Acta Universitatis Carolinae. Mathematica et Physica

J. Kvasil; F. Štěrba; P. Holan

Correlation between some structural effects in odd deformed nuclei and single particle transfer reactions

Acta Universitatis Carolinae. Mathematica et Physica, Vol. 20 (1979), No. 2, 11--26

Persistent URL: http://dml.cz/dmlcz/142437

1979 ACTA UNIVERSITATIS CAROLINAE – MATHEMATICA ET PHYSICA VOL. 20. NO. 2. Pag. 11—26

Correlation Between Some Structural Effects in Odd Deformed Nuclei and Single Particle Transfer Reactions

J. KVASIL, F. ŠTĚRBA, P. HOLAN
Department of Nuclear Physics, Charles University, Prague*)

Received 2 October 1978

Effect of the $\Delta N = 2$ and phonon-phonon interaction on the Coriolis interaction in deformed nuclei is examined. An attempt is done to explain by the phonon admixture the observed reduction of the Coriolis interaction matrix elements, calculated from Nilsson model. Effect of change of deformation parameters on nuclear structure is also considered. Obtained results are applied to the one nucleon transfer reactions and corresponding relations for differential cross sections are given. The possibility of exploitation of the standard DWBA codes for computation is also discussed.

Проводится анализ влияния $\Delta N = 2$ и фонон-фононного взаимодействия на взаимодействие Кориолиса. Известная редукция матричных элементов КВ, расчитанных по модели Нилссона, обясняется присутствием фононовых волновых функций в волновой функции возбужденных состояний. Внимание уделяется также изменению параметров деформации ядра На основе полученных результатов выводятся формулы для поперечного сечения реакций с переносом одного нуклона в рамках приступа метода приближения Борна с искаженными волнами (МИВ). В заключение показывается возможность переписать формулы для сечения реакции на форму, позволяющую воспользовать для численных расчетов известные программы для МИВ.

V práci je zkoumán vliv $\Delta N = 2$ a fonon-fononové interakce na Coriolisovu interakci v deformovaných jádrech. Ukazuje se možnost vysvětlit experimentálně pozorovanou redukcí maticových elementů Coriolisovy interakce, počítaných na základě Nilssonova modelu, pomocí fononových příměsí ve vlnových funkcích vzbuzených stavů, při čemž je brán také v úvahu možný vliv různých hodnot parametrů deformace jádra. Na základě získaných výsledků jsou dále studovány reakce s přenosem jednoho nukleonu. Jsou odvozeny vztahy pro diferenciální účinný průřez, zahrnující všechny uvažované efekty a vycházející z přístupu Bornovy aproximace s porušenými vlnami (DWBA). Na závěr je diskutována možnost úpravy vztahů do tvarů, umožňujících pro numerické výpočty využívat standartní DWBA programy.

1. Introduction

In last 15 – 20 years the direct reactions became the very useful tool for study of structure of deformed nuclei (e.g. [1, 2]). The Distorted Waves Born Approximation (DWBA) or more realistic Coupled Chanels (CC) methods are usually used

*) 180 00 Praha 8 - Troja, Pelc Tyrolka, Czechoslovakia.

for analysis of experimental results. Nevertheless, the CC one needs rather long computation time and therefore the DWBA theory, for which the computation codes are prepared [3, 4], is mainly used.

Detail information about the nuclear structure are necessary in both methods. In the DWBA one the nuclear structure affects first of all the absolute value of differential cross sections through the spectroscopic factors (for the one particle transfer reactions) or through the reduced transition probabilities (for inelastic scattering of charged particles) [2]. The angular distribution is determined mainly by the averaged interaction between incident and outgoing light particles and nucleus, represented usually by the optical potential.

In the first period the spectroscopic factors for one nucleon transfer reactions on deformed nuclei were calculated from simple unified model with Nilsson potential [5] (e.g. [6]). Nevertheless, obtained absolute values of cross sections are in bad agreement with experimental ones [7, 8] and therefore more accurate description for the nuclear structure was used. It was found that pairing effect [1, 9] and Coriolis interaction (CI) in deformed nuclei [10] can substantially change intensity of transitions to different members of rotational bands [1, 11], but the CI matrix elements calculated from the Nilsson model have to be artificially reduced [11, 12, 13]. More fine structural effects (e.g. phonon admixtures [14, 15] or $\Delta N = 2$ interaction [16, 17]) complicate substantially the analysis of experimental results by the DWBA method.

We analysed the structure of deformed nuclei including simultaneously the CI, $\Delta N = 2$ and phonon-phonon interactions and partly their competition in dependence on nuclear deformation. The results were applied to the one nucleon transfer reactions. In present paper we give the ideas of derivation of basic formulas in spirit of the DWBA theory. In first part we give scarce analysis of the nuclear structure including optimalization of the nuclear deformation parameters. The discussion of the differential cross section for the one nucleon transfer reactions and modification of general expressions to the form allowing use of the standard DWBA codes is given in second part.The examples of application of theoretical analysis to the experimental results are shown in last part.

2. Structure of Deformed Nuclei

We will assume that the deformed nucleus possess axially symmetric form with respect to axis 3. The principal moments of inertia, $\mathscr{P}_k$, are than $\mathscr{P}_1 = \mathscr{P}_2 = \mathscr{P} \neq \mathscr{P}_3$. In the unified model the hamiltonian of deformed nucleus can be expressed as [14, 18, 19, 20]

$$H = H_{\text{in}} + \frac{\hbar^2}{2\mathscr{P}} j^2 + \frac{\hbar^2}{2\mathscr{P}} (I^2 - I_3^2 - j_3^2) - \frac{\hbar^2}{2\mathscr{P}} (I_+ j_- + I_- j_+) . \qquad (1)$$

Here I and j are the operators of total and intrinsic impuls-moment of nucleus respectively, H_{in} is the part of hamiltonian describing intrinsic motion (including surface vibrations) and $I_\pm = I_+ \pm iI_2$, $j_\pm = j_1 \pm ij_2$. The last term

$$H_{\mathrm{Cor}} = -\frac{\hbar^2}{2\mathscr{P}}(I_+j_- + I_-j_+) \tag{2}$$

represents the coupling between intrinsic and rotational motion of nucleus (Coriolis interaction – CI). Neglecting this term the wave function of the nuclear states can be written [14, 18, 19, 20]:

$$\Psi(IMK\Omega) = \left(\frac{2I+1}{16\pi^2(1+\delta_{K0})}\right)^{1/2} [\mathscr{D}^I_{MK}(\vartheta_k)\,\chi_\Omega + (-1)^{I+K}\mathscr{D}^I_{M-K}(\vartheta_k)\,\chi_{-\Omega}] \tag{3}$$

where $\mathscr{D}^I_{MK}(\vartheta_k)$ are the Wigner functions depending on Euler angles and describing the rotation of intrinsic coordinate system 1, 2, 3 joined with nucleus with respect to external one (x, y, z), $M = I_z$, $K = I_3$. χ_Ω is the wave function of intrinsic nuclear motion with projection $j_3 = \Omega$ (for axially symmetric nucleus $\Omega = j_3 = I_3 = K$).

2.1. Intrinsic States of Deformed Nucleus

It is convenient to separate H_{in} into three parts [14, 20, 21]

$$H_{\mathrm{in}} = H_{\mathrm{av}} + H_{\mathrm{p}} + H_{\mathrm{Q}} \tag{4}$$

H_{av} represents motion of individual nucleons in averaged field of nuclear forces. Corresponding deformed potential can be expressed through the nuclear sourface which can be for axially symmetric nucleus written as [14]

$$\begin{aligned} R(\vartheta, \varphi) &= R_0[1 + \alpha_{20}Y_{20}(\vartheta, \varphi) + \alpha_{40}Y_{40}(\vartheta, \varphi) = \\ &= R_0 \,.\, f(\alpha_{20}, \alpha_{40}, \vartheta, \varphi)\,. \end{aligned} \tag{5}$$

Here $R_0 = r_0A^{1/3}$ is average radius of nucleus, ϑ and φ are polar angles related to the intrinsic coordinate system, $Y_{20}(\vartheta, \varphi)$ and $Y_{40}(\vartheta, \varphi)$ are spherical functions. α_{20} and α_{40} are parameters of quadrupole and hexadecapolare deformation of nucleus respectively.

The most often used potentials are that of corrected axially symmetric deformed oscillator with $l - s$ interaction included (the expanded Nilsson potential) [5, 22, 23]

$$V(r, \vartheta, \varphi) = -\frac{\hbar^2}{2\mathrm{M}} + \frac{\hbar\omega_0}{2}\left(\frac{r}{f(\alpha_{20}, \alpha_{40}, \vartheta, \varphi)}\right)^2 + \mathrm{C}(\boldsymbol{ls}) + \mathrm{D}(l^2 - \langle l^2\rangle) \tag{6}$$

or the Saxon-Woods deformed potential [24]

$$\begin{aligned} V_{\mathrm{S-W}} &= V(r, \alpha_{20}, \alpha_{40}, \vartheta, \varphi) + V_{ls}(r, \alpha_{20}, \alpha_{40}, \vartheta, \varphi) \\ V(r, \alpha_{20}, \alpha_{40}, \vartheta, \varphi) &= \frac{-V_0}{1 + \exp\{\xi[r - R_0 f(\alpha_{20}, \alpha_{40}, \vartheta, \varphi)]\}} \\ V_{ls}(r, \alpha_{20}, \alpha_{40}, \vartheta, \varphi) &= -\zeta[\boldsymbol{p} \times \boldsymbol{s}]\,\mathrm{grad}\, V(r, \alpha_{20}, \alpha_{40}, \vartheta, \varphi)\,. \end{aligned} \tag{7}$$

In (6) and (7) $\boldsymbol{p}$, $\boldsymbol{l}$ and $\boldsymbol{s}$ are impuls, orbital moment and spin of nucleon respectively, ω_0 is oscillator frequency, C, D, V_0, ξ and ζ are numerical parameters of potentials. If the single proton states are calculated, the Coulomb potential in form [14]

$$V_c(r, \alpha_{20}, \alpha_{40}, \vartheta, \varphi) = \frac{3(Z-1)}{4\pi R_0^3} \int \frac{d^3 r}{|\boldsymbol{r} - \boldsymbol{r}'|} \cdot$$

$$\cdot \{1 + \exp\{\xi[r' - R_0 f(\alpha_{20}, \alpha_{40}, \vartheta, \varphi)]\} \tag{8}$$

is added to Eq. (6) or (7).

The single particle wave functions, $|\varphi_\Omega\rangle$, $(\Omega = K)$ can be expressed as a superposition of wave functions $|Nlj\Omega\rangle$, the solutions of the Schrödinger equation for spherical part of the potential [5, 14, 20]

$$|\varphi_\Omega\rangle = \sum_{Nlj} C_{Nlj}(\Omega) |Nlj\Omega\rangle \tag{9}$$

N is the principal quantum number, $C_{Nlj}(\Omega)$ are coefficients depending on the form of potential and on nuclear deformation. Only the states of given parity are included in (9).

Usually only one N-shell is included in (9), but in some, rather special cases, few values of N have to be considered what is interpreted as the $\Delta N = 2$ interaction. (The interaction is important for some $N = 4$ and 6 even parity and $N = 3$, 5 and 7 odd parity states in rare-earth deformed nuclei – e.g. [14, 16, 17, 25]).

The nucleons in single particle states interact through residual interaction not included in average nuclear field. The short-range part, assigned in (4) as H_p, can be interpreted as pairing force [14, 26, 27]. Using formalism of second quantization, the particle system can be transformed to the quasiparticle one by canonical transformation [14, 20, 21]

$$\begin{aligned} \alpha_{s\tau}^+ &= U_s a_{s\tau}^+ + \tau V_s a_{s-\tau} \\ \alpha_{s\tau} &= U_s a_{s-\tau} + \tau V_s a_{s\tau}^+ \end{aligned} \tag{10}$$

Here $a_{s\tau}^+$ and $a_{s\tau}$ are the particle creation and annihilation operators, $\alpha_{s\tau}^+$ and $\alpha_{s\tau}$ are the corresponding operators for creation and annihilation of quasiparticles. As the vacuum function for the quasiparticles is used the correlated function Ψ_0 [14, 26] of even-even nucleus which originates from odd nucleus in mind after removing the odd nucleon (proton for odd-Z and neutron for odd-N nucleus). Index "s" denotes all quantum numbers of the state related to definite type of nucleon, $\tau = \pm 1$ is connected with time reflection. The numbers U_s and V_s are the amplitudes of probability for the state s in even-even nucleus to be vacant or occupied by pair of protons or neutrons respectively, so that $U_s^2 + V_s^2 = 1$.

The transformation (10) made it possible to consider system of independent quasiparticles instead of system particles interacting through short-range (pairing) forces.

The long-range residual interaction (H_Q in (4)) leads to correlated motion of many nucleons interpreted as surface vibration of nucleus. Corresponding potential is usually taken in form [14]

$$V_Q = V(|\boldsymbol{r}_i - \boldsymbol{r}_k|) + V_\varrho(|\boldsymbol{r}_i - \boldsymbol{r}_k|)\,\boldsymbol{\sigma}_i\boldsymbol{\sigma}_k \tag{11}$$

where $\boldsymbol{r}_i$, $\boldsymbol{r}_k$ are radius vectors and $\boldsymbol{\sigma}_i$ and $\boldsymbol{\sigma}_k$ represent spins of interacting nucleons. Expanding potential (11) into multipol and spin-multipol series it can be shown [14, 20] that only the lowest multipoles are important. Therefore the interaction is usually interpreted as multipol-multipol one.

It is possible to construct the quantum of vibrational motion with boson properties ("phonon") and corresponding operators $Q^+_{\lambda\mu}$ and $Q_{\lambda\mu}$, creating and annihilatting the phonon with angular momentum λ, parity $(-1)^\lambda$ and projection μ of the momentum onto nuclear symmetry axis ($\lambda_3 = \mu$). As a vacuum state for phonon operators is again taken the ground state function Ψ_0 of the neighbour even-even nucleus.

The intrinsic state of odd nucleus with projection K of angular momentum onto symmetry axis and parity π can be interpreted as superposition of the quasiparticle and phonon states with $K = |K_0 \pm \mu|$, where $K_0 = j_3$ is projection of the particle impulsmoment. After rather complicated calculations not given here (see e.g. Ref. [14]) the intrinsic part of wave function (3) for odd axially symmetric deformed nucleus can be written as (see Eq. (9.41) in Ref. [14])

$$\chi_\Omega = \chi_n(K^\pi, \varrho\tau) = C^n_\varrho\{\alpha^+_{\varrho s\tau} + \sum_{\lambda\mu i}\sum_{s} D^{\lambda\mu i n}_{\varrho s\tau}\alpha^+_s\, Q^+_i(\lambda\mu)\}\ \Psi_0\,. \tag{12}$$

Here ϱ and τ are the quantum numbers of quasiparticle state (for the odd proton or neutron system), C^n_ϱ and $D^{\lambda\mu i n}_{\varrho s\tau}$ are normalization coefficients, which can be determined, together with corresponding intrinsic energy $\varepsilon_{K_n}(\varrho)$ by minimalizing average value of hamiltonian H_{in} (Eq. (4)) in state (12). The value of $(C^n_\varrho \,.\, D^{\lambda\mu i n}_{\varrho s\tau})^2$ is the probability of the phonon-quasiparticle admixture $\alpha^+_s\, Q^+_i(\lambda\mu)\ \Psi_0$ in total intrinsic wave function $\chi_n(K^\pi, \varrho)$.

2.2. The Coriolis Interaction

If the term H_{Cor} is neglected in (1), the rotational motion of the nucleus as whole relative to the external coordinate system gives arise to the rotational bands built on each of intrinsic states $\chi(K^\pi, \varrho)$. The energy of different members of the band corresponding to the wave function (3) is then given by formula [10, 14, 19]

$$E(I, K) = \varepsilon_K(\varrho) + \frac{\hbar^2}{2\mathscr{P}}\langle\chi_n(K^\pi, \varrho)\,|j^2|\,\chi_n(K^\pi, \varrho)\rangle + $$
$$+ \frac{\hbar^2}{2\mathscr{P}}\{[I(I + 1) - 2K^2] + a_\varrho(-1)^{I+1/2}(I + 1/2)\,\delta_{K,1/2}\} \tag{13}$$

where a_ϱ is the decoupling parameter.

From the properties of operators $I_\pm$ and $j_\pm$ follows that the total hamiltonian (1) including H_{Cor} is not diagonalized by function (3) if $K > 1/2$. The nondiagonal matrix elements are [10]

$$\langle \Psi(IMK+1) \left| H_{\mathrm{Cor}} \right| \Psi(IMK) \rangle = \frac{-\hbar^2}{2\mathscr{P}} \, \mathrm{A}_K[(I-K)(I+K+1)]^{1/2} \tag{14}$$

with

$$\mathrm{A}_K = \langle \chi_{\mathrm{n}}(K^\pi, \varrho) \left| j_- \right| \chi_{\mathrm{n}}(K+1^\pi, \varrho) \rangle \tag{15}$$

For the $K = 1/2$ states the CI matrix elements are diagonal with respect to function (3) and are usually included in $E(I, K)$ (last term in (13)).

Presence of H_{Cor} in (1) leeds to the mixing of the states (3) and resulting wave function can be written as

$$\overline{\Psi}(IMK) = \sum_{K'} \mathrm{a}_{K'}^{IK} \, \Psi(IMK) \tag{16}$$

where summation includes all states for which CI is considered. Corresponding energy and coefficients $\mathrm{a}_{K'}^{IK}$ can be obtained by diagonalization of hamiltonian (1).

Matrix elements A_K (15) are calculated from nuclear intrinsic structure. Generally it is assumed that intrinsic state is pure quasiparticle one [11, 12, 20, 28], but the obtained matrix elements A_K are too high and have to be artificially reduced by "attenuation factor" η_{KK+1} so that matrix element (15) can be written in form

$$\begin{aligned} \mathrm{A}_K &= \langle \Psi_0 | \, \alpha_{\varrho\tau} j_- \, | \, \alpha_{\varrho'\tau}^+ \, | \Psi_0 \rangle \, \eta_{KK+1} = \\ &= (U_\varrho U_{\varrho'} + V_\varrho V_{\varrho'}) \sum_{Nj} \mathrm{C}_{Nlj}(K\varrho) \, \mathrm{C}_{Nlj}(K+1\varrho) \, [(j-K)(j+K+1)]^{1/2} \, \eta_{KK+1} \end{aligned} \tag{17}$$

All symbols have the same meaning as before.

We have done attempt to explain the reduction of CI by presence of the phonon admixtures in the intrinsic wave function $\chi_{\mathrm{n}}(K^\pi, \varrho)$ (Eq. (12)).

For matrix elements of the operators $j_\pm$ between single particle states hold the symmetry relations [14]

$$\begin{aligned} \langle \Psi_{00} | \, a_{\varrho\tau} j_+ a_{\varrho'\tau}^+ | \Psi_{00} \rangle &= \langle \varrho\tau | \, j_+ | \varrho'\tau \rangle = \\ = - \langle \varrho' - \tau | \, j_+ | \varrho - \tau \rangle &= - \langle \varrho - \tau | \, j_+ | \varrho' - \tau \rangle \end{aligned} \tag{18}$$

Here Ψ_{00} is the particle vacuum function defined by relation $a_{\varrho\tau} | \Psi_{00} \rangle = 0$. Operators $j_\pm$ can be expressed through creation and annihilation particle operators as

$$j_+ = \sum_{\nu\tau} \sum_{\nu'\tau'} \langle \nu\tau | \, j_+ | \nu'\tau' \rangle \, a_{\nu\tau}^+ a_{\nu'\tau'} \, . \tag{19}$$

After transformation (10) to quasiparticle operators and rearrangement we obtain for j_+ the relation

$$\begin{aligned} j_+ = \sum_{\nu\tau} \sum_{\nu'\tau'} \langle \nu\tau | \, j_+ | \nu'\tau' \rangle \, [&U_\nu U_{\nu'} \alpha_{\nu-\tau}^+ \alpha_{\nu'-\tau'} + \tau V_\nu U_{\nu'} \alpha_{\nu\tau} \alpha_{\nu'\tau'} + \\ &+ \tau' U_\nu V_{\nu'} \alpha_{\nu-\tau}^+ \alpha_{\nu'\tau'}^+ + \tau\tau' V_\nu V_{\nu'} \alpha_{\nu\tau} \alpha_{\nu'\tau'}^+] \end{aligned} \tag{20}$$

which can be directly used for evaluation of matrix elements A_K (Eq. (15)). In the Random phase approximation (RPA) [14] the phonon operators $Q_{\lambda\mu}$ commute with the quasiparticle operators $\alpha_{\nu\tau}$. Assuming that $Q_{\lambda\mu}$ and $\alpha_{\nu\tau}$ affect on the same vacuum function Ψ_0, the application of commutation relations for quasiparticles and symmetry relations (18) leed to following expression for A_K:

$$A_K = \langle \chi_n(K^\pi, \varrho_\tau) | j_+ | \chi_{n'}(K'\varrho'^{\pi'}, \varrho'\tau') = C_\varrho^n C_{\varrho'}^{n'} \cdot$$

$$\cdot \left[M_{\varrho\varrho'} \langle \varrho\tau | j_+ | \varrho'\tau' \rangle + \sum_{\lambda\mu i} \sum_{ss'} M_{ss'} D_{\varrho s\tau}^{\lambda\mu in} D_{\varrho's'\tau'}^{\lambda\mu in} \langle s\tau | j_+ | s'\tau' \rangle \right] . \tag{21}$$

Here symbol $M_{kk'} = U_k U_{k'} + V_k V_{k'}$ is used. Similar expression can be obtained for matrix elements of j_- operator.

Selection rules for matrix elements $\langle \nu\tau | j_\pm | \nu'\tau' \rangle$ indicate that in (21) are nonzero only that matrix elements for which $K_\nu = K'_{\nu'} \pm 1$.

The attenuation factor η_{KK+1} can be now obtained, in agreement with Eq. (17) by division the A_K calculated from (21) by the value A_K^0 calculated from the pure

Table 1. Theoretical and experimental values of $\eta_{KK\pm1}$ in ^{161}Dy

$K[Nn_3\Lambda]$	$K'[N'n'_3\Lambda']$	$(A_K^0)_N$	$(A_K^0)_{SW}$	A_K	$\eta_{KK\pm1}^{th}$	$\eta_{KK\pm1}^{exp}$
1/2 [530][a])	1/2 [530][a])	0.800	0.659	0.127	0.190	0.125
1/2 [521]	1/2 [530][a])	1.020	1.016	0.950	0.935	0.280[b])
3/2 [521]	1/2 [530][a])	3.476	3.287	0.447	0.147	0.310
3/2 [521]	1/2 [521]	—0.356	—0.425	—0.349	0.794	0.720
3/2 [532]	1/2 [521]	1.500	1.420	0.943	0.630	0.750
5/2 [512]	3/2 [521]	2.803	2.780	2.100	0.755	0.840
5/2 [512]	3/2 [532]	0.105	0.110	0.285	2.700	0.700[c])
5/2 [523]	3/2 [521]	0.836	0.600	0.393	0.700	0.800
5/2 [523]	3/2 [532]	2.350	2.400	2.230	0.950	0.830
1/2 [521]	1/2 [521]	0.650	0.484	0.390	0.600	0.640
3/2 [651]	1/2 [660]	6.046	6.395	4.830	0.770	0.720
3/2 [651]	1/2 [400]	—0.984	—0.775	—0.300	0.400	0.540
5/2 [642]	3/2 [651]	5.496	5.670	4.870	0.800	0.700
5/2 [642]	3/2 [402]	0.420	0.470	0.220	0.470	0.700
7/2 [633]	5/2 [642]	5.490	5.630	5.330	0.950	0.750
7/2 [404]	5/2 [642]	—0.017	—0.054	0.001	0.015	[c])
1/2 [660]	1/2 [660]	—5.594	—5.500	—4.900	0.750	0.640
1/2 [400]	1/2 [400]	—0.380	—0.176	—0.136	0.770	0.440

[a]) The states are close to the vibrational ones.

[b]) The extremally big difference between $\eta_{KK\pm1}^{th}$ and $\eta_{KK\pm1}^{exp}$ is not clear.

[c]) Because intensity of CI is low the energy of mixed states is weakly affected and $\eta_{KK\pm1}^{exp}$ is uncertain or cannot be extracted at all.

quasiparticle functions:

$$\eta^{\text{th}}_{KK+1} = \frac{A_K}{A^0_K} = \frac{\langle \chi_n(K^\pi_\varrho, \varrho\tau) | j_\pm | \chi_n(K_\varrho \pm 1^\pi, \varrho'\tau') \rangle}{\langle \Psi_0 | \alpha_{\varrho\tau} | j_\pm | \alpha^+_{\varrho'\tau'} | \Psi_0 \rangle} \tag{22}$$

The calculated values of η_{KK+1} for some rare-earth odd deformed isotopes were compared with the values obtained by optimalization of the CI matrix elements to experimentally observed energies of rotational bands. The results for the ^{161}Dy isotope are shown in Table 1. In first two columns the states $|\varrho\rangle$ and $|\varrho'\rangle$ are given in asymptotic notation. In next three columns are the CI matrix elements $(A^0_K)_N$, $(A^0_K)_{SW}$ and A_K calculated from Nilsson model, from model with deformed Saxon-Woods potential and from Eq. (21) respectively. The values of η^{th}_{KK+1}, presented in next column are that, obtained from Saxon-Woods potential (usually they are not remarcably different from values calculated from Nilsson model). The values of η^{exp}_{KK+1}, given in last column, were calculated from experimental energies taken from Ref. [29].

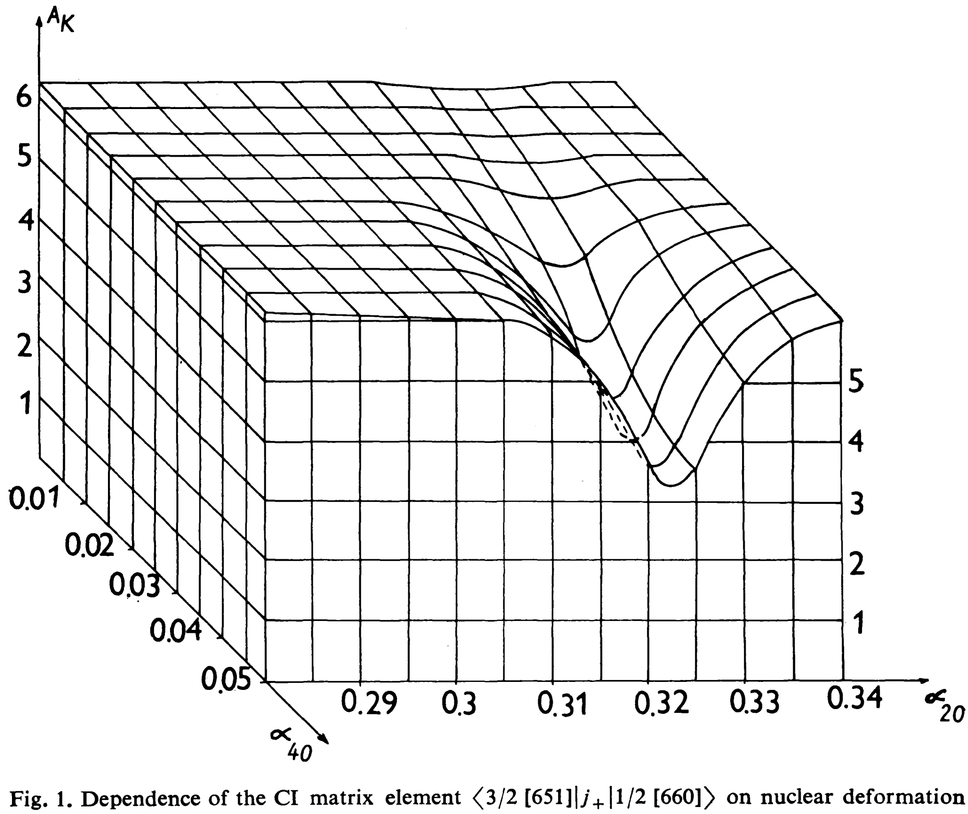

Fig. 1. Dependence of the CI matrix element $\langle 3/2\ [651] | j_+ | 1/2\ [660] \rangle$ on nuclear deformation (Overtaken from Ref. [30])

2.3. Deformation Parameters

According to relations (17) and (21) the CI matrix elements A_K depend on the nuclear deformation only through the coefficients $C_{Nlj}(\Omega)$ of progress (9). It might be shown that the dependence is weak if only one value of principal quantum number, N, is substantial in (9). Nevertheless, in some rare earth isotopes the $\Delta N = 2$ interaction become very important [16, 17, 25] and the dependence of the C-coefficients on deformation can be substantial. Performed analysis indicates [30] that in such cases also the CI matrix elements become strongly dependent on nuclear form in narrow region of deformation parameters. This evident from Fig. 1 overtaken from Ref. [30] on which the matrix elements between states close to the $3/2 +$ [651] and $1/2 +$ [660] quasiparticle states are given as a function of α_{20} and α_{40} parameters.

Strong dependence of the CI matrix elements on nuclear form indicates that for accurate theoretical model calculations a good knowledge of the deformation parameters is inevitably needed. The values of the quadrupole deformation parameter α_{20} can be determined with a good accuracy from the ground state electric quadrupole moment of nucleus, known with sufficient accuracy for many isotopes (e.g. [14, 31]).

The values of the hexadecapolare deformation parameter α_{40} are much less certain. Therefore we have tried to determine the parameter from experimentally known energies of low-lying levels in some rare-earth odd neutron nuclei. Using the experimental energies from survey papers [32] and [33], corresponding quasiparticle energies, $\varepsilon_K(\varrho)$, were extracted. For the isotopes, in which many excited states are known the CI was taken into account in this procedure while in other cases the quasiparticle energy was established by using Eq. (13). The single particle energies, corresponding to extracted quasiparticle ones, were then compared with the values, obtained for different hexadecapolar deformation by solution of the Schrödinger equation with the Nilsson potential (6). The values of α_{40}, for which the best agreement between calculated and experimental energies was achieved, were taken as corresponding to the nuclear hexadecapolar deformation.

The isotopes ^{153}Sm, ^{155}Gd, ^{157}Gd, ^{161}Dy, ^{163}Dy, ^{167}Er and ^{171}Yb were analysed by this manner. As an example the dependence of difference between calculated and experimental single particle energies on α_{40} in the ^{157}Gd isotope is shown on Fig. 2, from which the optimal value of $\alpha_{40} = 0.065$ can be extracted. (The states $1/2 -$ [530] and $3/2 -$ [523], for which the analysis is bad, are known to be of complex structure with substantial phonon admixture [25]). The results for all

Table 2. Parameters of hexadecapolar deformation for some rareearth deformed nuclei

Nucleus	^{153}Sm	^{155}Gd	^{157}Gd	^{161}Dy	^{163}Dy	^{167}Er	^{171}Yb
α_{40}	0.080	0.065	0.065	0.045	0.040	0.020	0.00

examined isotopes are collected in Table 2. Obtained values are in good agreement with parameters calculated theoretically by Solovjev [14].

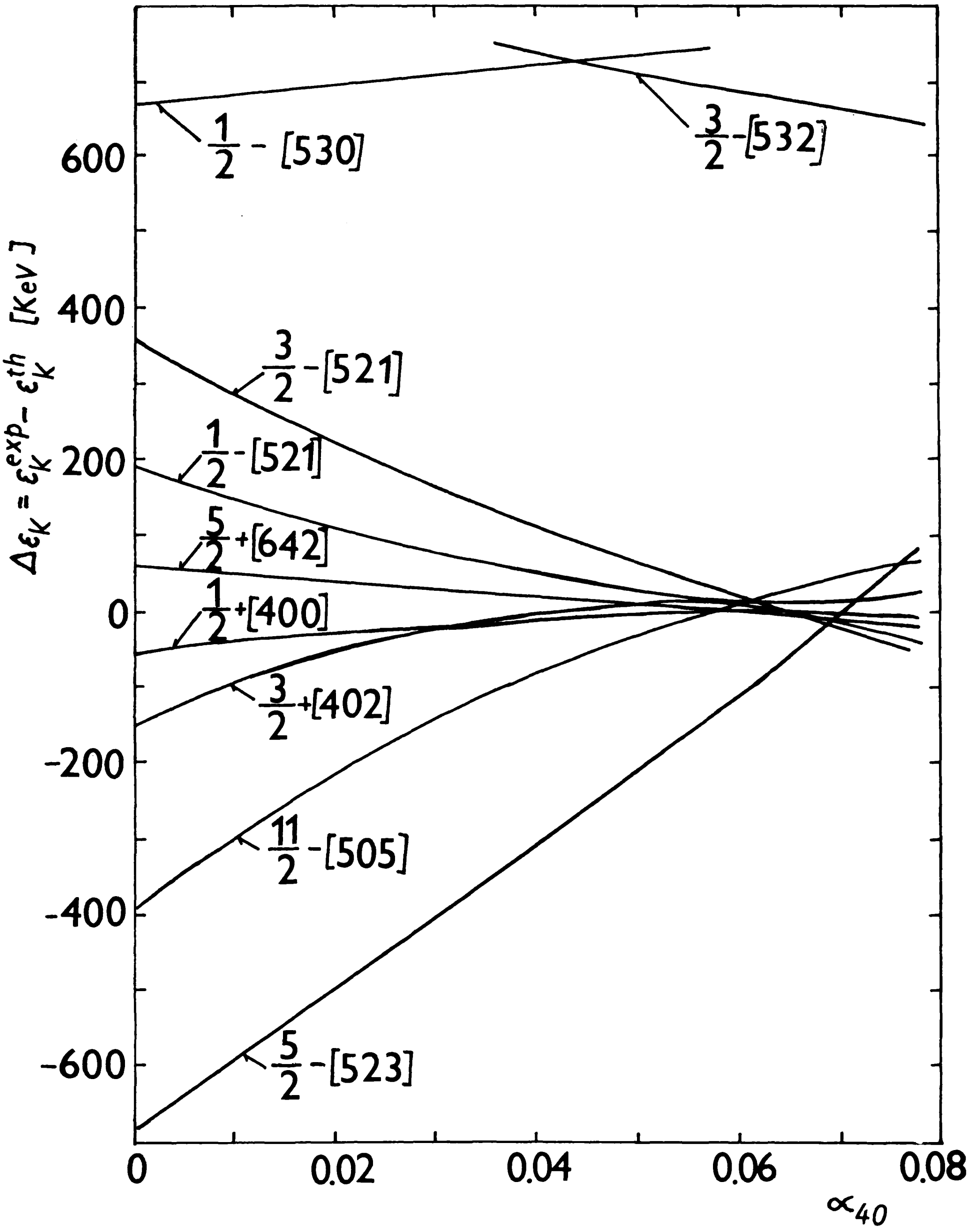

Fig. 2. Dependence of differences between calculated and experimental single particle energies on parameter α_{40}

The use of the values of the parameter α_{40} extracted from excitation energies made it possible substantially improve agreement between calculated and observed energy spectra of deformed rare-earth nuclei. It replaces in some extent artificial shifting of principal shells necessary in the Nilsson model calculations [14].

3. Nuclear Direct Reaction Cross Section

As was shown in [2] the nuclear structure is closely connected with direct nuclear reactions, the states close to the quasiparticle ones being excited mainly in the one nucleon stripping and pick-up processes. If the DWBA theory is used for description of the reaction, the differential cross section for reaction on even-even nucleus is usually taken in the form [2]

$$\frac{\mathrm{d}\sigma(\vartheta)}{\mathrm{d}\Omega} = \mathrm{N} \,.\, S_{lj} \,.\, \sigma_{lj}(\vartheta) \tag{23}$$

where N is normalisation factor, $\sigma_{lj}(\vartheta)$ is the one particle cross section calculated in the DWBA theory and S_{lj} is the spectroscopic factor in which all information about nuclear structure is included. Nevertheless, the simple expression (23) must be replaced by more complicate one if different N-shell have to be considered simultaneously.

In the present work we have taken into account, in addition to the pairing effect and CI, the $\Delta N = 2$ interaction in deformed odd isotopes and its influence onto reaction cross section is examined. We show briefly derivation of corresponding expressions keeping the spirit of the DWBA theory. Because the methods for the stripping and pick-up reactions are similar, we limit ourselves to the one nucleon stripping reaction while the resulting expressions for the pick-up process are given.

3.1. General Formulas

In the first order Born approximation the differential cross section for reaction $\mathrm{a} + \mathrm{A} \rightarrow \mathrm{B} + \mathrm{b}$ is [2, 6, 34]

$$\frac{\mathrm{d}\sigma_{\alpha\beta}}{\mathrm{d}\Omega} = \frac{\mu_\alpha \mu_\beta}{(2\pi\hbar^2)^2}\left(\frac{k_\beta}{k_\alpha}\right)|T_{\alpha\beta}|^2 \tag{24}$$

where μ_α, μ_β are the reduced masses in incident (α) and outgoing (β) chanels of reaction respectively, $\boldsymbol{k}_\alpha$, $\boldsymbol{k}_\beta$ are corresponding impulses.

In the DWBA theory the matrix element $T_{\alpha\beta}$ can be expressed in form [2]

$$T_{\alpha\beta} = \mathscr{J}\int \mathrm{d}^3 r_\alpha \int \mathrm{d}^3 r_\beta \chi_\beta^{(-)}(\boldsymbol{k}_\beta, \boldsymbol{r}_\beta)\, \langle \mathrm{bB}|V|\, \mathrm{aA}\rangle\, \chi_\alpha^{(+)}(\boldsymbol{k}_\alpha, \boldsymbol{r}_\alpha) \tag{25}$$

where $\chi_\alpha^{(+)}$ and $\chi_\beta^{(-)}$ are the distorted waves describing relative motion of particles in incident and outgoing chanels respectively and $\mathcal{J}$ is transformation Jacobian. The matrix element $\langle \mathrm{bB}|V|\,\mathrm{aA}\rangle$ represents the transition between states of the nuclei in both chanels and can be rewritten as

$$\langle \mathrm{bB}|V|\,\mathrm{aA}\rangle = \langle I_\mathrm{B} M_\mathrm{B} s_\mathrm{b} m_\mathrm{b}|V|\, I_\mathrm{A} M_\mathrm{A} s_\mathrm{a} m_\mathrm{a}\rangle \tag{26}$$

where in both chanels I and s are the spins of nucleus and light particle respectively, M and m are their projections onto quantum axis.

If we assume that in the stripping reaction the transfered particle x is captured by the target nucleus A we get, after some rearrangement, for the matrix element:

$$\langle I_\mathrm{B} M_\mathrm{B} s_\mathrm{b} m_\mathrm{b}|\, V|I_\mathrm{A} M_\mathrm{A} s_\mathrm{a} m_\mathrm{a}\rangle = \sum_{Nj} (I_\mathrm{A} M_\mathrm{A} j M_\mathrm{B} - M_\mathrm{A} \mid I_\mathrm{B} M_\mathrm{B})\,.$$

$$\cdot \int \varphi^{\alpha_\mathrm{A} I_\mathrm{A}}_{NjM_\mathrm{B}-M_\mathrm{A}}(\boldsymbol{r}_{\mathrm{xA}}, \sigma_\mathrm{x})\, \Phi_{s_\mathrm{b} m_\mathrm{b}}(\sigma_\mathrm{b})\, V(r_{\mathrm{bx}})\, \Psi_{s_\mathrm{a} m_\mathrm{a}}(\boldsymbol{r}_{\mathrm{bx}}, \sigma_\mathrm{b}, \sigma_\mathrm{x})\, \mathrm{d}\sigma_\mathrm{b}\, \mathrm{d}\sigma_\mathrm{x}\,. \tag{27}$$

$(I_\mathrm{A} M_\mathrm{A} j M_\mathrm{B} - M_\mathrm{A} \mid I_\mathrm{B} M_\mathrm{B})$ are the Clebsch-Gordon coefficients, $V(r_{\mathrm{bx}})$ is the interaction potential responsible for transition, $\Phi_{s_\mathrm{b} m_\mathrm{b}}(\sigma_\mathrm{b})$, $\Psi_{s_\mathrm{a} m_\mathrm{a}}(\boldsymbol{r}_{\mathrm{bx}}, \sigma_\mathrm{b}, \sigma_\mathrm{x})$ are the wave functions of light particles and σ_b, σ_x are intrinsic coordinates of particles b and transfered particle x respectively.

The function $\varphi^{\alpha_\mathrm{A} I_\mathrm{A}}_{NjM_\mathrm{B}-M_\mathrm{A}}(\boldsymbol{r}_{\mathrm{xA}}, \sigma_\mathrm{x})$ represents the singleparticle state in which the transfered nucleon x is captured. It depends on the model used for description of nuclear structure and is in the DWBA theory usually expressed over the single particle wave functions $\varphi_{NljM_\mathrm{B}-M_\mathrm{A}}(\boldsymbol{r}_{\mathrm{xA}}, \sigma_\mathrm{x})$ of the spherically symmetric part of the average nuclear potential. The model effects are concentrated to the spectroscopic amplitude $S^{1/2}_{Nlj}$ so that the function $\varphi^{\alpha_\mathrm{A} I_\mathrm{A}}_{NjM_\mathrm{B}-M_\mathrm{A}}(\boldsymbol{r}_{\mathrm{xA}}, \sigma_\mathrm{x})$ can be expressed as

$$\varphi^{\alpha_\mathrm{A} I_\mathrm{A}}_{NljM_\mathrm{B}-M_\mathrm{A}}(\boldsymbol{r}_{\mathrm{xA}}, \sigma_\mathrm{x}) = S^{1/2}_{Nlj} \cdot \varphi_{NljM_\mathrm{B}-M_\mathrm{A}}(\boldsymbol{r}_{\mathrm{xA}}, \sigma_\mathrm{x}) \tag{28}$$

The differential cross section for the one nucleon stripping reaction on even-even target nucleus can be obtained by the same method as in the classical DWBA theory. But, if we consider the $\Delta N = 2$ interaction, we have to include different N-shells and the cross section in the zero-range approximation is given by formula

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{2I_\mathrm{B}+1}{2I_\mathrm{A}+1} \sum_{lj} \left\{ \frac{\mu_\alpha \mu_\beta}{(2\pi\hbar^2)^2} \frac{k_\beta}{k_\alpha} \sum_m \left| \frac{1}{(2l+1)^{1/2}} \,. \right.\right. \tag{29}$$

$$\left.\left. \cdot \sum_N S^{1/2}_{Nlj}\, \chi_\beta^{(-)} \boldsymbol{k}_\beta, \frac{\mathrm{A}}{\mathrm{B}}\, \boldsymbol{r}\, R_{Nlj}(r)\, Y_{lm}(\vartheta, \varphi)\, \chi_\alpha^{(+)}(\boldsymbol{k}_\alpha, \boldsymbol{r})\, \mathrm{d}^3 r \right|^2 \mathrm{D}^2 \right\}.$$

Here $R(r)$ and $Y(\vartheta, \varphi)$ are the radial and angular parts of the function (28) and D is numerical constant.

Similar expression can be derived for the one nucleon pick-up reaction, in which the transferred nucleon, x, is assumed to be extracted from the target nucleus A.

It is seen form (29) that if different N-shells are simultaneously considered the cross section cannot be written simply as a product of the structural part and part describing relative motion as it is in the clasical DWBA theory (Eq. (23)). With respect to structure of (29) the described theory represents intermedial state between the DWBA and CC methods.

3.2. The Spectroscopic Factors with the $\Delta N = 2$ Interaction

The spectroscopic amplitude defined by Eq. (28) can be written in the following form [2]:

$$S_{Nlj}^{1/2} = \sum_{M_A} (I_A M_A j M_B - M_A \mid I_B M_B) \langle \Psi(I_A M_A K_A) \, \varphi_{NljM_B - M_A} | \Psi(I_B M_B K_B) \rangle \quad (30)$$

where $\Psi(I_A M_A K_A)$ and $\Psi(I_B M_B K_B)$ are the wave functions of target and residual nucleus respectively, including the CI (see Eq. 16)).

Further we assume that the wave function of deformed nucleus is given by Eq. (3) and that the ground state of the target even-even nucleus is not affected by the CI. Using the symmetry properties of Clebsch-Gordon coefficients and Wigner functions [35] after some rearrangement can be the spectroscopic amplitude written in the form

$$S_{Nlj}^{1/2} = \frac{1}{[2(2I_B + 1)]^{1/2}} \sum_{K_B'} a_{K_B'}^{I_B K_B} \, .$$

$$. \{\langle \Psi_{0A} \varphi_{NljK_B} | \, \chi_{K_B'} \rangle + \langle \chi_{0A} \varphi_{Nlj-K_B} | \, \chi_{-K_B'} \rangle \, (-1)^{I_B + K_B}\} \, \delta_{jI_B} \, . \quad (31)$$

Here Ψ_{0A} is the wave function of quasiparticle vacuum (ground state wave function of nucleus A) and χ_{K_B} is the quasiparticle part of the intrinsic wave function (12). Using the Nilsson model or model with Saxon-Woods potential for description of the nucleus and the properties of creation and annihilation operators (10) [35] the final form of $S_{Nlj}^{1/2}$ is:

$$S_{Nlj}^{1/2} = \frac{2}{[2(2I_B + 1)]^{1/2}} \sum_{K_B'} a_{K_B'}^{I_B K_B} U_{K'_B} C_{Nlj}(K'_B) \, \delta_{jI_B} \, . \quad (32)$$

By the same method the expression for the one nucleon pick-up process on even-even deformed target nucleus is obtained in the form

$$(S_{Nlj}^{1/2})_{p-u} = (2)^{1/2} \sum_{K_B'} a_{K_B'}^{I_B K_B} V_{K_B'} C_{Nlj}(K'_B) \, \delta_{jI_B} \, . \quad (33)$$

In (32) and (33) the symbols $a_{K'}^{IK}$, U_K, V_K and $C_{Nlj}(K)$ have the same meaning as in section 2.1.

3.3. Approximate formulas

If the general formula (29) for differential cross section is used, the standard DWBA computation codes cannot be applied to the numerical calculations because the terms with mixed m-values cannot be evaluated. Nevertheless, if the mixed terms in (29) may be neglected the expression can be replaced by approximate relation

$$\frac{d\sigma}{d\Omega} = \frac{2I_B + 1}{2I_A + 1} \sum_{lj} [\sum_N S_{Nlj}^{1/2} \sigma_{Nlj}^{1/2}(\vartheta)]^2 \tag{34}$$

with

$$\sigma_{Nlj}^{1/2}(\vartheta) = \frac{\mu_\alpha \mu_\beta}{(2\pi\hbar^2)^2} \frac{k_\beta}{k_\alpha} \left| \sum_m \frac{1}{(2l+1)^{1/2}} \chi_\beta^{(-)} R_{Nlj} Y_{lm} \chi_\alpha^{(+)} \, d^3r \right|^2 D^2 \tag{35}$$

For the pick-up reaction the approximate relation is:

$$\left(\frac{d\sigma}{d\Omega}\right)_{p-u} = \frac{2s_b + 1}{2s_a + 1} \sum_{lj} [\sum_N (S_{Nlj}^{1/2})_{p-u} \sigma_{Nlj}^{1/2}(\vartheta)]^2 \tag{36}$$

with $(S_{Nlj}^{1/2})_{p-u}$ given by Eq. (33).

The formulas (34) and (35) are similar to that used in Ref. [16, 17].

The validity of approximate relations can be expected for small values of transferred angular momentum, l, [36]. In other cases in which the $\Delta N = 2$ interaction is substantial the more detail analysis has to be done before the approximate relations (34) and (35) are used for calculation of the differential cross sections.

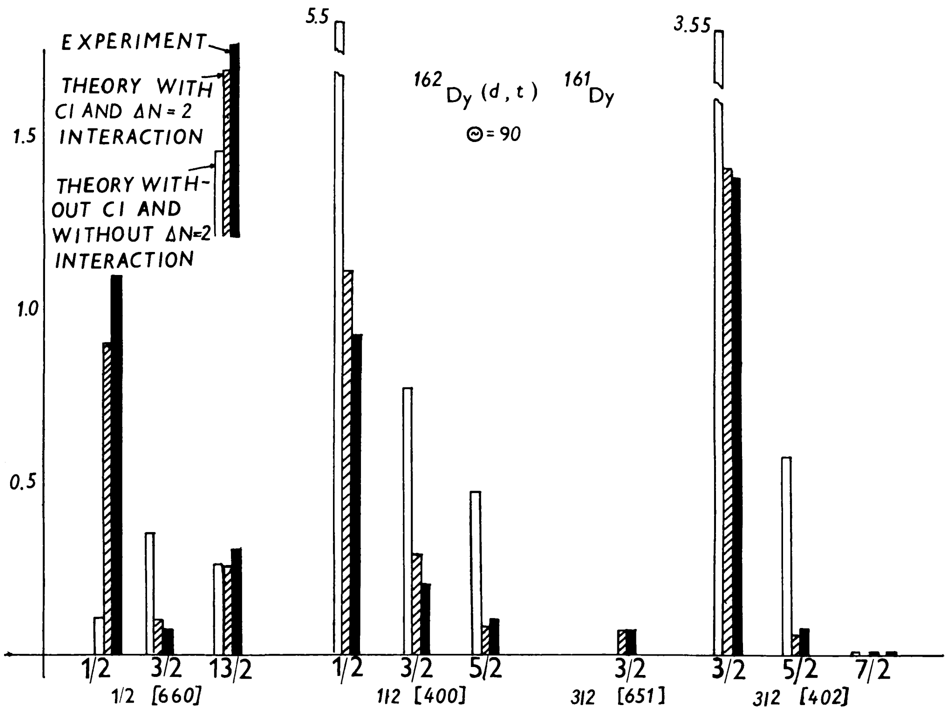

Fig. 3. The relative cross section for excitation of some even parity states in ^{161}Dy by the ^{162}Dy(d, t)^{161}Dy reaction

3.4. Comparison with experiment

Theoretical considerations presented in the paper were applicated to the analysis of experimentally studied (d, p) and (d, t) reactions on rare-earth deformed isotopes. As an example the analysis of excitation of some even parity states in ^{161}Dy by the ^{162}Dy(d, t) ^{161}Dy reaction is presented on Fig. 3. The experimental relative cross sections, taken from Ref. [29], are compared with that calculated theoretically (the cross section is normalised to the odd parity state not present on Fig. 3). Cross sections calculated from the simple Nilsson model, from the Nilsson model with CI and that calculated using expression (36) are shown. It is seen that the $\Delta N = 2$ mixing affects substantially the transition intensity and expressively improves the agreement between the calculated and experimental results.

Performed analysis and its application to the experimentally obtained results indicates that rather fine effects in nuclear structure can substantially affect nuclear reaction cross section. Nevertheless, if accurate models and optimalized parameters are used for description of excited states, experimental material can be satisfactorily analysed in frame of DWBA theory, although in some cases the computation codes have to be modified.

References

[1] Elbek B., Tjøm P. O.: in Advances in Nuclear Physics, (ed. by M. Baranger and Vogt) Plenum, N. York 1969, Vol. III, p. 259.
[2] Austern N.: Direct Nuclear Reaction Theory. J. Wiley a Sons, N. York 1970.
[3] Bassel R. H., Drisko R. M., Satchler R. G.: ORNL Report 3240 1962 (Unpublished).
[4] Kunz P. D.: University of Colorado, Boulder, Colorado 80302 (Unpublished).
[5] Nilsson S. G.: Mat. Fys. Medd. Dan. Vid. Selsk., 29 1958, No. 16.
[6] Sachtler R. G.: Ann. Phys., 3 1958, 275.
[7] Gastebois J., Fernandez B., Laget J. M.: Nucl. Phys., A 125 1969, 351.
[8] Siemssen R. H., Erskine J. R.: Phys. Rev. Let., 19 1967, 90.
[9] Burke D. G. et al.: Mat. Fys. Medd. Dan. Vid. Selsk., 35 1966, No. 2.
[10] Kerman A. K.: Mat. Fys. Medd. Dan. Vid. Selsk., 30 1956, No. 15.
[11] Kaneström I., Tjøm P. O.: Nucl. Phys., A138 1969, 177.
[12] Hultberg et al.: Nucl. Phys., A205 1973, 321.
[13] Kvasil J., Štěrba F., Holan P.: Czech. Journ. Phys., B28 1978, 291.
[14] Solovjev V. G.: Těorija složnych jaděr. Nauka, Moskva 1971. (In Russian).
[15] Kvasil J. et al.: Czech. Journ. Phys., B28 1978, 843.
[16] Andersen B. L.: Nucl. Phys., A112 1968, 443.
[17] Andersen B. L.: Nucl. Phys., A196 1972, 547.
[18] Bohr A.: Mat. Fys. Medd. Dan. Vid. Selsk., 26 1952, No. 4.
[19] Bohr A., Mottelson B. R.: Mat. Fys. Medd. Dan. Vid. Selsk., 27 1953, No. 16.
[20] Bohr A., Mottelson B. R.: Nuclear Structure, Vol. II. Benjamin Inc., 1974.
[21] Kisslinger L. S., Sorensen R. A.: Mat. Fys. Medd. Dan. Vid. Selsk., 32 1960, No. 9.
[22] Gustafson C. et al.: Ark. Fys., 36 1967, 613.
[23] Nilsson S. G., Prior O.: Mat. Fys. Medd. Dan. Vid. Selsk., 32 1961, No. 16.
[24] Gareev F. A., Ivanova S. P.: Preprint JINR Dubna, P4-5221, 1961.
[25] Kvasil J.: Thesis. MFF UK, Prague 1977.

[26] BARDEEN J., COOPER L., SCHRIFFER R.: Phys. Rev., 108 1957, 1175.
[27] BOGOLJUBOV N. N.: DAN USSR, 119 1958, 224.
[28] STEPHENS F. S. et al.: Nucl. Phys., A111 1968, 129.
[29] HOLAN P., ROZKOŠ M., ŠTĚRBA F.: Czech. Journ. Phys., B25 1975, 963.
[30] KVASIL J., ŠTĚRBA F.: (To be published).
[31] ELBEK B.: Disssertation, Munsgaards Forlag, Copenhagen 1963.
[32] BUNKER M. E., REICH C. W.: Rev. Mod. Phys., 43 1971, 348.
[33] OGLE W., WAHLBORN S., PIEPENBRINGS T.: Rev. Mod. Phys., 43 1971, 424.
[34] AUSTERN N. et al.: Phys. Rev., B133 1964, 3.
[35] BOHR A., MOTTELSON B. R.: Nuclear structure, Vol. I. Benjamin Inc., 1974.
[36] SAFAROV R. CH.: Private communication.