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TABLES OF THE BRILLOUIN FUNCTION

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The Brillouin function $B_J(x)$ is tabulated for $J = n/2$ ($n = 1, 2, \dots, 20; \infty$).

The origin of these tables was stimulated by the need of the authors of sufficiently comprehensive tables of the Brillouin function

$$B_J(x) = \frac{2J+1}{2J} \coth \frac{2J+1}{2J} x - \frac{1}{2J} \coth \frac{x}{2J}$$

when calculating the spontaneous magnetization of thin films and other magnetic materials.

Nevertheless, such tables are useful not only in ferromagnetism, but also for analogous problems in ferroelectricity, in the theory of electric and magnetic susceptibilities, in the theory of order-disorder phenomena, magnetochemistry etc.

Since there was a lack of numerically evaluated data of $B_J(x)$ (despite of some partial results of insufficient extent (see e.g. /1/, /2/) for what is frequently needed) and only a few numerical data and graphs may be found dispersed in literature (see e.g. /3/, /4/), the authors decided in 1965 to evaluate $B_J(x)$ for $J = n/2$ ($n = 1, 2, \dots, 20; \infty$) for sufficiently small intervals of x and a sufficient number of decimal places.

The calculations have been performed by the computer URAL II with the accuracy of eight places. The results have been rounded to five decimal places for practical use.

To get some more insight in the dependence on x for a wide range of x the results have been plotted graphically and some of them added to the tables presented in this work.

In practice, mostly the values of $J = n + 1/2$ for $n = 1, 2, \dots, 7$ and ∞ are needed. Therefore, a more dense selection of values for $B_J(x)$ has been realized for those J 's and the values of $B_J(x)$ for J from $J = 4$ to $J = 10$ are presented for a reduced selection of the argument x .

In these tables the mostly used name of "Brillouin function" has been used for $B_J(x)$. Nevertheless, let us remark that other names, namely the "Langevin function" ($L_{1/2}(x), L_1(x), \dots, L(x)$) or the "Generalized Langevin function" or the "Brillouin-Debye" function are used.

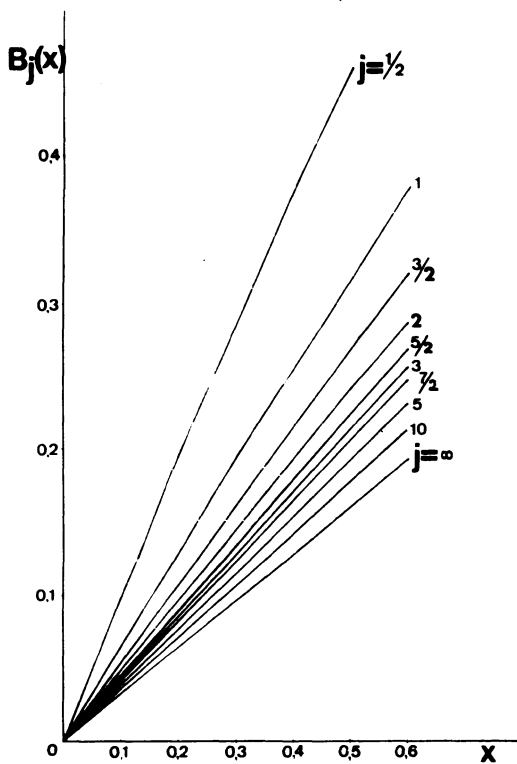


Fig. 1.

The tables which were finished in autumn 1965, were originally destined for the use in the authors' research work and were given only at the disposal of some research fellows of the authors from other laboratories.

In the time between the completion of our tables and their edition some further numerical data on $B_J(x)$ have been published by J.S. Smart /5/ and M.I. Darby /6/. The comparison of those tables with ours, similarly as the comparison with the table of $B_\infty(x)$ in /2/ shows that our tables remain still the most complete ones as to the density of x 's and the range of J 's. Moreover, our work contains some graphs for $B_J(x)$ which are not to be found in the

papers and books mentioned above. This is one of other reasons why we decided to publish our tables.

Comparing our values of $B_j(x)$ for x 's with these which may be found in the tables of other authors we have found no differences.

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x	$B_{1/2}(x)$	$B_1(x)$	$B_{3/2}(x)$	$B_2(x)$
0.00	0.00000	0.00000	0.00000	0.00000
.01	.01000	.00667	.00556	.00500
.02	.02000	.01333	.01111	.01000
.03	.02999	.02000	.01666	.01500
.04	.03998	.02666	.02222	.02000
.05	.04996	.03332	.02777	.02499
.06	.05993	.03998	.03332	.02999
.07	.06989	.04663	.03886	.03498
.08	.07983	.05328	.04441	.03997
.09	.08976	.05992	.04995	.04496
0.10	0.09967	0.06656	0.05549	0.04995
.11	.10956	.07319	.06102	.05493
.12	.11943	.07981	.06655	.05991
.13	.12927	.08642	.07207	.06488
.14	.13909	.09303	.07759	.06985
.15	.14889	.09963	.08310	.07482
.16	.15865	.10621	.08860	.07978
.17	.16838	.11279	.09410	.08474
.18	.17808	.11936	.09959	.08969
.19	.18775	.12591	.10508	.09463
0.20	0.19738	0.13245	0.11056	0.09957
.21	.20697	.13898	.11602	.10450
.22	.21652	.14550	.12148	.10943
.23	.22603	.15200	.12693	.11435
.24	.23550	.15848	.13238	.11926
.25	.24492	.16495	.13781	.12416
.26	.25430	.17141	.14323	.12906
.27	.26362	.17785	.14864	.13395
.28	.27291	.18427	.15404	.13882
.29	.28213	.19067	.15943	.14370
0.30	0.29131	0.19706	0.16481	0.14856
.31	.30044	.20342	.17017	.15341
.32	.30951	.20977	.17552	.15825
.33	.31852	.21610	.18086	.16308
.34	.32748	.22241	.18619	.16791
0.35	0.33638	0.22869	0.19151	0.17272

x	$B_{5/2}(x)$	$B_3(x)$	$B_{7/2}(x)$	$L(x)$
0.00	0.00000	0.00000	0.00000	0.00000
.01	.00467	.00446	.00428	.00333
.02	.00933	.00889	.00857	.00667
.03	.01400	.01333	.01285	.01000
.04	.01866	.01778	.01714	.01333
.05	.02333	.02222	.02142	.01666
.06	.02799	.02666	.02571	.02000
.07	.03265	.03110	.02999	.02333
.08	.03731	.03553	.03427	.02666
.09	.04197	.03997	.03854	.02998
0.10	0.04662	0.04440	0.04282	0.03331
.11	.05127	.04883	.04709	.03664
.12	.05592	.05326	.05136	.03996
.13	.06057	.05769	.05563	.04328
.14	.06521	.06211	.05990	.04661
.15	.06985	.06653	.06416	.04993
.16	.07448	.07094	.06842	.05324
.17	.07911	.07535	.07267	.05656
.18	.08373	.07976	.07692	.05987
.19	.08835	.08416	.08117	.06318
0.20	0.09297	0.08856	0.08541	0.06649
.21	.09758	.09295	.08965	.06980
.22	.10218	.09734	.09388	.07310
.23	.10678	.10172	.09811	.07640
.24	.11137	.10610	.10234	.07969
.25	.11595	.11047	.10656	.08299
.26	.12053	.11484	.11077	.08628
.27	.12510	.11920	.11498	.08957
.28	.12967	.12355	.11918	.09285
.29	.13422	.12790	.12337	.09613
0.30	0.13877	0.13224	0.12756	0.09941
.31	.14331	.13657	.13174	.10268
.32	.14785	.14089	.13592	.10595
.33	.15237	.14521	.14008	.10921
.34	.15689	.14952	.14425	.11247
0.35	0.16139	0.15382	0.14840	0.11572

x	$B_{1/2}(x)$	$B_1(x)$	$B_{3/2}(x)$	$B_2(x)$
0.35	0.33638	0.22869	0.19151	0.17272
.36	.34521	.23496	.19681	.17752
.37	.35399	.24120	.20209	.18231
.38	.36271	.24742	.20736	.18709
.39	.37136	.25362	.21262	.19186
0.40	0.37995	0.25979	0.21786	0.19661
.41	.38847	.26594	.22309	.20136
.42	.39693	.27207	.22830	.20609
.43	.40532	.27817	.23350	.21081
.44	.41364	.28425	.23867	.21552
.45	.42190	.29030	.24384	.22021
.46	.43008	.29632	.24898	.22489
.47	.43820	.30232	.25411	.22956
.48	.44624	.30830	.25922	.23421
.49	.45422	.31424	.26431	.23885
0.50	0.46212	0.32016	0.26939	0.24347
.51	.46995	.32605	.27445	.24808
.52	.47770	.33191	.27948	.25268
.53	.48538	.33774	.28450	.25726
.54	.49299	.34354	.28950	.26183
.55	.50052	.34931	.29448	.26638
.56	.50798	.35506	.29945	.27091
.57	.51536	.36077	.30439	.27543
.58	.52267	.36646	.30931	.27993
.59	.52990	.37211	.31421	.28442
0.60	0.53705	0.37773	0.31909	0.28889
.61	.54413	.38332	.32395	.29335
.62	.55113	.38888	.32879	.29779
.63	.55805	.39441	.33361	.30221
.64	.56490	.39991	.33841	.30662
.65	.57167	.40537	.34318	.31100
.66	.57836	.41080	.34794	.31537
.67	.58498	.41620	.35267	.31973
.68	.59152	.42157	.35738	.32406
.69	.59798	.42690	.36207	.32838
0.70	0.60437	0.43220	0.36673	0.33268

x	$B_{5/2}(x)$	$B_3(x)$	$B_{7/2}(x)$	$L(x)$
0.35	0.16139	0.15382	0.14840	0.11572
.36	.16589	.15811	.15255	.11898
.37	.17038	.16240	.15668	.12222
.38	.17486	.16667	.16081	.12546
.39	.17932	.17094	.16494	.12870
0.40	0.18378	0.17520	.16905	0.13193
.41	.18823	.17945	.17316	.13516
.42	.19267	.18369	.17725	.13838
.43	.19710	.18792	.18134	.14160
.44	.20151	.19214	.18542	.14481
.45	.20592	.19635	.18949	.14801
.46	.21031	.20055	.19355	.15121
.47	.21469	.20474	.19760	.15441
.48	.21906	.20891	.20164	.15760
.49	.22342	.21308	.20567	.16078
0.50	0.22777	0.21724	0.20969	0.16395
.51	.23210	.22139	.21370	.16712
.52	.23642	.22552	.21770	.17029
.53	.24073	.22964	.22169	.17344
.54	.24503	.23376	.22567	.17660
.55	.24931	.23786	.22964	.17974
.56	.25358	.24194	.23359	.18288
.57	.25784	.24602	.23754	.18601
.58	.26208	.25008	.24147	.18913
.59	.26631	.25413	.24540	.19225
0.60	0.27052	0.25817	0.24931	0.19536
.61	.27472	.26220	.25321	.19846
.62	.27891	.26621	.25709	.20156
.63	.28308	.27021	.26097	.20465
.64	.28724	.27420	.26483	.20773
.65	.29138	.27817	.26868	.21080
.66	.29551	.28213	.27252	.21387
.67	.29962	.28608	.27635	.21692
.68	.30372	.29001	.28016	.21997
.69	.30780	.29393	.28396	.22302
0.70	0.31187	0.29784	0.28774	0.22605

x	$B_{1/2}(x)$	$B_1(x)$	$B_{3/2}(x)$	$B_2(x)$
0.70	0.60437	0.43220	0.36673	0.33268
.71	.61068	.43747	.37137	.33696
.72	.61691	.44270	.37599	.34123
.73	.62307	.44790	.38059	.34547
.74	.62915	.45306	.38517	.34970
.75	.63515	.45820	.38972	.35391
.76	.64108	.46329	.39425	.35810
.77	.64693	.46836	.39875	.36227
.78	.65271	.47339	.40323	.36642
.79	.65841	.47838	.40769	.37056
0.80	0.66404	0.48334	0.41212	0.37467
.81	.66959	.48827	.41653	.37877
.82	.67507	.49316	.42092	.38284
.83	.68048	.49801	.42528	.38690
.84	.68581	.50283	.42962	.39094
.85	.69107	.50762	.43393	.39495
.86	.69626	.51237	.43822	.39895
.87	.70137	.51709	.44249	.40293
.88	.70642	.52177	.44673	.40689
.89	.71139	.52641	.45095	.41083
0.90	0.71630	0.53102	0.45514	0.41474
.91	.72113	.53560	.45930	.41864
.92	.72590	.54014	.46345	.42252
.93	.73059	.54465	.46756	.42638
.94	.73522	.54912	.47166	.43022
.95	.73978	.55355	.47573	.43403
.96	.74428	.55796	.47977	.43783
.97	.74870	.56232	.48379	.44161
.98	.75307	.56665	.48778	.44537
.99	.75736	.57095	.49175	.44910
1.00	0.76159	0.57521	0.49569	0.45282
.01	.76576	.57944	.49961	.45651
.02	.76987	.58363	.50351	.46019
.03	.77391	.58779	.50737	.46384
.04	.77789	.59191	.51122	.46747
1.05	0.78181	0.59600	0.51504	0.47109

x	$B_{5/2}(x)$	$B_3(x)$	$B_{7/2}(x)$	$L(x)$
0.70	0.31187	0.29784	0.28774	0.22605
.71	.31592	.30173	.29152	.22908
.72	.31995	.30560	.29528	.23209
.73	.32397	.30947	.29903	.23511
.74	.32797	.31332	.30276	.23811
.75	.33196	.31715	.30648	.24110
.76	.33593	.32097	.31019	.24409
.77	.33989	.32477	.31388	.24706
.78	.34383	.32856	.31756	.25003
.79	.34775	.33234	.32123	.25299
0.80	0.35165	0.33609	0.32488	0.25594
.81	.35554	.33984	.32852	.25888
.82	.35942	.34357	.33214	.26182
.83	.36327	.34728	.33575	.26474
.84	.36711	.35098	.33934	.26766
.85	.37093	.35466	.34292	.27056
.86	.37473	.35833	.34649	.27346
.87	.37852	.36198	.35004	.27635
.88	.38229	.36562	.35357	.27923
.89	.38604	.36923	.35710	.28210
0.90	0.38978	0.37284	0.36060	0.28496
.91	.39349	.37643	.36409	.28781
.92	.39719	.38000	.36757	.29065
.93	.40087	.38355	.37103	.29348
.94	.40454	.38709	.37448	.29630
.95	.40818	.39062	.37791	.29912
.96	.41181	.39412	.38133	.30192
.97	.41542	.39762	.38473	.30471
.98	.41902	.40109	.38812	.30750
.99	.42259	.40455	.39149	.31027
1.00	0.42615	0.40799	0.39484	0.31304
.01	.42969	.41142	.39818	.31579
.02	.43321	.41482	.40151	.31853
.03	.43671	.41822	.40482	.32127
.04	.44019	.42159	.40811	.32399
.05	0.44366	0.42495	0.41139	0.32671

x	$B_{1/2}(x)$	$B_1(x)$	$B_{3/2}(x)$	$B_2(x)$
1.05	0.78181	0.59600	0.51504	0.47109
.06	.78566	.60005	.51883	.47468
.07	.78946	.60407	.52260	.47825
.08	.79320	.60806	.52634	.48180
.09	.79688	.61201	.53006	.48533
1.10	0.80050	0.61593	0.53376	0.48884
.11	.80406	.61981	.53743	.49233
.12	.80757	.62366	.54107	.49580
.13	.81102	.62748	.54469	.49925
.14	.81441	.63126	.54829	.50267
.15	.81775	.63501	.55186	.50608
.16	.82104	.63873	.55540	.50947
.17	.82427	.64241	.55892	.51283
.18	.82745	.64606	.56242	.51618
.19	.83058	.64968	.56589	.51950
1.20	0.83365	0.65326	0.56934	0.52280
.21	.83668	.65681	.57276	.52609
.22	.83965	.66033	.57616	.52935
.23	.84258	.66382	.57954	.53259
.24	.84546	.66728	.58289	.53581
.25	.84828	.67070	.58621	.53902
.26	.85106	.67409	.58952	.54220
.27	.85380	.67745	.59279	.54536
.28	.85648	.68078	.59605	.54850
.29	.85913	.68408	.59928	.55162
1.30	0.86172	0.68735	0.60249	0.55472
.31	.86428	.69059	.60567	.55780
.32	.86678	.69379	.60883	.56086
.33	.86925	.69697	.61196	.56390
.34	.87167	.70011	.61508	.56692
.35	.87405	.70323	.61816	.56992
.36	.87639	.70631	.62123	.57290
.37	.87869	.70937	.62427	.57586
.38	.88095	.71240	.62729	.57880
.39	.88317	.71539	.63029	.58173
1.40	0.88535	0.71836	0.63326	0.58463

x	$B_{5/2}(x)$	$B_3(x)$	$B_{7/2}(x)$	L(x)
1.05	0.44366	0.42495	0.41139	0.32671
.06	.44711	.42829	.41465	.32941
.07	.45054	.43162	.41790	.33211
.08	.45395	.43493	.42113	.33479
.09	.45734	.43822	.42434	.33747
1.10	0.46072	0.44150	0.42754	0.34013
.11	.46407	.44476	.43073	.34278
.12	.46741	.44800	.43389	.34543
.13	.47073	.45122	.43705	.34806
.14	.47404	.45443	.44018	.35068
.15	.47732	.45762	.44330	.35330
.16	.48058	.46080	.44641	.35590
.17	.48383	.46396	.44950	.35849
.18	.48706	.46710	.45257	.36107
.19	.49027	.47022	.45563	.36364
1.20	0.49346	0.47333	0.45867	0.36620
.21	.49664	.47642	.46170	.36875
.22	.49979	.47949	.46471	.37129
.23	.50293	.48255	.46770	.37382
.24	.50605	.48559	.47068	.37634
.25	.50915	.48861	.47365	.37885
.26	.51224	.49162	.47659	.38135
.27	.51530	.49461	.47952	.38384
.28	.51835	.49759	.48244	.38631
.29	.52138	.50054	.48534	.38878
1.30	0.52439	0.50348	0.48822	0.39123
.31	.52738	.50641	.49109	.39368
.32	.53035	.50931	.49394	.39611
.33	.53331	.51220	.49678	.39854
.34	.53625	.51508	.49960	.40095
.35	.53917	.51793	.50241	.40335
.36	.54207	.52077	.50520	.40575
.37	.54496	.52360	.50797	.40813
.38	.54783	.52640	.51073	.41050
.39	.55068	.52920	.51348	.41286
1.40	0.55351	0.53197	0.51620	0.41521

x	$B_{1/2}(x)$	$B_1(x)$	$B_{3/2}(x)$	$B_2(x)$
1.40	0.88535	0.71836	0.63326	0.58463
.41	.88749	.72130	.63621	.58751
.42	.88960	.72421	.63914	.59037
.43	.89167	.72709	.64205	.59322
.44	.89370	.72995	.64493	.59604
.45	.89569	.73277	.64779	.59884
.46	.89765	.73557	.65063	.60163
.47	.89958	.73834	.65345	.60440
.48	.90147	.74108	.65624	.60714
.49	.90332	.74380	.65902	.60987
1.50	0.90515	0.74648	0.66177	0.61258
.51	.90694	.74915	.66450	.61527
.52	.90870	.75178	.66721	.61795
.53	.91042	.75439	.66989	.62060
.54	.91212	.75697	.67256	.62323
.55	.91379	.75953	.67520	.62585
.56	.91542	.76206	.67783	.62845
.57	.91703	.76456	.68043	.63103
.58	.91860	.76704	.68301	.63359
.59	.92015	.76949	.68557	.63613
1.60	0.92167	0.77192	0.68811	0.63866
.61	.92316	.77433	.69063	.64117
.62	.92462	.77671	.69313	.64366
.63	.92606	.77906	.69561	.64613
.64	.92747	.78139	.69807	.64858
.65	.92886	.78370	.70051	.65102
.66	.93022	.78599	.70293	.65344
.67	.93155	.78825	.70533	.65584
.68	.93286	.79048	.70771	.65823
.69	.93415	.79270	.71007	.66059
1.70	0.93541	0.79489	0.71241	0.66924
.71	.93665	.79705	.71473	.66528
.72	.93786	.79920	.71703	.66759
.73	.93906	.80132	.71932	.66989
.74	.94023	.80342	.72158	.67218
1.75	0.94138	0.80550	0.72383	0.67444

x	$B_{5/2}(x)$	$B_3(x)$	$B_{7/2}(x)$	L(x)
1.40	0.55351	0.53197	0.51620	0.41521
.41	.55632	.53473	.51892	.41755
.42	.55912	.53747	.52161	.41988
.43	.56190	.54020	.52429	.42219
.44	.56466	.54291	.52696	.42450
.45	.56741	.54560	.52961	.42680
.46	.57013	.54828	.53225	.42909
.47	.57284	.55094	.53487	.43136
.48	.57554	.55358	.53747	.43363
.49	.57821	.55621	.54006	.43588
1.50	0.58087	0.55882	0.54264	0.43812
.51	.58351	.56142	.54520	.44036
.52	.58614	.56400	.54774	.44258
.53	.58874	.56657	.55027	.44479
.54	.59134	.56912	.55278	.44700
.55	.59391	.57165	.55528	.44919
.56	.59647	.57417	.55777	.45137
.57	.59901	.57667	.56024	.45354
.58	.60153	.57916	.56269	.45570
.59	.60404	.58163	.56513	.45785
1.60	0.60653	0.58409	0.56756	0.45999
.61	.60900	.58653	.56997	.46212
.62	.61146	.58895	.57237	.46424
.63	.61390	.59136	.57475	.46634
.64	.61633	.59376	.57712	.46844
.65	.61874	.59614	.57947	.47053
.66	.62113	.59850	.58181	.47261
.67	.62351	.60085	.58413	.47468
.68	.62587	.60319	.58644	.47673
.69	.62822	.60551	.58874	.47878
1.70	0.63055	0.60781	0.59102	0.48082
.71	.63286	.61010	.59329	.48284
.72	.63516	.61238	.59554	.48486
.73	.63744	.61464	.59778	.48686
.74	.63971	.61689	.60001	.48886
1.75	0.64196	0.61912	0.60222	0.49085

x	$B_{1/2}(x)$	$B_1(x)$	$B_{3/2}(x)$	$B_2(x)$
1.75	0.94138	0.80550	0.72383	0.67444
.76	.94250	.80756	.72606	.67669
.77	.94361	.80960	.72827	.67892
.78	.94470	.81161	.73046	.68114
.79	.94576	.81360	.73263	.68334
1.80	0.94681	0.81558	0.73479	0.68553
.81	.94783	.81753	.73692	.68769
.82	.94884	.81946	.73904	.68985
.83	.94983	.82137	.74115	.69198
.84	.95080	.82326	.74323	.69410
.85	.95175	.82513	.74530	.69621
.86	.95268	.82699	.74735	.69830
.87	.95359	.82882	.74938	.70037
.88	.95449	.83063	.75139	.70243
.89	.95537	.83242	.75339	.70447
1.90	0.95624	0.83420	0.75538	0.70650
.91	.95709	.83595	.75734	.70851
.92	.95792	.83769	.75929	.71051
.93	.95873	.83941	.76122	.71250
.94	.95953	.84111	.76314	.71446
.95	.96032	.84279	.76504	.71642
.96	.96109	.84446	.76692	.71836
.97	.96185	.84610	.76879	.72028
.98	.96259	.84773	.77064	.72219
.99	.96331	.84934	.77248	.72409
2.00	0.96403	0.85094	0.77430	0.72597
.05	.96740	.85866	.78318	.73517
.10	.97045	.86597	.79169	.74403
.15	.97323	.87291	.79985	.75256
.20	.97574	.87948	.80766	.76078
.25	.97803	.88570	.81515	.76869
.30	.98010	.89160	.82232	.77631
.35	.98197	.89718	.82920	.78364
.40	.98367	.90247	.83579	.79071
.45	.98522	.90749	.84211	.79751
2.50	0.98661	0.91223	0.84816	0.80406

x	$B_{5/2}(x)$	$B_3(x)$	$B_{7/2}(x)$	L(x)
1.75	0.64196	0.61912	0.60222	0.49085
.76	.64420	.62134	.60442	.49282
.77	.64642	.62354	.60660	.49479
.78	.64863	.62573	.60877	.49674
.79	.65082	.62790	.61093	.49869
1.80	0.65300	0.63007	0.61307	0.50063
.81	.65516	.63221	.61520	.50255
.82	.65731	.63435	.61732	.50447
.83	.65944	.63647	.61942	.50638
.84	.66156	.63857	.62152	.50827
.85	.66366	.64066	.62359	.51016
.86	.66575	.64274	.62566	.51204
.87	.66783	.64480	.62771	.51390
.88	.66989	.64685	.62975	.51576
.89	.67193	.64889	.63177	.51761
1.90	0.67397	0.65092	0.63378	0.51945
.91	.67598	.65293	.63578	.52128
.92	.67799	.65492	.63777	.52310
.93	.67998	.65691	.63974	.52491
.94	.68195	.65888	.64171	.52671
.95	.68392	.66084	.64365	.52850
.96	.68587	.66278	.64559	.53028
.97	.68780	.66471	.64751	.53205
.98	.68972	.66663	.64943	.53382
.99	.69163	.66854	.65133	.53557
2.00	0.69353	0.67043	0.65321	0.53731
.05	.70281	.67972	.66247	.54590
.10	.71177	.68869	.67143	.55426
.15	.72042	.69738	.68011	.56239
.20	.72878	.70577	.68851	.57031
.25	.73684	.71389	.69664	.57802
.30	.74463	.72174	.70451	.58553
.35	.75214	.72933	.71213	.59283
.40	.75940	.73667	.71951	.59993
.45	.76641	.74377	.72665	.60684
2.50	0.77317	0.75063	0.73356	0.61357

x	$B_{1/2}(x)$	$B_1(x)$	$B_{3/2}(x)$	$B_2(x)$
2.50	0.98661	0.91223	0.84816	0.80406
.55	.98788	.91673	.85396	.81037
.60	.98903	.92100	.85952	.81645
.65	.99007	.92504	.86486	.82231
.70	.99101	.92886	.86997	.82795
.75	.99186	.93249	.87487	.83339
.80	.99263	.93593	.87958	.83863
.85	.99333	.93919	.88409	.84368
.90	.99396	.94227	.88841	.84855
.95	.99454	.94520	.89257	.85325
3.00	0.99505	0.94797	0.89655	0.85778
.05	.99552	.95061	.90037	.86214
.10	.99595	.95310	.90405	.86636
.15	.99633	.95547	.90757	.87043
.20	.99668	.95771	.91095	.87435
.25	.99700	.95984	.91421	.87814
.30	.99728	.96185	.91733	.88180
.35	.99754	.96377	.92033	.88534
.40	.99777	.96559	.92321	.88875
.45	.99799	.96731	.92598	.89205
3.50	0.99818	0.96894	0.92865	0.89523
.55	.99835	.97050	.93121	.89831
.60	.99851	.97197	.93367	.90129
.65	.99865	.97337	.93604	.90417
.70	.99878	.97470	.93831	.90695
.75	.99889	.97595	.94050	.90965
.80	.99900	.97715	.94261	.91225
.85	.99909	.97829	.94464	.91477
.90	.99918	.97936	.94659	.91721
.95	.99926	.98039	.94847	.91957
4.00	0.99933	0.98136	0.95028	0.92185
.05	.99939	.98228	.95202	.92407
.10	.99945	.98316	.95370	.92621
.15	.99950	.98399	.95532	.92829
.20	.99955	.98479	.95687	.93030
4.25	0.99959	0.98554	0.95837	0.93224

x	$B_{5/2}(x)$	$B_3(x)$	$B_{7/2}(x)$	L(x)
2.50	0.77317	0.75063	0.73356	0.61357
.55	.77971	.75727	.74025	.62011
.60	.78602	.76370	.74673	.62648
.65	.79211	.76991	.75300	.63267
.70	.79800	.77592	.75908	.63870
.75	.80369	.78174	.76497	.64457
.80	.80918	.78736	.77067	.65028
.85	.81449	.79281	.77620	.65584
.90	.81963	.79809	.78155	.66125
.95	.82459	.80319	.78674	.66651
3.00	0.82939	0.80814	0.79177	0.67164
.05	.83403	.81293	.79664	.67663
.10	.83852	.81757	.80137	.68149
.15	.84286	.82206	.80596	.68622
.20	.84706	.82642	.81041	.69083
.25	.85113	.83064	.81473	.69532
.30	.85507	.83474	.81892	.69969
.35	.85888	.83871	.82298	.70396
.40	.86257	.84256	.82693	.70811
.45	.86615	.84629	.83076	.71216
3.50	0.86962	0.84992	0.83449	0.71611
.55	.87297	.85344	.83811	.71996
.60	.87623	.85685	.84162	.72372
.65	.87938	.86017	.84504	.72738
.70	.88244	.86339	.84836	.73095
.75	.88541	.86652	.85159	.73444
.80	.88829	.86956	.85473	.73784
.85	.89108	.87251	.85778	.74117
.90	.89379	.87538	.86076	.74441
.95	.89642	.87817	.86365	.74758
4.00	0.89897	0.88089	0.86647	0.75067
.05	.90145	.88353	.86921	.75369
.10	.90386	.88610	.87188	.75665
.15	.90620	.88859	.87448	.75953
.20	.90847	.89103	.87701	.76235
4.25	0.91068	0.89339	0.87948	0.76511

x	$B_{1/2}(x)$	$B_1(x)$	$B_{3/2}(x)$	$B_2(x)$
4.25	0.99959	0.98554	0.95837	0.93224
.30	.99963	.98625	.95981	.93413
.35	.99967	.98693	.96121	.93596
.40	.99970	.98758	.96255	.93774
.45	.99973	.98819	.96384	.93946
4.50	0.99975	0.98877	0.96509	0.94112
.55	.99978	.98932	.96629	.94274
.60	.99980	.98985	.96744	.94431
.65	.99982	.99035	.96856	.94583
.70	.99983	.99082	.96964	.94731
.75	.99985	.99127	.97068	.94874
.80	.99986	.99170	.97168	.95013
.85	.99988	.92211	.97264	.95148
.90	.99989	.99250	.97358	.95279
.95	.99990	.92287	.97447	.95406
5.00	0.99991	0.99322	0.97534	0.95530
.10	.99993	.99387	.97699	.95766
.20	.99994	.99445	.97852	.95989
.30	.99995	.99498	.97994	.96199
.40	.99996	.99546	.98127	.96398
.50	.99997	.99590	.98251	.96586
.60	.99997	.99629	.98367	.96763
.70	.99998	.99664	.98475	.96930
.80	.99998	.99696	.98575	.97089
.90	.99998	.99725	.98669	.97239
6.00	0.99999	0.99752	0.98756	0.97380
.10	.99999	.99775	.98838	.97514
.20	.99999	.99797	.98914	.97641
.30	.99999	.99816	.98985	.97762
.40	.99999	.99834	.99051	.97875
.50	1.00000	0.99849	0.99113	0.97983
.60		.99864	.99171	.98085
.70		.99877	.99225	.98182
.80		.99888	.99276	.98274
.90		.99899	.99323	.98361
7.00		0.99909	0.99367	0.98443

x	$B_{5/2}(x)$	$B_3(x)$	$B_{7/2}(x)$	$L(x)$
4.25	0.91068	0.89339	0.87948	0.76511
.30	.91283	.89570	.88188	.76781
.35	.91492	.89794	.88422	.77045
.40	.91694	.90012	.88651	.77303
.45	.91892	.90225	.88874	.77555
4.50	0.92084	0.90433	0.89091	0.77802
.55	.92270	.90635	.89303	.78044
.60	.92452	.90831	.89510	.78281
.65	.92628	.91023	.89712	.78513
.70	.92800	.91210	.89909	.78740
.75	.92968	.91393	.90101	.78962
.80	.93131	.91571	.90289	.79180
.85	.93289	.91744	.90472	.79394
.90	.93444	.91914	.90651	.79603
.95	.93594	.92079	.90826	.79808
5.00	0.93741	0.92240	0.90997	0.80009
.10	.94023	.92551	.91327	.80400
.20	.94290	.92847	.91643	.80775
.30	.94545	.93130	.91944	.81137
.40	.94786	.93400	.92233	.81486
.50	.95016	.93657	.92509	.81822
.60	.95235	.93903	.92773	.82146
.70	.95443	.94138	.93026	.82458
.80	.95641	.94363	.93269	.82760
.90	.95830	.94577	.93501	.83052
6.00	0.96009	0.94783	0.93725	0.83335
.10	.96181	.94980	.93939	.83608
.20	.96344	.95168	.94144	.83872
.30	.96500	.95349	.94342	.84128
.40	.96649	.95522	.94532	.84376
.50	.96791	.95687	.94714	.84616
.60	.96926	.95846	.94890	.84849
.70	.97056	.95999	.95059	.85075
.80	.97179	.96145	.95221	.85294
.90	.97297	.96286	.95378	.85507
7.00	0.97410	0.96421	0.95528	0.85714

x	$B_{1/2}(x)$	$B_1(x)$	$B_{3/2}(x)$	$B_2(x)$
7.00		0.99909	0.99367	0.98443
.10		.99917	.99408	.98521
.20		.99925	.99447	.98595
.30		.99932	.99483	.98666
.40		.99939	.99516	.98732
.50		.99945	.99548	.98796
.60		.99950	.99577	.98856
.70		.99955	.99605	.98913
.80		.99959	.99630	.98967
.90		.99963	.99654	.99018
8.00		0.99966	0.99677	0.99067
.20		.99973	.99717	.99157
.40		.99978	.99753	.99239
.60		.99982	.99784	.99312
.80		.99985	.99811	.99379
9.00		0.99988	0.99834	0.99438
.20		.99990	.99855	.99492
.40		.99992	.99873	.99541
.60		.99993	.99889	.99585
.80		.99994	.99903	.99625
10.00		0.99995	0.99915	0.99661
.50		.99997	.99939	.99736
11.00		.99998	.99956	.99795
.50		.99999	.99969	.99840
12.00		.99999	.99978	.99876
.50		1.00000	0.99984	0.99903
13.00			.99989	.99925
.50			.99992	.99941
14.00			.99994	.99954
.50			.99996	.99964
15.00			0.99997	0.99972
.50			.99998	.99978
16.00			.99998	.99983
.50			.99999	.99987
17.00			.99999	.99990
.50			.99999	.99992
18.00			1.00000	0.99994

x	$B_{5/2}(x)$	$B_3(x)$	$B_{7/2}(x)$	L(x)
7.00	0.97410	0.96421	0.95528	0.85714
.10	.97518	.96550	.95673	.85916
.20	.97621	.96674	.95813	.86111
.30	.97720	.96794	.95948	.86301
.40	.97814	.96909	.96077	.86487
.50	.97904	.97019	.96203	.86667
.60	.97990	.97125	.96323	.86842
.70	.98073	.97227	.96440	.87013
.80	.98152	.97326	.96552	.87180
.90	.98228	.97420	.96661	.87342
8.00	0.98300	0.97511	0.96765	0.87500
.20	.98436	.97683	.96964	.87805
.40	.98561	.97842	.97149	.88095
.60	.98675	.97989	.97323	.88372
.80	.98780	.98126	.97484	.88636
9.00	0.98876	0.98253	0.97636	0.88889
.20	.98965	.98372	.97777	.89130
.40	.99046	.98481	.97910	.89362
.60	.99121	.98584	.98034	.89583
.80	.99190	.98678	.98150	.89796
10.00	0.99254	0.98767	0.98259	0.90000
.50	.99391	.98962	.98503	.90476
11.00	.99503	.99125	.98711	.90909
.50	.99594	.99263	.98889	.91304
12.00	.99668	.99378	.99042	.91667
.50	0.99729	0.99475	0.99173	0.92000
13.00	.99778	.99557	.99286	.92308
.50	.99819	.99626	.99383	.92593
14.00	.99852	.99684	.99467	.92857
.50	.99879	.99733	.99539	.93103
15.00	0.99901	0.99774	0.99601	0.93333
.50	.99919	.99809	.99655	.93548
16.00	.99933	.99838	.99701	.93750
.50	.99946	.99863	.99742	.93939
17.00	.99955	.99884	.99776	.94118
.50	.99963	.99902	.99806	.94286
18.00	0.99970	0.99917	0.99832	0.94444

x	$B_2(x)$	$B_{5/2}(x)$	$B_3(x)$	$B_{7/2}(x)$	$L(x)$
18.00	0.99994	0.99970	0.99917	0.99832	0.94444
.50	.99995	.99976	.99930	.99855	.94595
19.00	.99996	.99980	.99941	.99874	.94737
.50	.99997	.99984	.99950	.99891	.94872
20.00	0.99998	0.99987	0.99958	0.99905	0.95000

x	$L(x)$	x	$L(x)$
20.00	0.95000	50.00	0.98000
22.00	.95455	55.00	.98182
24.00	.95833	60.00	.98333
26.00	.96154	65.00	.98462
28.00	.96429	70.00	.98571
30.00	0.96667	75.00	0.98667
32.00	.96875	80.00	.98750
34.00	.97059	85.00	.98824
36.00	.97222	90.00	.98889
38.00	.97368	95.00	.98947
40.00	0.97500	100.00	0.99000
42.00	.97619	500.00	.99800
44.00	.97727	10^3	.99900
46.00	.97826	10^4	.99990
48.00	.97917	10^5	.99999
50.00	0.98000	10^6	1.00000

x	$B_4(x)$	$B_{9/2}(x)$	$B_5(x)$	$B_{11/2}(x)$
0.0	0.00000	0.00000	0.00000	0.00000
0.1	.04163	.04071	.03997	.03936
0.2	.08305	.08121	.07974	.07854
0.3	.12405	.12132	.11913	.11734
0.4	.16443	.16084	.15796	.15560
0.5	.20401	.19959	.19605	.19314
0.6	.24263	.23743	.23325	.22983
0.7	.28014	.27420	.26944	.26553
0.8	.31641	.30980	.30448	.30012
0.9	.35135	.34412	.33830	.33352
1.0	0.38489	0.37709	0.37081	0.36566
1.1	.41696	.40865	.40197	.39647
1.2	.44753	.43878	.43173	.42592
1.3	.47660	.46746	.46009	.45401
1.4	.50417	.49469	.48703	.48072
1.5	.53026	.52049	.51259	.50607
1.6	.55489	.54489	.53678	.53008
1.7	.57812	.56792	.55964	.55279
1.8	.60000	.58963	.58121	.57424
1.9	.62057	.61008	.60155	.59448
2.0	0.63990	0.62931	0.62070	0.61355
2.2	.67510	.66441	.65568	.64842
2.4	.70612	.69540	.68662	.67932
2.6	.73344	.72276	.71400	.70669
2.8	.75753	.74694	.73823	.73095
3.0	0.77882	0.76835	0.75972	0.75249
3.5	.82214	.81207	.80371	.79667
4.0	.85478	.84517	.83714	.83034
4.5	.87991	.87077	.86309	.85655
5.0	0.89964	0.89098	0.88364	0.87736
6.0	.92820	.92046	.91380	.90804
7.0	.94742	.94055	.93454	.92927
8.0	.96087	.95480	.94941	.94461
9.0	.97055	.96522	.96039	.95604
10.0	0.97764	0.97299	0.96870	0.96477
15.0	.99398	.99178	.98952	.98728
20.0	0.99830	0.99736	0.99627	0.99508

x	$B_6(x)$	$B_{13/2}(x)$	$B_7(x)$	$B_{15/2}(x)$
0.0	0.00000	0.00000	0.00000	0.00000
0.1	.03886	.03843	.03807	.03775
0.2	.07753	.07669	.07596	.07533
0.3	.11585	.11459	.11350	.11256
0.4	.15363	.15197	.15054	.14930
0.5	.19072	.18867	.18691	.18539
0.6	.22698	.22456	.22249	.22069
0.7	.26227	.25950	.25713	.25507
0.8	.29648	.29340	.29075	.28845
0.9	.32953	.32615	.32324	.32071
1.0	0.36134	0.35768	0.35454	0.35181
1.1	.39187	.38796	.38460	.38168
1.2	.42106	.41693	.41338	.41029
1.3	.44892	.44459	.44086	.43762
1.4	.47542	.47092	.46704	.46367
1.5	.50060	.49594	.49193	.48843
1.6	.52446	.51967	.51554	.51194
1.7	.54704	.54214	.53791	.53422
1.8	.56838	.56338	.55907	.55531
1.9	.58853	.58345	.57906	.57524
2.0	0.60753	0.60238	0.59794	0.59406
2.2	.64230	.63706	.63254	.62858
2.4	.67314	.66786	.66328	.65928
2.6	.70050	.69519	.69059	.68656
2.8	.72477	.71946	.71486	.71083
3.0	0.74635	0.74106	0.73647	0.73245
3.5	.79067	.78549	.78097	.77701
4.0	.82452	.81947	.81507	.81119
4.5	.85092	.84603	.84174	.83796
5.0	0.87193	0.86720	0.86304	0.85936
6.0	.90301	.89859	.89469	.89121
7.0	.92463	.92052	.91686	.91359
8.0	.94034	.93653	.93311	.93003
9.0	.95213	.94860	.94542	.94253
10.0	0.96119	0.95794	0.95497	0.95227
15.0	.98510	.98300	.98101	.97913
20.0	0.99383	0.99256	0.99130	0.99004

x	$B_8(x)$	$B_{17/2}(x)$	$B_9(x)$	$B_{19/2}(x)$	$B_{10}(x)$
0.0	0.00000	0.00000	0.00000	0.00000	0.00000
0.1	.03747	.03723	.03701	.03681	.03664
0.2	.07477	.07429	.07385	.07347	.07312
0.3	.11174	.11102	.11037	.10980	.10928
0.4	.14822	.14726	.14641	.14565	.14497
0.5	.18405	.18287	.18183	.18089	.18004
0.6	.21911	.21772	.21648	.21537	.21438
0.7	.25327	.25168	.25026	.24900	.24785
0.8	.28643	.28465	.28307	.28165	.28037
0.9	.31850	.31655	.31481	.31325	.31185
1.0	0.34942	0.34730	0.34542	0.34373	0.34221
1.1	.37912	.37686	.37484	.37304	.37141
1.2	.40758	.40519	.40305	.40114	.39942
1.3	.43478	.43226	.43002	.42801	.42620
1.4	.46070	.45808	.45575	.45365	.45176
1.5	.48537	.48265	.48023	.47806	.47610
1.6	.50878	.50599	.50349	.50126	.49924
1.7	.53098	.52812	.52556	.52326	.52119
1.8	.55200	.54907	.54646	.54411	.54199
1.9	.57187	.56889	.56623	.56384	.56168
2.0	0.59065	0.58762	0.58492	0.58250	0.58030
2.2	.62510	.62201	.61924	.61676	.61452
2.4	.65575	.65262	.64981	.64730	.64502
2.6	.68301	.67985	.67703	.67449	.67219
2.8	.70727	.70411	.70127	.69872	.69641
3.0	0.72890	0.72573	0.72290	0.72034	0.71803
3.5	.77349	.77036	.76756	.76502	.76273
4.0	.80775	.80467	.80191	.79941	.79715
4.5	.83460	.83159	.82888	.82643	.82421
5.0	0.85608	0.85314	0.85049	0.84809	0.84591
6.0	.88810	.88530	.88277	.88047	.87837
7.0	.91064	.90798	.90557	.90337	.90136
8.0	.92725	.92473	.92243	.92033	.91840
9.0	.93991	.93752	.93534	.93333	.93149
10.0	0.94981	0.94755	0.94547	0.94356	0.94180
15.0	.97736	.97569	.97413	.97266	.97128
20.0	0.98882	0.98764	0.98650	0.98540	0.98435

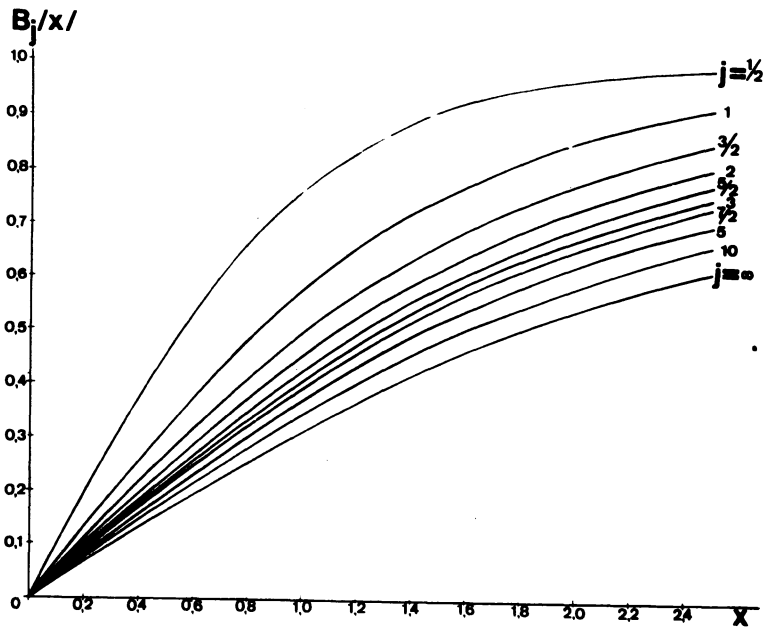


Fig. 2.

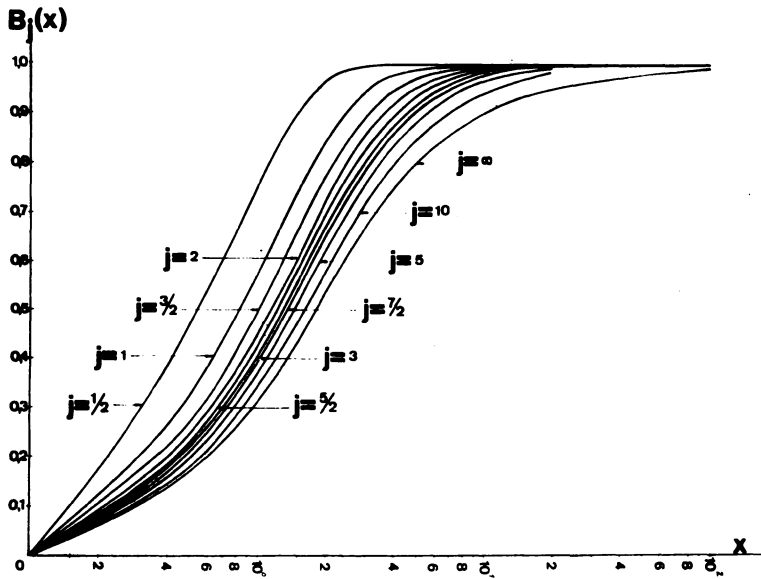


Fig. 3.