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SWITCHED MODIFIED FUNCTION PROJECTIVE SYNCHRONIZATION BETWEEN TWO COMPLEX NONLINEAR HYPERCHAOTIC SYSTEMS BASED ON ADAPTIVE CONTROL AND PARAMETER IDENTIFICATION

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This paper investigates adaptive switched modified function projective synchronization between two complex nonlinear hyperchaotic systems with unknown parameters. Based on adaptive control and parameter identification, corresponding adaptive controllers with appropriate parameter update laws are constructed to achieve switched modified function projective synchronization between two different complex nonlinear hyperchaotic systems and to estimate the unknown system parameters. A numerical simulation is presented to demonstrate the validity and feasibility of the proposed controllers and update laws.

Keywords: modified function projective synchronization, switched state, hyperchaotic system, complex variable, adaptive control

Classification: 34C28, 34D06, 34H10

1. INTRODUCTION

Since Fowler etc. [9] introduced a complex Lorenz model to generalize the real Lorenz model in 1982, complex chaotic and hyperchaotic systems have attracted increasing attention due to the fact that the systems with complex variables can be used to describe the physics of a detuned laser, rotating fluids, disk dynamos, electronic circuits, and particle beam dynamics in high energy accelerators [17]. When applying the complex systems in communications, the complex variables will double the number of variables and can increase the content and security of the transmitted information. Many more complex chaotic and hyperchaotic systems have been proposed after the complex Lorenz model. In ref. [18], the authors studied chaotic unstable limit cycles of complex Van der Pol oscillators. The rich dynamical behaviors of the complex Chen and complex Lü systems were investigated in [19]. By adding state feedback controllers to their complex chaotic systems, complex hyperchaotic Chen, Lorenz, Lü systems were introduced and studied in [20] – [22], respectively. The authors [23] constructed a complex nonlinear hyperchaotic system by adding a cross-product nonlinear term to the complex Lorenz

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system. A complex modified hyperchaotic Lü system [24] were proposed by introducing complex variables to its real counterpart.

In 1990 [27], Pecora and Carroll introduced the concept of synchronizing two identical chaotic systems starting from different initial conditions. Over the last two decades, synchronization in chaotic systems has been extensively investigated. A wide variety of synchronization phenomena have been proposed, which include complete synchronization [15], lag synchronization [1], generalized synchronization [10], phase synchronization [15], anti-synchronization [14], partial synchronization [30], practical synchronization [16], projective synchronization [8], etc. Among the above-mentioned synchronization phenomena, projective synchronization has been investigated with increasing interest in recent years due to the fact that it can obtain faster communication with its proportional feature [7, 8, 31, 33]. The concept of projective synchronization was first introduced by Mainieri and Rehacek in 1999 [26], in which the drive and response systems could be synchronized up to a constant scaling factor. Later on, Li [12] proposed a new synchronization scheme called modified projective synchronization (MPS), in which the drive and response dynamical states synchronize up to a constant scaling matrix. Afterwards, Chen et al. [2] extended the modified projective synchronization and proposed function projective synchronization (FPS), in which the drive and response dynamical states synchronize up to a scaling function matrix, but not a constant one. Recently, Du et al. [6] discussed a new type of synchronization phenomenon, modified function projective synchronization (MFPS), in which the drive and response systems could be synchronized up to a desired scaling function matrix. In [13], the authors discussed the MFPS of general complex nonlinear chaotic systems.

Lately, Sudheer and Sabir [29] reported switched modified function projective synchronization (SMFPS) of a hyperchaotic system using adaptive control method, in which a state variable of the drive system synchronize with a different state variable of the response system up to a desired scaling function matrix. In [32], the authors achieved SMFPS of a five-term three-dimensional autonomous chaotic system. The unpredictability of the switched states and scaling function matrix in SMFPS can provide additional security in securess communication. In the following, we investigate the SMFPS between two complex nonlinear hyperchaotic systems.

This paper is organized as follows. Section 2 introduces the switched modified function projective synchronization scheme and two complex nonlinear hyperchaotic systems. In Section 3, we propose appropriate adaptive controllers and parameter update laws for the adaptive switched modified function projective synchronization of two different complex nonlinear hyperchaotic systems. Section 4 presents a numerical example to illustrate the effectiveness of the proposed method. Finally, conclusions are given in Section 5.

2. A BRIEF DESCRIPTION OF SMFPS DEFINITION AND TWO COMPLEX NONLINEAR HYPERCHAOTIC SYSTEMS

Consider the following two nonlinear dynamical systems:

$$\dot{X} = F(X), \tag{1}$$

$$\dot{Y} = G(Y) + U(X, Y, t) \tag{2}$$

where system (1) is the drive system and system (2) is the response system. $X = (x_1, x_2, \ldots, x_n)^T$, $Y = (Y_1, Y_2, \ldots, Y_n)^T$ are state vectors of the systems (1) and (2), respectively. $F, G : R^n \to R^n$ are two continuous vector functions. $U(X, Y, t)$ is the controller vector to be designed.

If we define the error state as

$$e_i = y_i - \phi_i(t)x_j, \ (i, j = 1, 2, \ldots, n, \ i \neq j) \tag{3}$$

where $\Phi(t) = \mathrm{d}iag(\phi_1(t), \ \phi_2(t), \ldots, \ \phi_n(t))$ is an n-order diagonal matrix, and $\phi_i(t)$ is a continuously differentiable function, $\phi_i(t) \neq 0$ for all t.

Definition 2.1. The drive system (1) and the response system(2) are said to be switched modified function projective synchronization (SMFPS) if there exists a scaling function matrix $\Phi(t)$ such that

$$\lim_{t\to\infty} \|e_i\| = \lim_{t\to\infty} \|y_i - \phi_i(t)x_j\| = 0, (i, j = 1, 2, \ldots, n, i \neq j). \tag{4}$$

Remark 1. In the definition of SMFPS, if let $i = j$, then the SMFPS is simplified to the MFPS [6].

Recently, the authors [23] constructed a new complex nonlinear hyperchaotic system by adding a cross-product nonlinear term to the first equation of the complex Lorenz system [9]. This new system takes the following form

$$\begin{cases} \dot{x} = a(y - x) + yz, \\ \dot{y} = cx - y - xz, \\ \dot{z} = -bz + \dfrac{1}{2}(\bar{x}y + x\bar{y}), \end{cases} \tag{5}$$

where a, b and c are positive parameters, x and y are complex variables, and z is a real variable and the overbar denotes a complex conjugate variable. Compared with the complex Lorenz system [9], this system possesses certain distinctive characteristics, such as having two circles of equilibria and large parameter intervals of hyperchaotic and chaotic behavior [23].

Lately, a novel hyperchaotic complex system [25] which generates 2-, 3- and 4-scroll attractors is introduced and described by

$$\begin{cases} \dot{x} = y - ax + byz, \\ \dot{y} = cy - xz + z, \\ \dot{z} = \dfrac{d}{2}(\bar{x}y + x\bar{y}) - hz, \end{cases} \tag{6}$$

where $a, b, c, d,$ and h are positive parameters, x and y are complex variables, and z is a real variable and the overbar denotes a complex conjugate variable. This system's hyperchaotic attractors exist for large ranges of system parameters.

For more details about the dynamical behaviors of these two complex nonlinear hyperchaotic systems (5) and (6) , please refer to [23] and [25]. Since the real counterparts of these two systems only exhibit chaotic behaviors while these two systems with complex variables exhibit hyperchaotic behaviors, it's very important to investigate their applications. In the following, we'd like to investigate SMPFS between them.

3. ADAPTIVE SMPFS BETWEEN TWO COMPLEX NONLINEAR HYPERCHAOTIC SYSTEMS WITH UNKNOWN PARAMETERS

In this section, we investigate the adaptive switched modified function projective synchronization between the two complex nonlinear hyperchaotic systems (5) and (6) with unknown parameters.

Suppose that system (5) is the drive system whose three variables are denoted by subscript 1 and system (6) is the response system whose variables are denoted by subscript 2. Therefore, the drive and response systems are described, respectively, by the following equations:

$$\begin{cases} \dot{x}_1 = a_1(y_1 - x_1) + y_1 z_1, \\ \dot{y}_1 = c_1 x_1 - y_1 - x_1 z_1, \\ \dot{z}_1 = -b_1 z_1 + \dfrac{1}{2}(\bar{x}_1 y_1 + x_1 \bar{y}_1), \end{cases} \tag{7}$$

and

$$\begin{cases} \dot{x}_2 = y_2 - a_2 x_2 + b_2 y_2 z_2 + U_1, \\ \dot{y}_2 = c_2 y_2 - x_2 z_2 + z_2 + U_2, \\ \dot{z}_2 = \dfrac{d_2}{2}(\bar{x}_2 y_2 + x_2 \bar{y}_2) - h_2 z_2 + U_3, \end{cases} \tag{8}$$

where $x_1 = v_1 + iv_2,\ y_1 = v_3 + iv_4,\ z_1 = v_5$, and $x_2 = u_1 + iu_2,\ y_2 = u_3 + iu_4,\ z_2 = u_5$, v_i and $u_i (i = 1, 2, 3, 4, 5)$ are real functions, and $U_1 = \mu_1 + i\mu_2,\ U_2 = \mu_3 + i\mu_4,\ U_3 = \mu_5$, $\mu_i (i = 1, 2, 3, 4, 5)$ are real control functions to be determined later.

In order to achieve SMFPS between the drive and response systems, we define error states between systems (7) and (8) as

$$\begin{cases} e_1 + ie_2 = x_2 - \phi_1(t) y_1, \\ e_3 + ie_4 = y_2 - \phi_2(t) x_1, \\ \qquad\ e_5 = z_2 - \phi_3(t) z_1, \end{cases} \tag{9}$$

where $\phi_i(t) (i = 1, 2, 3)$ are scaling functions, such that

$$\begin{cases} \lim\limits_{t\to\infty} \|x_2 - \phi_1(t) y_1\| = 0, \\ \lim\limits_{t\to\infty} \|y_2 - \phi_2(t) x_1\| = 0, \\ \lim\limits_{t\to\infty} \|z_2 - \phi_3(t) z_1\| = 0. \end{cases} \tag{10}$$

Thus, we have the following error dynamical system

$$\begin{cases} \dot{e}_1 + i\dot{e}_2 = \dot{x}_2 - \phi_1(t)\dot{y}_1 - \dot{\phi}_1(t) y_1, \\ \dot{e}_3 + i\dot{e}_4 = \dot{y}_2 - \phi_2(t)\dot{x}_1 - \dot{\phi}_2(t) x_1, \\ \qquad\ \dot{e}_5 = \dot{z}_2 - \phi_3(t)\dot{z}_1 - \dot{\phi}_3(t) z_1. \end{cases} \tag{11}$$

Separating the real and imaginary parts of the above equation, yields

$$\begin{cases} \dot{e}_1 = u_3 - a_2 u_1 + b_2 u_3 u_5 - \phi_1(t)(c_1 v_1 - v_1 v_5 - v_3) - \dot{\phi}_1(t) v_3 + \mu_1, \\ \dot{e}_2 = u_4 - a_2 u_2 + b_2 u_4 u_5 - \phi_1(t)(c_1 v_2 - v_2 v_5 - v_4) - \dot{\phi}_1(t) v_4 + \mu_2, \\ \dot{e}_3 = c_2 u_3 - u_1 u_5 + u_5 - \phi_2(t)(a_1(v_3 - v_1) + v_3 v_5) - \dot{\phi}_2(t) v_1 + \mu_3, \\ \dot{e}_4 = c_2 u_4 - u_2 u_5 - \phi_2(t)(a_1(v_4 - v_2) + v_4 v_5) - \dot{\phi}_2(t) v_2 + \mu_4, \\ \dot{e}_5 = d_2(u_1 u_3 + u_2 u_4) - h_2 u_5 - \phi_3(t)(-b_1 v_5 + (v_1 v_3 + v_2 v_4)) - \dot{\phi}_3(t) v_5 + \mu_5. \end{cases} \tag{12}$$

Our aim is to find appropriate controllers $\mu_i (i = 1, 2, 3, 4, 5)$ to stabilize the error states of system (12) at the origin. For this purpose, we propose the following controllers

$$\begin{cases} \dot{\mu}_1 = -u_3 + \tilde{a}_2 u_1 - \tilde{b}_2 u_3 u_5 + \phi_1(\tilde{c}_1 v_1 - v_1 v_5 - v_3) + \dot{\phi}_1 v_3 - k_1 e_1, \\ \dot{\mu}_2 = -u_4 + \tilde{a}_2 u_2 - \tilde{b}_2 u_4 u_5 + \phi_1(\tilde{c}_1 v_2 - v_2 v_5 - v_4) + \dot{\phi}_1 v_4 - k_2 e_2, \\ \dot{\mu}_3 = -\tilde{c}_2 u_3 + u_1 u_5 - u_5 + \phi_2(\tilde{a}_1(v_3 - v_1) + v_3 v_5) + \dot{\phi}_2 v_1 - k_3 e_3, \\ \dot{\mu}_4 = -\tilde{c}_2 u_4 + u_2 u_5 + \phi_2(\tilde{a}_1(v_4 - v_2) + v_4 v_5) + \dot{\phi}_2 v_2 - k_4 e_4, \\ \dot{\mu}_5 = -\tilde{d}_2(u_1 u_3 + u_2 u_4) + \tilde{h}_2 u_5 + \phi_3(-\tilde{b}_1 v_5 + (v_1 v_3 + v_2 v_4)) + \dot{\phi}_3 v_5 - k_5 e_5, \end{cases} \tag{13}$$

and update laws for the unknown parameters $a_1, b_1, c_1, a_2, b_2, c_2, d_2$ and h_2 are given as follows

$$\begin{cases} \dot{\tilde{a}}_1 = -\phi_2(t)((v_3 - v_1)e_3 + (v_4 - v_2)e_4) - k_6(\tilde{a}_1 - a_1), \\ \dot{\tilde{b}}_1 = \phi_3(t) v_5 e_5 - k_7(\tilde{b}_1 - b_1), \\ \dot{\tilde{c}}_1 = -\phi_1(t)(v_1 e_1 + v_2 e_2) - k_8(\tilde{c}_1 - c_1), \\ \dot{\tilde{a}}_2 = -u_1 e_1 - u_2 e_2 - k_9(\tilde{a}_2 - a_2), \\ \dot{\tilde{b}}_2 = u_3 u_5 e_1 + u_4 u_5 e_2 - k_{10}(\tilde{b}_2 - b_2), \\ \dot{\tilde{c}}_2 = u_3 e_3 + u_4 e_4 - k_{11}(\tilde{c}_2 - c_2), \\ \dot{\tilde{d}}_2 = (u_1 u_3 + u_2 u_4) e_5 - k_{12}(\tilde{d}_2 - d_2), \\ \dot{\tilde{h}}_2 = -u_5 e_5 - k_{13}(\tilde{h}_2 - h_2), \end{cases} \tag{14}$$

where $\tilde{a}_1, \tilde{b}_1, \tilde{c}_1, \tilde{a}_2, \tilde{b}_2, \tilde{c}_2,\ \tilde{d}_2$ and $\tilde{h}_2$ are the estimate values for these unknown parameters, respectively, and $k_i > 0$ $(k = 1, 2, \ldots, 13)$. Then, we arrive at the following result.

Theorem 3.1. For a given continuous differential scaling function matrix $\Phi(x) = = \mathrm{diag}\{\phi_1, \phi_2, \phi_3\}$, and any initial values, the SMFPS between systems (7) and (8) can be achieved by the adaptive controllers (13) and the parameter update laws (14), and satisfying

$$\begin{aligned} &\lim_{t\to\infty} |\tilde{a}_1 - a_1| = \lim_{t\to\infty} |\tilde{b}_1 - b_1| = \lim_{t\to\infty} |\tilde{c}_1 - c_1| = \lim_{t\to\infty} |\tilde{a}_2 - a_2| = \lim_{t\to\infty} |\tilde{b}_2 - b_2| \\ &\qquad = \lim_{t\to\infty} |\tilde{c}_2 - c_2| = \lim_{t\to\infty} |\tilde{d}_2 - d_2| = \lim_{t\to\infty} |\tilde{h}_2 - h_2| = 0. \end{aligned} \tag{15}$$

P r o o f. Construct the following Lyapunov function

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_5^2 + e_{a_1}^2 + e_{b_1}^2 + e_{c_1}^2 + e_{a_2}^2 + e_{b_2}^2 + e_{c_2}^2 + e_{d_2}^2 + e_{h_2}^2) \tag{16}$$

where $e_{a_1} = a_1 - \tilde{a}_1,\ e_{b_1} = b_1 - \tilde{b}_1,\ e_{c_1} = c_1 - \tilde{c}_1,\ e_{a_2} = a_2 - \tilde{a}_2,\ e_{b_2} = b_2 - \tilde{b}_2,\ e_{c_2} = c_2 - \tilde{c}_2,\ e_{d_2} = d_2 - \tilde{d}_2,\ e_{h_2} = h_2 - \tilde{h}_2$.

Taking the time derivative of V along the trajectory of the error dynamical system (12) yields

$$\begin{aligned} \dot{V} =& \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 + \dot{e}_4 e_4 + \dot{e}_5 e_5 + \dot{e}_{a_1} e_{a_1} + \dot{e}_{b_1} e_{b_1} + \dot{e}_{c_1} e_{c_1} \\ & + \dot{e}_{a_2} e_{a_2} + \dot{e}_{b_2} e_{b_2} + \dot{e}_{c_2} e_{c_2} + \dot{e}_{d_2} e_{d_2} + \dot{e}_{h_2} e_{h_2} \\ =& \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 + \dot{e}_4 e_4 + \dot{e}_5 e_5 \\ & + e_{a_1}(-\dot{\tilde{a}}_1) + e_{b_1}(-\dot{\tilde{b}}_1) + e_{c_1}(-\dot{\tilde{c}}_1) + e_{a_2}(-\dot{\tilde{a}}_2) + e_{b_2}(-\dot{\tilde{b}}_2) \\ & + e_{c_2}(-\dot{\tilde{c}}_2) + e_{d_2}(-\dot{\tilde{d}}_2) + e_{h_2}(-\dot{\tilde{h}}_2). \end{aligned} \tag{17}$$

Substituting Eqs. (13) and (14) into Eq. (17) yields

$$\begin{aligned} \dot{V} =& -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 - k_5 e_5^2 \\ & - k_6 e_{a_1}^2 - k_7 e_{b_1}^2 - k_8 e_{c_1}^2 - k_9 e_{a_2}^2 - k_{10} e_{b_2}^2 - k_{11} e_{c_2}^2 - k_{12} e_{d_2}^2 - k_{13} e_{h_2}^2 \\ =& -e^T K e \end{aligned} \tag{18}$$

where $e = (e_1, e_2, e_3, e_4, e_5, e_{a_1}, e_{b_1}, e_{c_1}, e_{a_2}, e_{b_2}, e_{c_2}, e_{d_2}, e_{h_2})^T$ and $K = diag\{k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}, k_{11}, k_{12}, k_{13}\}$.

Since $\dot{V} \le 0$, we have $e_1, e_2, e_3, e_4, e_5, e_{a_1}, e_{b_1}, e_{c_1}, e_{a_2}, e_{b_2}, e_{c_2}, e_{d_2}, e_{h_2} \to 0$ as $t \to \infty$, i.e. $\lim_{t\to\infty} \| e \| = 0$. Therefore, the drive system (7) and response system (8) achieve SMFPS.

This completes the proof. □

4. A NUMERICAL SIMULATION

In this section, we perform a numerical simulation to demonstrate the effectiveness of the previous theoretical analysis. In the following numerical simulation, the fourth-order Runge–Kutta method [11] is used to solve the systems with time step size 0.01. The true values of the "unknown" parameters of systems (7) and (8) are chosen as $a_1 = 30,\ b_1 = 11,\ c_1 = 90,\ a_2 = 3.5,\ b_2 = 0.599,\ c_2 = 3,\ d_2 = 2$ and $h_2 = 9$, so that the two systems exhibit hyperchaotic behavior, respectively. The initial values for the drive and response systems are $(x_1(0), y_1(0), z_1(0)) = (2 + 4i, 1 + 3i, 2)$ and $(x_2(0), y_2(0), z_2(0)) = (5 + 2i, -1 + i, -4)$, thus $(v_1(0), v_2(0), v_3(0), v_4(0), v_5(0)) = (2, 4, 1, 3, 2)$ and $(u_1(0), u_2(0), u_3(0), u_4(0), u_5(0)) = (5, 2, -1, 1, -4)$, respectively. The initial values of the parameter estimation update laws are $\tilde{a}_1(0) = \tilde{b}_1(0) = \tilde{c}_1(0) = \tilde{a}_2(0) = \tilde{b}_2(0) = \tilde{c}_2(0) = \tilde{d}_2(0) = \tilde{h}_2(0) = 0.1$. The function factors are given as $\phi_1(t) = 0.1\sin(-0.2\pi t) + 0.3,\ \phi_2(t) = 0.2\sin(0.3\pi t) - 0.1,\ \phi_3(t) = -0.2\sin(0.4\pi t) + 0.2$, the control gains are chosen as $k_i = 1\ (i = 1, 2, \dots, 13)$. The simulation results are shown in Figures 1−3. Figure 1 demonstrates the SMFPS errors between the drive system (7) and response system (8). From this figure, it can be seen that the SMFPS errors converge to zero, i. e., these two systems achieve SMFPS. And Figures 2 – 3 show that the unknown system parameters approach the true values.

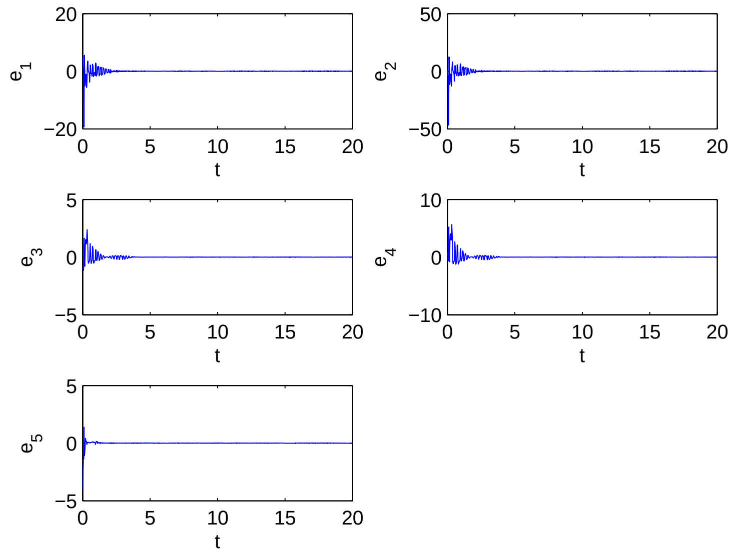

Fig. 1. The time evolution of SMFPS errors for the drive system (7) and response system (8) with controllers (13) and parameter update laws (14), where $e_1 = u_1 - (0.1\sin(-0.2\pi t) + 0.3)v_3,\ e_2 = u_2 - (0.1\sin(-0.2\pi t) + 0.3)v_4,\ e_3 = u_3 - (0.2\sin(0.3\pi t) - 0.1)v_1,\ e_4 = u_4 - (0.2\sin(0.3\pi t) - 0.1)v_2,\ e_5 = u_5 - (-0.2\sin(0.4\pi t) + 0.2)v_5$.

5. CONCLUSIONS

In this paper, we have investigated switched modified function projective synchronization between two different complex nonlinear hyperchaotic systems with fully unknown parameters, in which a state variable of the drive system synchronizes with a different state variable of the response system up to a scaling function matrix. Based on adaptive control and parameter identification, the appropriate adaptive controllers with parameter update laws are proposed to achieve SMFPS between the two different complex nonlinear hyperchaotic systems and to estimate the unknown system parameters. A numerical simulation is conducted to illustrate the validity and feasibility of the proposed adaptive controllers and parameter update laws. Recently, many researchers have begun to give their attention to the multiscroll chaotic systems [28] and the networked chaotic systems [3] – [5]. Consequently, we will investigate the SMFPS of these systems in our future work.

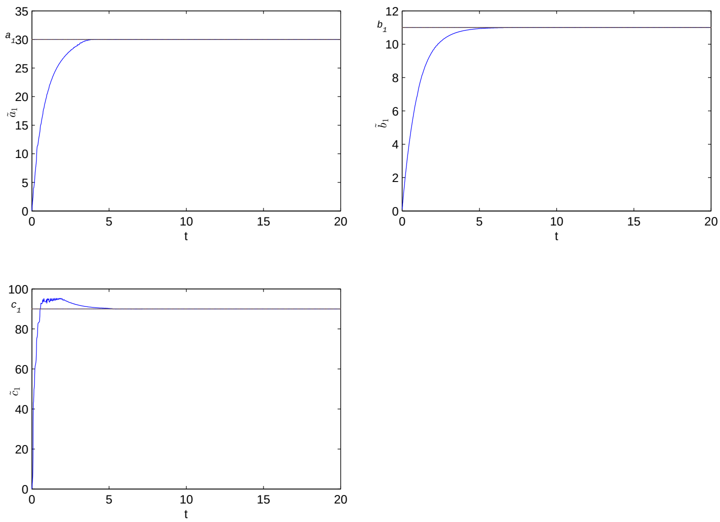

Fig. 2. The time evolution of the estimated unknown parameters of system (7).

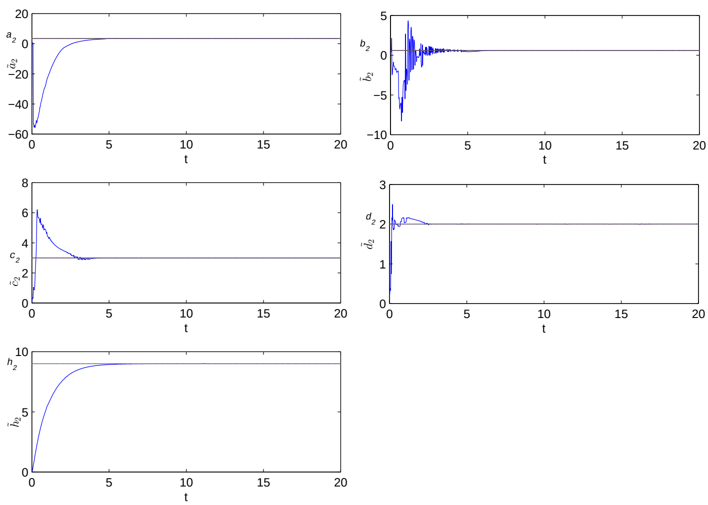

Fig. 3. The time evolution of the estimated unknown parameters of system (8).

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