation of system (2) it follows that

$$u(n+1) \leqslant u(n)\gamma(n) + a(n)\bar{x} \quad \text{for all } n \geqslant N_1 + \tau.$$

By using Lemma 2 and the comparison theorem for the difference equation, we obtain that there exists a constant $\bar{u} > 0$ such that

$$\limsup_{n\to\infty} u(n) < \bar{u}. \tag{19}$$

By $(\mathrm{H}_2)$ we can choose a constant $\varepsilon_1 > 0$ and an integer $N_2 \geqslant N_1$ such that

$$\sum_{s=n}^{n+\omega} (r(s) - \varepsilon_1 c(s)) \geqslant \varepsilon_1 \quad \text{for all } n \geqslant N_2. \tag{20}$$

Consider the auxiliary equation

$$v(n+1) = v(n)\gamma(n) + a(n)\alpha_0, \tag{21}$$

where $\alpha_0$ is a parameter. By Lemma 3, for $\varepsilon_1 > 0$ and $\bar{u} > 0$ given above there exist constants $\hat{\delta}_0 = \hat{\delta}_0(\varepsilon_1)$ and $\hat{n}_0 = \hat{n}_0(\varepsilon_1, \bar{u})$ such that for any $n_0 \in \mathbb{N}$ and $0 \leqslant v_0 \leqslant \bar{u}$, when $\alpha_0 a(n) < \hat{\delta}_0$ for all $n \geqslant n_0$, we have

$$v(n, n_0, v_0) < \varepsilon_1 \quad \text{for all } n \geqslant n_0 + \hat{n}_0, \tag{22}$$

where $v(n, n_0, v_0)$ is the solution of equation (21) with the initial condition $v(n_0, n_0, v_0) = v_0$.

It follows from (20) that there exists a positive constant $\alpha_0 \leqslant \min\{\varepsilon_1, \hat{\delta}_0/(a^u+1)\}$ such that

$$\sum_{s=n}^{n+\omega}\Big(r(n) - \sum_{i=1}^{m} a_i(n)\alpha_0 - \varepsilon_1 c(n)\Big) \geqslant \alpha_0 \quad \text{for all } n \geqslant N_2. \tag{23}$$

We first prove

$$\limsup_{n\to\infty} x(n) \geqslant \alpha_0. \tag{24}$$

Otherwise, there exist a positive solution $(x(n), u(n))$ of system (2) and an integer $\hat{n}_1 > 0$ such that $x(n) < \alpha_0$ for all $n \geqslant \hat{n}_1$. Further, by (18) and (19) there exists an integer $\hat{n}_2 \geqslant \hat{n}_1$ such that

$$x(n) \leqslant \bar{x}, \ \ u(n) \leqslant \bar{u} \quad \text{for all } n \geqslant \hat{n}_2. \tag{25}$$

Hence, from the second equation of system (2) we have

$$u(n+1) \leqslant \gamma(n)u(n) + a(n)\alpha_0 \quad \text{for all } n \geqslant \hat{n}_1 + \tau. \tag{26}$$

Let $v(n)$ be the solution of equation (21) with the initial value $v(\hat{n}_2+\tau) = u(\hat{n}_2+\tau)$. Then by the comparison theorem for the difference equation and inequality (26) we obtain

$$u(n) \leqslant v(n) \quad \text{for all } n \geqslant \hat{n}_2 + \tau.$$

In (22) we set $n_0 = \hat{n}_2 + \tau$ and $v_0 = u(\hat{n}_2+\tau)$. Since $\alpha_0 a(n) < \hat{\delta}_0$ for all $n \geqslant \hat{n}_2 + \tau$, we get

$$v(n) = v(n, \hat{n}_0, v_0)) < \varepsilon_1 \quad \text{for all } n \geqslant \hat{n} + \hat{n}_2 + \tau.$$

Hence, we further have

$$u(n) < \varepsilon_1 \quad \text{for all } n \geqslant \hat{n} + \hat{n}_2 + \tau.$$

Thus, for any $n \geqslant \hat{n} + \hat{n}_2 + N_2 + 2\tau$, from system (2) and (23) we find that

$$\begin{aligned} x(n+\omega+1) &\geqslant x(n)\exp\Big\{\sum_{s=n}^{n+\omega}\Big[r(n) - \sum_{i=1}^{m} a_i(n)\alpha_0 - c(n)\varepsilon_1\Big]\Big\} \\ &\geqslant x(n)\exp\{\alpha_0\}. \end{aligned}$$

Consequently, we further obtain

$$x(\bar{n} + k(\omega+1)) \geqslant x(\bar{n})\exp\{k\alpha_0\} \quad \text{for all } k \in \mathbb{N},$$

where $\bar{n} = \hat{n} + \hat{n}_2 + N_2 + 2\tau$. Therefore, we finally have $x(\bar{n} + k(\omega + 1)) \to \infty$ as $k \to \infty$, which contradicts (25). Hence, (24) holds.

Next, we prove there exists a constant $\underline{x} > 0$ such that

$$\liminf_{n\to\infty} x(n) \geqslant \underline{x}$$

for any positive solution $(x(n), u(n))$ of system (2). If this is not true, then there is a sequence of initial values $z^{(k)} = (\varphi^{(k)}, \psi^{(k)})$ of system (2) such that

$$\liminf_{n\to\infty} x(n, z^{(k)}) < \frac{\alpha_0}{k^2} \quad \text{for all } k = 1, 2, \ldots, \tag{27}$$

where $(x(n, z^{(k)}), u(n, z^{(k)}))$ is the solution of system (2) with the initial condition

$$x(n) = \varphi^{(k)}(n), \quad u(n) = \psi^{(k)}(n), \quad n \in [-\tau, 0].$$

By (24) and (27), for each $k \in \mathbb{N}$ there exist two sequences of positive integers $\{s_q^{(n)}\}$ and $\{t_q^{(n)}\}$ such that

$$0 < s_1^{(k)} < t_1^{(k)} < s_2^{(k)} < t_2^{(k)} < \ldots < s_q^{(k)} < t_q^{(k)} < \ldots$$

and

$$s_q^{(k)} \to \infty \quad \text{as } q \to \infty, \tag{28}$$

such that

$$x(s_q^{(k)}, z^{(k)}) > \frac{\alpha_0}{k}, \quad x(t_q^{(k)}, z^{(k)}) < \frac{\alpha_0}{k^2} \tag{29}$$

and

$$\frac{\alpha_0}{k^2} \leqslant x(n, z^{(k)}) \leqslant \frac{\alpha_0}{k} \qquad \text{for all } n \in [s_q^{(k)} + 1, t_q^{(k)} - 1]. \tag{30}$$

Obviously, from (28) we first have $t_q^{(k)} - s_q^{(k)} \geqslant 1$ for all $k \geqslant 1$. Next, (18) and (19) imply that for each $k \in \mathbb{N}$ there exists an integer $\hat{n}_2^{(k)} > 0$ such that

$$x(n, z^{(k)}) \leqslant \bar{x}, \;\; u(n, z^{(k)}) \leqslant \bar{u} \qquad \text{for all } n \geqslant \hat{n}_2^{(k)}.$$

It follows from (28) that there exists an integer $N_1^{(k)} > 0$ such that $s_q^{(k)} > \hat{n}_2^{(k)} + \tau$ for all $q \geqslant N_1^{(k)}$. For any $n \in [s_q^{(k)}, t_q^{(k)}]$ and $q \geqslant N_1^{(k)}$, we have

$$\begin{aligned} &x(n+1, z^{(k)}) \\ &\quad = x(n, z^{(k)}) \exp\Big\{r(n) - \sum_{j=1}^{m} a_j(n) x(n - \tau_j(n), z^{(k)}) - c(n) u(n - \delta(n), z^{(k)})\Big\} \\ &\quad \geqslant x(n, z^{(k)}) \exp\{-\theta\}, \end{aligned}$$

where $\theta = r^u + \sum\limits_{i=1}^{m} a^u M_2 + c^u M_2$. Hence,

$$x(t_q^{(k)}, z^{(k)}) \geqslant x(s_q^{(k)}, z^{(k)}) \exp\{-\theta(t_q^{(k)} - s_q^{(k)})\},$$

which implies

$$t_q^{(k)} - s_q^{(k)} > \frac{\ln k}{\theta} \quad \text{for all } q \geqslant N_1^{(k)},\ k \in \mathbb{N}.$$

Choose an integer $K_0 > 0$ such that

$$t_q^{(k)} - s_q^{(k)} \geqslant \hat{n}_0 + \omega + 2\tau + 1 \quad \text{for all } k \geqslant K_0,\ q \geqslant N_1^{(k)}.$$

For any $k \geqslant K_0$, $q \geqslant N_1^{(k)}$ and $n \in [s_q^{(k)} + \tau + 1, t_q^{(k)}]$, from the second equation of system (2) we have

$$u(n+1, z^{(k)}) \leqslant \gamma(n) u(n, z^{(k)}) + \alpha_0 a(n). \tag{31}$$

Let $v(n)$ be the solution of equation (21) with the initial value $v(s_q^{(k)} + \tau + 1) = u(s_q^{(k)} + \tau + 1)$. Then using the comparison theorem and inequality (31), we obtain

$$u(n) \leqslant v(n) \quad \text{for all } n \in [s_q^{(k)} + \tau + 1, t_q^{(k)}]. \tag{32}$$

In (22) we set $n_0 = s_q^{(k)} + \tau + 1$ and $v_0 = u(s_q^{(k)} + \tau + 1)$. Since $\alpha_0 a(n) < \hat{\delta}_0$ for all $n \in [s_q^{(k)} + \tau + 1, t_q^{(k)}]$, we get

$$v(n) = v(n, s_q^{(k)} + \tau + 1, u(s_q^{(k)} + \tau + 1)) < \varepsilon_1$$

for all $n \in [s_q^{(k)} + \hat{n}_0 + \tau + 1, t_q^{(k)}]$. Thus, from (32) we further have

$$u(n, z^{(k)}) < \varepsilon_1 \quad \text{for all } n \in [s_q^{(k)} + \hat{n}_0 + \tau + 1, t_q^{(k)}],\ k \geqslant K_0,\ q \geqslant N_1^{(k)}.$$

For any $n \in [s_q^{(k)} + \hat{n}_0 + \tau + 1, t_q^{(k)}]$, $k \geqslant K_0$ and $q \geqslant N_1^{(k)}$, system (2) yields

$$x(n+1, z^{(k)}) \geqslant x(n, z^{(k)}) \exp\left\{ r(n) - \sum_{i=1}^{m} a_i(n)\alpha_0 - c(n)\varepsilon_1 \right\}.$$

Hence, we further obtain

$$x(n+\omega+1, z^{(k)}) \geqslant x(n, z^{(k)}) \exp\left\{ \sum_{s=n}^{n+\omega} \left[ r(n) - \sum_{i=1}^{m} a_i(n)\alpha_0 - c(n)\varepsilon_1 \right] \right\}.$$

It follows from (23), (29), and (30) that

$$\begin{aligned}\frac{\alpha_0}{k^2} &> x(t_q^{(k)}, z^{(k)}) \\ &\geqslant x(t_q^{(k)} - \omega - 1, z^{(k)}) \exp\Bigg\{\sum_{s=n}^{n+\omega}\Bigg[r(n) - \sum_{i=1}^{m} a_i(n)\alpha_0 - c(n)\varepsilon_1\Bigg]\Bigg\} \\ &> \frac{\alpha_0}{k^2}\exp\{\alpha_0\},\end{aligned}$$

which leads to a contradiction. This completes the proof of Theorem 1. □

**Theorem 2.** *Assume that* ($H_1$), ($H_3$), *and* ($H_4$) *hold. Then species* $x$ *in system* (2) *is extinct.*

P r o o f. We first prove that there exists an integer $p_0 > 0$ such that

$$\liminf_{n\to\infty} \sum_{s=n}^{n+p_0(\omega+1)-1} \sum_{j=1}^{m} a_j(s) > 0. \tag{33}$$

In fact, by ($H_4$) there exist a constant $\beta > 0$ and an integer $S_0 > 0$ such that

$$\sum_{s=n}^{n+\lambda}\sum_{j=1}^{m} a_j(s) > \beta \quad \text{for all } n \geqslant S_0. \tag{34}$$

For any integers $n \geqslant S_0$ and $p > 0$ we can choose an integer $q_p \geqslant 0$ such that

$$n + p(\omega+1) - 1 \in [n + q_p(\lambda+1), n + (q_p+1)(\lambda+1)),$$

hence from (34) we obtain

$$\begin{aligned}\sum_{s=n}^{n+p(\omega+1)-1} \sum_{j=1}^{m} a_j(s) &= \sum_{s=n}^{n+q_p(\lambda+1)-1} \sum_{j=1}^{m} a_j(s) + \sum_{s=n+q_p(\lambda+1)}^{n+p(\omega+1)-1} \sum_{j=1}^{m} a_j(s) \\ &\geqslant q_p\beta - (\lambda+1)\sum_{j=1}^{m} a_j^u.\end{aligned} \tag{35}$$

Since $q_p \to \infty$ as $p \to \infty$, there exists an integer $p_0 > 0$ such that

$$q_{p_0}\beta - (\lambda+1)\sum_{j=1}^{m} a_j^u \geqslant \beta.$$

Hence, from (35) we find that

$$\sum_{s=n}^{n+p_0(\omega+1)-1} \sum_{j=1}^{m} a_j(s) \geqslant \beta \quad \text{for all } n \geqslant S_0.$$

This shows that (33) is true.

On the other hand, by ($H_3$) we get

$$\limsup_{n\to\infty} \sum_{s=n}^{n+p_0(\omega+1)-1} r(s) \leqslant 0. \tag{36}$$

From (33) and (36) we obtain that for any constant $\varepsilon \in (0,1)$ there exist a constant $\eta > 0$ and an integer $S_1 \geqslant S_0$ such that

$$\sum_{s=n}^{n+p_0(\omega+1)-1} \left[ r(s) - \sum_{j=1}^{m} a_j(s) \exp\{-r^u \tau\} \varepsilon \right] \leqslant -\eta \quad \text{for all } n \geqslant S_1. \tag{37}$$

Let $(x(n), u(n))$ be any positive solution of system (2). If $x(n) \geqslant \varepsilon$ for all $n \geqslant S_1 + \tau$, let $n_0 = S_1 + \tau$. Then from (37) we have

$$\begin{aligned} x(n_0 + p_0(\omega+1)) &\leqslant x(n_0) \exp\left\{ \sum_{s=n_0}^{n_0+p_0(\omega+1)-1} \left[ r(s) - \sum_{j=1}^{m} a_j(s) x(s - \tau_j(s)) \right] \right\} \\ &\leqslant x(n_0) \exp\left\{ \sum_{s=n_0}^{n_0+p_0(\omega+1)-1} \left[ r(s) - \sum_{j=1}^{m} a_j(s) \exp\{-r^u \tau\} x(s) \right] \right\} \\ &\leqslant x(n_0) \exp\left\{ \sum_{s=n_0}^{n_0+p_0(\omega+1)-1} \left[ r(s) - \sum_{j=1}^{m} a_j(s) \exp\{-r^u \tau\} \varepsilon \right] \right\} \\ &\leqslant x(n_0) \exp\{-\eta\}. \end{aligned}$$

Hence, we further obtain

$$x(n_0 + kp_0(\omega+1)) \leqslant x(n_0) \exp\{-k\eta\} \quad \text{for all } k \in \mathbb{N},$$

which implies $x(n_0 + kp_0(\omega+1)) \to 0$ as $k \to \infty$. This leads to a contradiction. Therefore, there exists an integer $n_1 \geqslant n_0$ such that $x(n_1) < \varepsilon$.

Now, we claim that

$$x(n) \leqslant \varepsilon \exp\{p_0(\omega+1) r^u\} \quad \text{for all } n \geqslant n_1. \tag{38}$$

In fact, if it is not true, then there exists $n_2 \geqslant n_1$ such that $x(n) \leqslant \varepsilon \exp\{p_0(\omega+1)r^u\}$ for all $n_1 \leqslant n \leqslant n_2$ and

$$x(n_2+1) > \varepsilon \exp\{p_0(\omega+1)r^u\}. \tag{39}$$

In the case of $n_2 - n_1 < p_0(\omega+1)$, we have

$$\begin{aligned} x(n_2+1) &\leqslant x(n_1)\exp\biggl\{\sum_{s=n_1}^{n_2}\biggl[r(s) - \sum_{j=1}^{m} a_j(s)x(s-\tau_j(s))\biggr]\biggr\} \\ &\leqslant x(n_1)\exp\biggl\{\sum_{s=n_1}^{n_2} r(s)\biggr\} \\ &\leqslant x(n_1)\exp\{(n_2-n_1+1)r^u\} \leqslant \varepsilon\exp\{p_0(\omega+1)r^u\}, \end{aligned}$$

which leads to a contradiction with (39).

In the case of $n_2 - n_1 \geqslant p_0(\omega+1)$, let $n_2 = n_1 + kp_0(\omega+1) + \varrho$, where $k \in \mathbb{N}$ and $0 \leqslant \varrho < p_0(\omega+1)$. Then it follows from (36) that

$$\begin{aligned} x(n_2+1) &\leqslant x(n_1)\exp\biggl\{\sum_{s=n_1}^{n_2}\biggl[r(s) - \sum_{j=1}^{m} a_j(s)x(s-\tau_j(s))\biggr]\biggr\} \\ &\leqslant x(n_1)\exp\biggl\{\sum_{s=n_1}^{n_1+kp_0(\omega+1)-1} r(s) + \sum_{s=n_1+kp_0(\omega+1)}^{n_2} r(s)\biggr\} \\ &\leqslant x(n_1)\exp\biggl\{\sum_{s=n_1+kp_0(\omega+1)}^{n_2} r(s)\biggr\} \\ &\leqslant \varepsilon\exp\{p_0(\omega+1)r^u\}. \end{aligned}$$

This also leads to a contradiction. According to the arguments of the two cases above, we have shown that (38) is true.

Since $\varepsilon \in (0,1)$ is arbitrary, let $\varepsilon \to 0$. Then from (39) we finally obtain $x(n) \to 0$ as $n \to \infty$. Therefore, species $x$ in system (2) is extinct. This completes the proof of Theorem 2. □

Remark 1. From Theorems 1 and 2 we see that for system (2) under some quite weak assumptions the feedback control and delays do not affect the permanence and extinction of species $x$. This is a very important and interesting fact for a discrete-time single-species logistic system. It shows that in a discrete-time single-species logistic system introducing the feedback control to factitiously control the permanence and extinction of a species is useless.

Now, we consider system (1) which is a special case of system (2). To establish criteria for the permanence of species of system (1), we need to introduce the following assumptions:

($A_1$) $r(n)$, $c(n)$, $a(n)$, and $b(n)$ are nonnegative bounded sequences defined on $\mathbb{N}$, $m$ is a nonnegative integer.

($A_2$) There exists an integer $\omega > 0$ such that

$$\liminf_{n\to\infty} \sum_{s=n}^{n+\omega} r(s) > 0.$$

($A_3$) Sequence $k(n)$ is defined on $\mathbb{N}$ and is bounded from above and below by positive constants.

($A_4$) There exists an integer $\sigma > 0$ such that

$$\limsup_{n\to\infty} \prod_{s=n}^{n+\sigma} (1 - a(s)) < 1.$$

Applying Theorem 1, we have the following result.

**Corollary 1.** *Assume that* ($A_1$), ($A_2$), ($A_3$), *and* ($A_4$) *hold. Then species* $N$ *in system* (1) *is permanent.*

Remark 2. Obviously, Corollary 1 is a very good improvement of the corresponding result obtained by Chen in [7] (see Theorem 2.1 in [7]). In fact, in Theorem 2.1 we easily see that the feedback control has a negative influence on the permanence of system (1). However, in Corollary 1 the feedback control does not affect the permanence of species $N$.

Next, we establish criteria for the permanence and extinction of species in system (2) with periodic coefficients. When system (2) degenerates into the $\omega$-periodic case, then corresponding to assumptions ($H_1$)–($H_5$) we have the following assumptions:

($H_1'$) $r(n)$ is an $\omega$-periodic sequence defined on $\mathbb{N}$, $a_i(n)$ $(i = 1, 2, \ldots, m)$, $c(n)$, $a(n)$, and $\gamma(n)$ are nonnegative $\omega$-periodic sequences defined on $\mathbb{N}$, and $\tau_i(n)$ $(i = 1, 2, \ldots, m)$, $\delta(n)$, and $\sigma(n)$ are nonnegative $\omega$-periodic integer sequences defined on $\mathbb{N}$,

($H_2'$) $\sum\limits_{k=0}^{\omega-1} r(k) > 0,$

($H_3'$) $\sum\limits_{k=0}^{\omega-1} r(k) \leqslant 0,$

($H_4'$) $\sum\limits_{k=0}^{\omega-1} \sum\limits_{j=1}^{m} a_j(k) > 0,$

($H_5'$) $\prod\limits_{k=0}^{\omega-1} \gamma(k) < 1.$

As consequences of Theorems 1 and 2, for system (2) with periodic coefficients we have the following results.

**Corollary 2.** *Assume that* $(\mathrm{H}_1')$, $(\mathrm{H}_2')$, $(\mathrm{H}_4')$, *and* $(\mathrm{H}_5')$ *hold. Then species* $x$ *in system* (2) *is permanent.*

**Corollary 3.** *Assume that* $(\mathrm{H}_1')$, $(\mathrm{H}_3')$, *and* $(\mathrm{H}_4')$ *hold. Then species* $x$ *in system* (2) *is extinct.*

R e m a r k 3. Actually, for system (2) with periodic coefficients, the above Corollaries 2 and 3 provide sufficient and necessary conditions for the permanence and extinction of species.

In system (2), if the coefficient $c(n) \equiv 0$ for all $n \in \mathbb{N}$ then system (2) becomes the discrete single-species system without feedback controls

$$x(n+1) = x(n) \exp\left\{ r(n) - \sum_{j=1}^{m} a_j(n) x(n - \tau_j(n)) \right\}. \tag{40}$$

Using arguments similar to the proofs of Theorems 1 and 2, we can prove the following results.

**Theorem 3.** *Assume that* $(\mathrm{H}_1)$, $(\mathrm{H}_2)$, *and* $(\mathrm{H}_4)$ *hold. Then species* $x$ *in system* (40) *is permanent.*

**Theorem 4.** *Assume that* $(\mathrm{H}_1)$, $(\mathrm{H}_3)$, *and* $(\mathrm{H}_4)$ *hold. Then species* $x$ *in system* (40) *is extinct.*

Particularly, as consequences of Theorems 3 and 4, for system (40) with periodic coefficients we have the following corollaries.

**Corollary 4.** *Assume that system* (40) *is periodic and* $(\mathrm{H}_1')$ *and* $(\mathrm{H}_4')$ *hold. Then*

(1) *Species* $x$ *is permanent if and only if* $(\mathrm{H}_2')$ *holds.*
(2) *Species* $x$ *is extinct if and only if* $(\mathrm{H}_3')$ *holds.*

R e m a r k 4. Comparing Theorems 3 and 4 and Corollary 4 with the corresponding results which are obtained for the continuous time single-species logistic systems (see Theorems 2–4 in [35]), we easily see that Theorems 3 and 4, and Corollary 4 are extensions of the corresponding results for the continuous time single-species systems to discrete time single-species systems.

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