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ROBUST SAMPLED-DATA OBSERVER DESIGN FOR LIPSCHITZ NONLINEAR SYSTEMS

Yu Yu and Yanjun Shen

In this paper, a robust sampled-data observer is proposed for Lipschitz nonlinear systems. Under the minimum-phase condition, it is shown that there always exists a sampling period such that the estimation errors converge to zero for whatever large Lipschitz constant. The optimal sampling period can also be achieved by solving an optimal problem based on linear matrix inequalities (LMIs). The design methods are extended to Lipschitz nonlinear systems with large external disturbances as well. In such a case, the estimation errors converge to a small region of the origin. The size of the region can be small enough by selecting a proper parameter. Compared with the existing results, the design parameters can be easily obtained by solving LMIs.

Keywords: sampled-data observer, nonlinear systems, Lipschitz, sampling period, LMIs

Classification: 93C57, 93B51

1. INTRODUCTION

The problem of estimating the states of nonlinear systems is a hot topic and has received extensive attention for decades. In [11], the authors gave a comprehensive overview of this topic. It is shown that many approaches for nonlinear observer design have been proposed. Since observer design for the nonlinear systems with Lipschitz condition is presented in [26], further works have been done [2, 4, 5, 9, 10, 17, 29, 31, 32]. In [5, 10, 17, 32], the system dynamic is divided into two parts: a linear and observable part and a nonlinear and Lipschitz part. Sufficient conditions in terms of LMIs are proposed to ensure the convergence of the observer errors. An observer is also proposed for time-delayed systems with polynomial nonlinearities [20]. However, some of the proposed approaches suffer some problems such as too many assumptions and constraints. Therefore, they are hard to be applied into practice. For example, a large Lipschitz constant may lead to infeasibility of LMIs. To overcome these problems, further works have been done in [2, 4, 9, 29, 31, 34]. For instance, the linear parameter varying (LPV) approach is served for observer design of Lipschitz nonlinear systems [29]. The observer gain is carried out via LMI conditions. This method doesn't suppose small-Lipschitz-constant condition, and provides a solution with less restrictive conditions. An adaptive observer for Lipschitz nonlinear systems is derived in [9]. The convergence property

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can be guaranteed even there exists some unknown parameters. In [4], robust nonlinear observers are presented for Lipschitz nonlinear systems with whatever large Lipschitz constant. The design parameter is selected by the method of stepwise schedule. The authors in [2, 31, 34], propose high-gain observers for nonlinear systems. But, high-gain observers may lead to peaking phenomenon [12].

In the above literatures, the observers are all designed based on continuous time analysis. With the development of science and technology, digital control systems have attracted more and more attention in the application of modern industry. Thus, researchers have paid great enthusiasm and energy into sampled-data systems. Recently, there are three main approaches of sampled-data observer design for nonlinear systems, such as continuous designed and then discretization [1, 7, 16], design based on a discrete-time approximation model [18, 33], continuous and discrete design [6, 19, 22, 23, 25, 27, 30]. Inspired by [4], we want to extend observer design for Lipschitz nonlinear systems with outputs sampled at discrete instants by the third method, i. e., continuous and discrete design. The sampled outputs are directly applied to construct the observer without discretizing the nonlinear systems. Under the minimum-phase condition, it is shown that, for the nonlinear systems considered in [4] with whatever large Lipschitz constant, there always exists a sampling period such that the estimation errors converge to zero. The optimal sampling period can also be achieved by solving a LMI-based optimal problem. Then, we investigate sampled-data observer design for Lipschitz nonlinear systems with large external disturbances as well. Under the same conditions as in [4], there also exists a sampling period such that the estimation errors converge to a small region of the origin. We can obtain the optimal sampling period by solving a LMI-based optimal problem as well. The size of the region can be small enough by selecting a proper parameter. Compared with the existing results, the design parameters can be easily obtained by solving LMIs through Matlab LMI Toolbox.

This paper is organized as follows. In Section 2, we present some preliminary results and our main aim of this paper. Sampled-data observers are designed for nonlinear systems with large Lipschitz constant, and for nonlinear systems with large Lipschitz constant and large external disturbances, in Section 3 and 4, respectively. In Section 5, an example is given to illustrate the efficiency and validity of the proposed method. Finally, Section 6 concludes the paper.

2. PRELIMINARIES

In this paper, we consider a Lipschitz nonlinear system in the form of

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + G_1 f(x(t)) \\ y(t) = Cx(t), \end{cases} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$ and $y(t) \in R^p$ are the system unknown state vector, the control input vector, and the system measured output vector, respectively, the matrices $A \in R^{n\times n}$, $B \in R^{n\times m}$, $G_1 \in R^{n\times q}$ and $C \in R^{p\times n}$. The nonlinear function $f(x(t)) \in R^q$ satisfies the following Lipschitz condition

$$\|f(x(t) - f(\hat{x}(t))\| \leq \gamma \|x(t) - \hat{x}(t)\|, \tag{2}$$

for a positive constant γ. In this paper, $\hat{x}(t)$ indicates the observer states. As in [4], we assume that (A, C) is observable, (A, G_1) is controllable, and $(A+\alpha I, G_1, C)$ is minimum-phase for a positive parameter α. Moreover, let $\{t_k\}$ denote a strictly increasing sequence and satisfy $t_k = t_{k-1}+T$, where T is the sampling period. We make the assumption that the system output $y(t)$ is sampled at time instant t_k. Our main purpose is to estimate the system states based on the sampled and measured output vector $y(t_k)$.

In order to derive our main results, the following lemmas are needed.

Lemma 2.1. (Shen et al. [23]) (Jensen Inequality) Let M denote a symmetric positive definite matrix, λ denote a positive real number, and $\omega(t)$ is an integrable vector function defined on the interval $[0, \lambda]$. Then, the following inequality holds

$$\left[\ \int_0^\lambda \omega(s)\,\mathrm{d}s\ \right]^T M \left[\ \int_0^\lambda \omega(s)\,\mathrm{d}s\ \right] \le \lambda \int_0^\lambda \omega^T(s) M \omega(s)\,\mathrm{d}s.$$

Lemma 2.2. (Yu [28]) (Schur Lemma) Let $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$ denote a symmetric matrix, where $S_{11} \in R^{n\times n}$. Then the following three conditions are equivalent to each other:

(1) $S < 0$;

(2) $S_{11} < 0,\ S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(3) $S_{22} < 0,\ S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

3. SAMPLED-DATA NONLINEAR OBSERVER

In this section, we will propose sufficient conditions to ensure exponential stability of the estimation errors for whatever large Lipschitz constant γ in (2). The sampled observer for the system (1) is given as follows

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t_k) - C\hat{x}(t_k)) + G_1 f(\hat{x}(t)), \\ \hat{x}(t_{k+1}) = \lim\limits_{t\to t_{k+1}^-} \hat{x}(t),\ t \in [t_k, t_{k+1}), \end{cases} \tag{3}$$

where $L = QC^T \in R^{n\times p}$, and the positive definite symmetric matrix Q can be obtained by solving the LQ Riccati equation

$$Q(A+\alpha I)^T + (A+\alpha I)Q - QC^TCQ + \pi G_1 G_1^T = 0,\ \alpha > 0,\ \pi > 0. \tag{4}$$

It is obvious that the LQ Riccati equation (4) is equivalent to

$$(A+\alpha I)^T Q^{-1} + Q^{-1}(A+\alpha I) - C^TC + \pi Q^{-1} G_1 G_1^T Q^{-1} = 0,\ \alpha > 0,\ \pi > 0. \tag{5}$$

We can guarantee that there exists a positive definite symmetric matrix Q satisfying the LQ Riccati equation (4) or (5) for any $\alpha > 0$ and $\pi > 0$, under the conditions that (A, C) is observable, and (A, G_1) is controllable [14, 21]. From (4) or (5), it is followed that the positive definite symmetric matrix Q is determined by the parameter π. As in [4], we summarize the relationship between $Q(\pi)$ and π as follows.

Theorem 3.1. (Chen and Chen [4], Doyle and Stein [8], Kwakernaak and Sivan [13]) If the system $(A+\alpha I, G_1, C)$ is minimum-phase, the matrices C and G_1 have full rank, and $\operatorname{rank}(C^T) = \operatorname{rank}(G_1)$, then, we have

$$\lim_{\pi\to\infty} \frac{Q(\pi)}{\pi} = 0. \tag{6}$$

Recently, a great number of problems originating in system and control theory can be reduced to some standard convex or quasiconvex optimization problems involving linear matrix inequalities (LMIs). These optimization problems can be solved numerically tractable and efficiently by Matlab LMI Toolbox [25]. The connection between the matrix equation (5) and LMIs is given in the following theorem.

Theorem 3.2. (Boyd et al. [3]) There exists a positive definite symmetric matrix Q^{-1} satisfying the matrix equation (5), if and only if the following LMI

$$\begin{bmatrix} (A+\alpha I)^T Q^{-1} + Q^{-1}(A+\alpha I) - C^T C & Q^{-1}G_1 \\ G_1^T Q^{-1} & -\frac{1}{\pi} I \end{bmatrix} \le 0, \tag{7}$$

in the variable $Q^{-T} = Q^{-1}$ is feasible.

From Theorem 3.1 and Theorem 3.2, we also have the following results.

Theorem 3.3. If the system $(A+\alpha I, G_1, C)$ is minimum-phase, the matrices C and G_1 have full rank, and $\operatorname{rank}(C^T) = \operatorname{rank}(G_1)$, then, the positive definite symmetric matrix Q solved by (7) satisfying (6) as well.

Remark 3.4. On the interval $[t_k, t_{k+1})$, $y(t_k) - C\hat{x}(t_k)$ is a constant vector and the nonlinear function $f(x(t))$ satisfies the Lipschitz condition (2). Then, the differential equation (3) has an unique solution $\hat{x}(t)$ on $[t_k, t_{k+1})$. Since $\hat{x}(t_{k+1}) = \lim_{t\to t_{k+1}^-} \hat{x}(t)$, we have $\hat{x}(t_{k+1}) = \lim_{t\to t_{k+1}^-} \hat{x}(t) = \lim_{t\to t_{k+1}^+} \hat{x}(t)$. Thus, the solution $\hat{x}(t)$ to the the differential equation (3) is continuous on $[t_k, t_{k+1}]$ for any $k \ge 0$. Therefore, the solution $\hat{x}(t)$ to the differential equation (3) is unique and continuous on $[t_0, \infty)$, and is dependent on the initial state x_0 and the sampled instants t_k.

Define the state estimation error $e(t) = x(t) - \hat{x}(t)$. Form (1) and (3), we have

$$\begin{cases} \dot{e}(t) = Ae(t) - LCe(t_k) + G_1(f(x(t)) - f(\hat{x}(t))), \\ e(t_{k+1}) = \lim\limits_{t\to t_{k+1}^-} e(t), \quad t \in [t_k, t_{k+1}). \end{cases} \tag{8}$$

Next, we give one of our main results.

Theorem 3.5. For the error dynamics system (8), with full rank matrices C and G_1, if, $\operatorname{rank}(C^T) = \operatorname{rank}(G_1)$, and $(A+\alpha I, G_1, C)$ is minimum-phase, then, there exists a positive definite symmetric matrix Q^{-1}, and two parameters $\pi > 0$, $\delta > 1$ satisfy the following conditions,

$$\begin{bmatrix} (A+\alpha I)^T Q^{-1} + Q^{-1}(A+\alpha I) - C^T C & Q^{-1}G_1 \\ G_1^T Q^{-1} & -\frac{1}{\pi} I \end{bmatrix} \le 0, \tag{9}$$

$$-\alpha Q^{-1}+\frac{\gamma^2\delta}{\pi}I<0. \tag{10}$$

Moreover, for the matrix Q and the parameter π solved by (9) and (10), if there exists a sampling period T such that the condition

$$\begin{bmatrix} -\alpha Q^{-1}-C^TC+\frac{\gamma^2\delta}{\pi}I & 0 & C^TC & 0 & (A-QC^TC)^T \\ 0 & -\pi I & 0 & I & 0 \\ C^TC & 0 & -(\frac{1}{T}-\alpha)Q^{-1} & 0 & (QC^TC)^T \\ 0 & I & 0 & -\frac{\delta}{\pi}I & G_1^T \\ A-QC^TC & 0 & QC^TC & G_1 & -\frac{Q}{T} \end{bmatrix}<0, \tag{11}$$

holds, then the estimation error system (8) is globally exponentially stable for whatever large Lipschtiz constant γ.

Proof. According to Theorem 3.3, there exists a positive definite symmetric matrix Q and a parameter $\pi>0$ such that the conditions (9) and (6) hold for any $\pi>0$. From (6), we also have that there exists π such that $\frac{\gamma^2\delta}{\pi}Q<\alpha I$. Therefore, the condition (10) holds.

Next, we will prove that if there exists a sampling period T such that the condition (11) is satisfied, then estimation error system (8) is globally exponentially stable. In fact, the error system (8) can be rewritten as

$$\begin{cases} \dot{e}(t)=(A-LC)e(t)+LC\displaystyle\int_{t_k}^{t}\dot{e}(\rho)\,\mathrm{d}\rho+G_1\tilde{f}(x,\hat{x}), \\ e(t_{k+1})=\displaystyle\lim_{t\to t_{k+1}^-}e(t), \quad t\in[t_k,t_{k+1}), \end{cases} \tag{12}$$

where $\tilde{f}(x,\hat{x})=f(x(t))-f(\hat{x}(t))$. Construct a Lyapunov–Krasovskii function in the form of

$$V(t)=V_1(t)+V_2(t), \tag{13}$$

where

$$V_1(t)=e^T(t)Q^{-1}e(t), \tag{14}$$

$$V_2(t)=\int_{-T}^{0}\int_{t+\beta}^{t}\dot{e}^T(\alpha)Q^{-1}\dot{e}(\alpha)\,\mathrm{d}\alpha\mathrm{d}\beta. \tag{15}$$

Calculate the derivative of $V_1(t)$ along the trajectory of the system (12), then

$$\begin{aligned} \dot{V}_1(t)&=\dot{e}^T(t)Q^{-1}e(t)+e^T(t)Q^{-1}\dot{e}(t) \\ &=e^T(t)[(A-LC)^TQ^{-1}+Q^{-1}(A-LC)]e(t)+\int_{t_k}^{t}\dot{e}^T(\rho)\,\mathrm{d}\rho(LC)^TQ^{-1}e(t) \\ &\quad+e^T(t)Q^{-1}LC\int_{t_k}^{t}\dot{e}(\rho)\,\mathrm{d}\rho+\tilde{f}^T(x,\hat{x})G_1^TQ^{-1}e(t)+e^T(t)Q^{-1}G_1\tilde{f}(x,\hat{x}), \\ &\quad t\in[t_k,t_{k+1}). \end{aligned} \tag{16}$$

According to the inequality (2), one obtains that

$$\frac{\gamma^2\delta}{\pi}e^T(t)e(t)-\frac{\delta}{\pi}\tilde{f}^T(x,\hat{x})\tilde{f}(x,\hat{x})\geq 0. \tag{17}$$

From (5), (16) and (17), it follows that

$$\begin{aligned}\dot{V}_1(t)\leq{}& e^T(t)[(A-LC)^TQ^{-1}+Q^{-1}(A-LC)]e(t)+2\int_{t_k}^{t}\dot{e}^T(\rho)\,\mathrm{d}\rho(LC)^TQ^{-1}e(t)\\ &+2\tilde{f}^T(x,\hat{x})G_1^TQ^{-1}e(t)+\frac{\gamma^2\delta}{\pi}e^T(t)e(t)-\frac{\delta}{\pi}\tilde{f}^T(x,\hat{x})\tilde{f}(x,\hat{x}),\\ ={}&\xi^T\begin{bmatrix}-2\alpha Q^{-1}-C^TC+\frac{\gamma^2\delta}{\pi}I & 0 & C^TC & 0\\ 0 & -\pi I & 0 & I\\ C^TC & 0 & 0 & 0\\ 0 & I & 0 & -\frac{\delta}{\pi}I\end{bmatrix}\xi,\ t\in[t_k,t_{k+1}),\end{aligned} \tag{18}$$

where $\xi=\begin{bmatrix}e(t)\\ G_1^TQ^{-1}e(t)\\ \int_{t_k}^{t}\dot{e}(\rho)\,\mathrm{d}\rho\\ \tilde{f}(x,\hat{x})\end{bmatrix}$. The derivative of $V_2(t)$ is given as

$$\dot{V}_2(t)=T\dot{e}^T(t)Q^{-1}\dot{e}(t)-\int_{t-T}^{t}\dot{e}^T(\rho)Q^{-1}\dot{e}(\rho)\,\mathrm{d}\rho.$$

Since $t\in[t_k,t_{k+1})$, then $t-t_k\leq T$, which implies that $t-T\leq t_k$. Using Lemma 2.1, we have

$$\begin{aligned}\dot{V}_2(t)={}&T\dot{e}^T(t)Q^{-1}\dot{e}(t)-\int_{t-T}^{t}\dot{e}^T(\rho)Q^{-1}\dot{e}(\rho)\,\mathrm{d}\rho\\ \leq{}&T\dot{e}^T(t)Q^{-1}\dot{e}(t)-\alpha T\int_{t-T}^{t}\dot{e}^T(\rho)Q^{-1}\dot{e}(\rho)\,\mathrm{d}\rho-(1-\alpha T)\int_{t_k}^{t}\dot{e}^T(\rho)Q^{-1}\dot{e}(\rho)\,\mathrm{d}\rho\\ \leq{}&\xi^T\begin{bmatrix}(A-QC^TC)^T\\ 0\\ (QC^TC)^T\\ G_1^T\end{bmatrix}TQ^{-1}\begin{bmatrix}(A-QC^TC)^T\\ 0\\ (QC^TC)^T\\ G_1^T\end{bmatrix}^T\xi-\frac{1-\alpha T}{t-t_k}\left(\int_{t_k}^{t}\dot{e}^T(\rho)\,\mathrm{d}\rho\right)\\ &Q^{-1}\left(\int_{t_k}^{t}\dot{e}(\rho)\,\mathrm{d}\rho\right)-\alpha T\int_{t-T}^{t}\dot{e}^T(\rho)Q^{-1}\dot{e}(\rho)\,\mathrm{d}\rho\\ \leq{}&\xi^T\begin{bmatrix}(A-QC^TC)^T\\ 0\\ (QC^TC)^T\\ G_1^T\end{bmatrix}TQ^{-1}\begin{bmatrix}(A-QC^TC)^T\\ 0\\ (QC^TC)^T\\ G_1^T\end{bmatrix}^T\xi-\frac{1-\alpha T}{T}\left(\int_{t_k}^{t}\dot{e}^T(\rho)\,\mathrm{d}\rho\right)\\ &Q^{-1}\left(\int_{t_k}^{t}\dot{e}(\rho)\,\mathrm{d}\rho\right)-\alpha T\int_{t-T}^{t}\dot{e}^T(\rho)Q^{-1}\dot{e}(\rho)\,\mathrm{d}\rho,\qquad t\in[t_k,t_{k+1}).\end{aligned} \tag{19}$$

From (18) and (19), we obtain

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) \leq \xi^T \Pi \xi - \alpha T \int_{t-T}^{t} \dot{e}^T(\rho) Q^{-1} \dot{e}(\rho)\, \mathrm{d}\rho, t \in [t_k, t_{k+1}), \tag{20}$$

where

$$\Pi = \begin{bmatrix} -2\alpha Q^{-1} - C^T C + \frac{\gamma^2 \delta}{\pi} I & 0 & C^T C & 0 \\ 0 & -\pi I & 0 & I \\ C^T C & 0 & -\frac{(1-\alpha T) Q^{-1}}{T} & 0 \\ 0 & I & 0 & -\frac{\delta}{\pi} I \end{bmatrix} + \begin{bmatrix} (A - QC^T C)^T \\ 0 \\ (QC^T C)^T \\ G_1^T \end{bmatrix} T Q^{-1} \begin{bmatrix} (A - QC^T C)^T \\ 0 \\ (QC^T C)^T \\ G_1^T \end{bmatrix}^T .$$

On the other hand, according to Lemma 2, the condition (11) is equivalent to

$$\begin{bmatrix} -\alpha Q^{-1} - C^T C + \frac{\gamma^2 \delta}{\pi} I & 0 & C^T C & 0 \\ 0 & -\pi I & 0 & I \\ C^T C & 0 & -\frac{(1-\alpha T) Q^{-1}}{T} & 0 \\ 0 & I & 0 & -\frac{\delta}{\pi} I \end{bmatrix} + \begin{bmatrix} (A - QC^T C)^T \\ 0 \\ (QC^T C)^T \\ G_1^T \end{bmatrix} T Q^{-1} \begin{bmatrix} (A - QC^T C)^T \\ 0 \\ (QC^T C)^T \\ G_1^T \end{bmatrix}^T < 0. \tag{21}$$

Note that

$$V_2(t) \leq T \int_{t-T}^{t} \dot{e}^T(\rho) Q^{-1} \dot{e}(\rho)\, \mathrm{d}\rho. \tag{22}$$

From (20) – (21) and (22), it follows that

$$\dot{V}(t) \leq \xi^T \Pi \xi \leq -\alpha V_1(t) - \alpha T \int_{t-T}^{t} \dot{e}^T(\rho) Q^{-1} \dot{e}(\rho)\, \mathrm{d}\rho \leq -\alpha V(t), t \in [t_k, t_{k+1}).$$

Note that $e(t)$ is continuous, then,

$$\dot{V}(t) \leq -\alpha V(t), \qquad t \in [t_0, \infty),$$

which means that the estimation error system (8) is globally exponentially stable. The proof is completed. □

Remark 3.6. From (9) and (10), we have

$$\begin{bmatrix} -\alpha Q^{-1} - C^T C + \frac{\gamma^2 \delta}{\pi} I & 0 & 0 \\ 0 & -\pi I & I \\ 0 & I & -\frac{\delta}{\pi} I \end{bmatrix} < 0.$$

By Lemma 2.2, the matrix inequality (11) is equivalent to the following matrix inequality

$$\begin{bmatrix} -\alpha Q^{-1} - C^TC + \frac{\gamma^2\delta}{\pi}I & 0 & 0 \\ 0 & -\pi I & I \\ 0 & I & -\frac{\delta}{\pi}I \end{bmatrix} + T \begin{bmatrix} C^TC & (A-QC^TC)^T \\ 0 & 0 \\ 0 & G_1^T \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{1-\alpha T}Q + \frac{T^3}{(1-\alpha T)^2}Q(QC^TC)^TX_1^{-1}(QC^TC)Q & \frac{T}{1-\alpha T}Q(QC^TC)^TX_1^{-1} \\ \frac{T}{1-\alpha T}X_1^{-1}(QC^TC)Q & X_1^{-1} \end{bmatrix}$$

$$\begin{bmatrix} C^TC & (A-QC^TC)^T \\ 0 & 0 \\ 0 & G_1^T \end{bmatrix}^T < 0,$$

where $X_1 = Q - \frac{T^2}{1-\alpha T}(QC^TC)Q(QC^TC)^T$. This means that if the conditions in Theorem 3.1 are satisfied, then, there exits a sufficient small sampling period T such that the condition (11) is satisfied. In other words, the nonlinear systems considered in [4] always have a sampled-data observer in the form of (3).

The conditions (9) and (10) are not LMIs with respect to the variables Q^{-1}, α, π. However, once the parameter α is fixed, then, these conditions can be transformed into LMI-based conditions.

Theorem 3.7. For the error dynamics system (8) and a given parameter α, with full rank matrices C and G_1, if, $\mathrm{rank}(C^T) = \mathrm{rank}(G_1)$, and $(A+\alpha I, G_1, C)$ is minimum-phase, then, there exists a positive definite symmetric matrix Q, and two parameters $\pi_1 > 0$, $\delta_1 > \pi_1$ satisfy the following conditions,

$$\begin{bmatrix} (A+\alpha I)^TQ^{-1} + Q^{-1}(A+\alpha I) - C^TC & Q^{-1}G_1 \\ G_1^TQ^{-1} & -\pi_1 I \end{bmatrix} \leq 0, \tag{23}$$

$$-\alpha Q^{-1} + \gamma^2\delta_1 I < 0. \tag{24}$$

Moreover, for the matrix Q, and the parameters π_1 and δ_1 solved by (23) and (24), if there exists a sampling period T such that the condition

$$\begin{bmatrix} -\alpha Q^{-1} - C^TC + \frac{\gamma^2\delta}{\pi}I & 0 & C^TC & 0 & (A-QC^TC)^T \\ 0 & -\pi I & 0 & I & 0 \\ C^TC & 0 & -(\frac{1}{T}-\alpha)Q^{-1} & 0 & (QC^TC)^T \\ 0 & I & 0 & -\frac{\delta}{\pi}I & G_1^T \\ A-QC^TC & 0 & QC^TC & G_1 & -\frac{Q}{T} \end{bmatrix} < 0, \tag{25}$$

holds, where $\pi = \frac{1}{\pi_1}$ and $\delta = \frac{\delta_1}{\pi_1}$, then the estimation error system (8) is globally exponentially stable for whatever large Lipschtiz constant γ.

P r o o f. From Theorem 3.5, the proof can be easily obtained. □

As mentioned in [4], a large value of π may lead to peaking phenomenon. In order to relieve the peaking phenomenon, a stepwise schedule is provided to set this design parameter [4]. Based on Theorem 3.7, we give the following algorithm sketch to derive the optimal design parameter π and the sampling period T.

Step 1: Select the parameter α such that $(A+\alpha I, G_1, C)$ is minimum-phase.

Step 2: Solve the following optimal problem

$$\min \quad -\pi_1$$
$$\text{s.t.} \begin{cases} \begin{bmatrix} (A+\alpha I)^T Q^{-1} + Q^{-1}(A+\alpha I) - C^T C & Q^{-1}G_1 \\ G_1^T Q^{-1} & -\pi_1 I \end{bmatrix} \le 0, \\ \\ -\alpha Q^{-1} + \gamma^2 \delta_1 I < 0. \end{cases} \tag{26}$$

Step 3: Calculate $\pi = \frac{1}{\pi_1}$ and $\delta = \frac{\delta_1}{\pi_1}$;

Step 4: Solve the following optimal problem

$$\min \quad T_1$$
$$\text{s.t.} \begin{bmatrix} -\alpha Q^{-1} - C^T C + \frac{\gamma^2 \delta}{\pi} I & 0 & C^T C & 0 & (A - QC^T C)^T \\ 0 & -\pi I & 0 & I & 0 \\ C^T C & 0 & -(T_1 - \alpha) Q^{-1} & 0 & (QC^T C)^T \\ 0 & I & 0 & -\frac{\delta}{\pi} I & G_1^T \\ A - QC^T C & 0 & QC^T C & G_1 & -T_1 Q \end{bmatrix} < 0. \tag{27}$$

The optimal sampling period $T^* = \frac{1}{T_1^*}$, where T_1^* is the optimal solution to the optimal problem (27).

Remark 3.8. In [4], the parameter π is determined by the method of stepwise schedule. Whereas, in this paper, it can be obtained by solving the optimal problem (26). The purpose of doing so is to avoid the peaking phenomenon. Because if the parameter π is chosen too large, then the observer we proposed will become high-gain which may lead to the peaking phenomenon [12]. Once the parameters Q, π_1 and δ_1 are solved, the optimal sampling period T can be obtained by solving the optimal problem (27).

4. ROBUST SAMPLED-DATA NONLINEAR OBSERVER

In practice, there usually exist unknown external disturbances in the nonlinear system (1). In this section, we will research the effect of external disturbances on the sampled-data observer. The model of Lipschitz nonlinear system with external disturbance is given by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + G_1 f(x(t)) + G_2 d(t), \\ y(t) = Cx(t), \end{cases} \tag{28}$$

where the external disturbance $d(t) \in R^r$, the matrix $G_2 \in R^{n \times r}$, the system output $y(t)$ is sampled at time instant t_k, the other state vectors and the system matrices are defined in section 3. Moreover, we make the assumption that $d(t)$ is bounded, i. e.,

$$\|d(t)\| \le \tilde{d}, \tag{29}$$

where $\tilde{d}$ is a positive real. As in section 3, we assume that (A, C) is observable, and $(A, [G_1, G_2])$ is controllable. Then, the robust sampled-data observer for the system (28) is given by

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t_k) - C\hat{x}(t_k)) + G_1 f(\hat{x}(t)), \\ \hat{x}(t_{k+1}) = \lim_{t \to t_{k+1}^-} \hat{x}(t), \ t \in [t_k, t_{k+1}), \end{cases} \tag{30}$$

where the observer gain $L = QC^T \in R^{n\times p}$, and $Q \in R^{n\times n}$ can be obtained by solving the following modified Riccati equation

$$Q(A + \alpha I)^T + (A + \alpha I)Q - QC^T CQ + \pi[G_1, G_2][G_1, G_2]^T = 0, \alpha > 0, \ \pi > 0, \tag{31}$$

which is equivalent to

$$\begin{aligned} (A + \alpha I)^T Q^{-1} + Q^{-1}(A + \alpha I) - C^T C + \pi Q^{-1}[G_1, G_2][G_1, G_2]^T Q^{-1} = 0, \\ \alpha > 0, \ \pi > 0. \end{aligned} \tag{32}$$

From the discussion in Section 3, we also have that (32) is equivalent to

$$\begin{bmatrix} (A + \alpha I)^T Q^{-1} + Q^{-1}(A + \alpha I) - C^T C & Q^{-1}G \\ G^T Q^{-1} & -\frac{1}{\pi} I \end{bmatrix} \le 0, \tag{33}$$

where $G = [G_1, G_2]$.

Thus, from (28) and (30), the dynamics of the estimation error $e(t) = x(t) - \hat{x}(t)$ are given as

$$\begin{cases} \dot{e}(t) = Ae(t) - LCe(t_k) + GF(t), \\ e(t_{k+1}) = \lim\limits_{t \to t_{k+1}^-} e(t), \quad t \in [t_k, t_{k+1}), \end{cases} \tag{34}$$

where $F(t) = \begin{bmatrix} f(x(t)) - f(\hat{x}(t)) \\ d(t) \end{bmatrix}$.

In the following theorem, we give LMI-based sufficient conditions such that the estimation error $x(t) - \hat{x}(t)$ converges to a small region of the origin even if there exists large disturbance.

Theorem 4.1. For the Lipschitz nonlinear system (28) with bounded disturbance $d(t)$, and full rank matrices C and G, if, $\text{rank}(C^T) = \text{rank}(G)$, and $(A+\alpha I, G, C)$ is minimum-phase, then, there exists a symmetric positive definite matrix Q^{-1} and two positive parameters $\pi > 0$, $\epsilon > 1$ such that the following conditions

$$\begin{bmatrix} (A + \alpha I)^T Q^{-1} + Q^{-1}(A + \alpha I) - C^T C & Q^{-1}G \\ G^T Q^{-1} & -\frac{1}{\pi} I \end{bmatrix} \le 0, \tag{35}$$

$$-\alpha Q^{-1} + \frac{\gamma^2 \epsilon}{\pi} I < 0, \tag{36}$$

hold. Moreover, for the matrix Q^{-1} and the parameter π solved by (35) and (36), if the sampling period T satisfies the following condition

$$\begin{bmatrix} -\alpha Q^{-1} - C^TC + \frac{\gamma^2\epsilon}{\pi}I & 0 & C^TC & 0 & (A-QC^TC)^T \\ 0 & -\pi I & 0 & I & 0 \\ C^TC & 0 & -(\frac{1}{T}-\alpha)Q^{-1} & 0 & (QC^TC)^T \\ 0 & I & 0 & -\frac{\epsilon}{\pi}I & G^T \\ A-QC^TC & 0 & QC^TC & G & -\frac{Q}{T} \end{bmatrix} < 0, \quad (37)$$

then, the estimation error system (34) converges to a small region of the origin. The size of the region approaches to zero when the design parameter π approaches infinity.

Proof. From the proof of Theorem 3.5, there always exists a symmetric positive definite matrix Q and two positive parameters $\pi > 0$, $\epsilon > 1$ such that the conditions (35) and (36) are satisfied.

Next, we will prove that the estimation error system (34) converges to a small region of the origin if the condition (37) is satisfied. In fact, the estimation error system (34) can be rewritten as

$$\begin{cases} \dot{e}(t) = (A-LC)e(t) + LC\displaystyle\int_{t_k}^{t} \dot{e}(\rho)\,\mathrm{d}\rho + GF(t), t \in [t_k, t_{k+1}), \\ e(t_{k+1}) = \displaystyle\lim_{t \to t_{k+1}^-} e(t), \quad t \in [t_k, t_{k+1}). \end{cases} \quad (38)$$

From the inequalities (2) and (29), we have

$$F^T(t)F(t) \leq \gamma^2 e^T(t)e(t) + \tilde{d}^2,$$

which implies that

$$\frac{\gamma^2\epsilon}{\pi}e^T(t)e(t) + \frac{\epsilon\tilde{d}^2}{\pi} - \frac{\epsilon}{\pi}F^T(t)F(t) \geq 0. \quad (39)$$

Consider the Lyapunov function (13) and calculate the derivatives of $V_1(t)$ and $V_2(t)$ along the system (38). Then,

$$\begin{aligned} \dot{V}_1(t) =& e^T(t)[(A-LC)^TQ^{-1} + Q^{-1}(A-LC)]e(t) + \int_{t_k}^{t} \dot{e}^T(\rho)\,\mathrm{d}\rho(LC)^TQ^{-1}e(t) \\ &+ e^T(t)Q^{-1}LC\int_{t_k}^{t}\dot{e}(\rho)\,\mathrm{d}\rho + F^T(t)G^TQ^{-1}e(t) + e^T(t)Q^{-1}GF(t),\ t \in [t_k, t_{k+1}). \end{aligned}$$

Applying (32) and (39), we have

$$\dot{V}_1(t) \leq \Gamma^T \begin{bmatrix} -\alpha Q^{-1} - C^TC + \frac{\gamma^2\epsilon}{\pi}I & 0 & C^TC & 0 \\ 0 & -\pi I & 0 & I \\ C^TC & 0 & 0 & 0 \\ 0 & I & 0 & -\frac{\epsilon}{\pi}I \end{bmatrix} \Gamma - \alpha V_1(t) + \frac{\epsilon\tilde{d}^2}{\pi},$$

$t \in [t_k, t_{k+1})$,

where $\Gamma = \begin{bmatrix} e(t) \\ G^T Q^{-1} e(t) \\ \int_{t_k}^t \dot{e}(\rho)\,\mathrm{d}\rho \\ F(t) \end{bmatrix}$, and

$$\dot{V}_2(t) \le T\Gamma^T \begin{bmatrix} (A - QC^TC)^T \\ 0 \\ (QC^TC)^T \\ G^T \end{bmatrix} Q^{-1} \begin{bmatrix} A - QC^TC & 0 & QC^TC & G \end{bmatrix} \Gamma$$
$$- (\frac{1}{T} - \alpha) \int_{t_k}^t \dot{e}^T(\rho)\,\mathrm{d}\rho Q^{-1} \int_{t_k}^t \dot{e}(\rho)\,\mathrm{d}\rho - \alpha V_2(t).$$

Thus,

$$\dot{V}(t) \le \Gamma^T \Omega \Gamma - \alpha \left(V_1(t) + V_2(t) - \frac{\epsilon \tilde{d}^2}{\alpha\pi} \right), t \in [t_k, t_{k+1}),$$

where

$$\Omega = \begin{bmatrix} -\alpha Q^{-1} - C^TC + \frac{\gamma^2 \epsilon}{\pi} I & 0 & C^TC & 0 \\ 0 & -\pi I & 0 & I \\ C^TC & 0 & -(\frac{1}{T} - \alpha) Q^{-1} & 0 \\ 0 & I & 0 & -\frac{\epsilon}{\pi} I \end{bmatrix}$$
$$+ \begin{bmatrix} (A - QC^TC)^T \\ 0 \\ (QC^TC)^T \\ G^T \end{bmatrix} TQ^{-1} \begin{bmatrix} A - QC^TC & 0 & QC^TC & G \end{bmatrix}.$$

By Lemma 2, we know that $\Omega < 0$ is equivalent to the condition (37). Hence,

$$\dot{V}(t) < -\alpha \left(V(t) - \frac{\epsilon \tilde{d}^2}{\alpha\pi} \right), t \in [t_k, t_{k+1}).$$

Since $e(t)$ is continuous on $[t_0, \infty)$, then,

$$\dot{V}(t) < -\alpha \left(V(t) - \frac{\epsilon \tilde{d}^2}{\alpha\pi} \right), t \in [t_0, \infty).$$

Note that $V(t) > \frac{\epsilon D^2}{\alpha\pi}$ implies that $\dot{V}(t) < 0$. Therefore,

$$\lim_{t\to\infty} V_1(t) \le \lim_{t\to\infty} V(t) \le \frac{\epsilon \tilde{d}^2}{\alpha\pi}.$$

On the other hand,

$$V_1(t) \ge \lambda_{\min}(Q^{-1}) e^T(t) e(t) = \frac{e^T(t) e(t)}{\lambda_{\max}(Q)}.$$

Thus,

$$\lim_{t\to\infty} e^T(t)e(t) \le \frac{\epsilon \tilde{d}^2}{\alpha} \cdot \frac{\lambda_{\max}(Q)}{\pi}. \tag{40}$$

Since $\lim_{\pi\to\infty} \frac{\lambda_{\max}(Q)}{\pi} = 0$, we conclude that the state estimation error $e(t)$ will become small enough as long as π is sufficiently large even if there exists large external disturbance. The proof is completed. □

The conditions (35) and (36) are not LMIs with respect to the variables Q^{-1}, α, π. However, once the parameter α is fixed, then, these conditions can be transformed into LMI-based conditions.

Theorem 4.2. For the Lipschitz nonlinear system (28) and a given parameter α with bounded disturbance $d(t)$, and full rank matrices C and G, if, $\mathrm{rank}(C^T) = \mathrm{rank}(G)$, and $(A+\alpha I, G, C)$ is minimum-phase, then, there exists a symmetric positive definite matrix Q^{-1} and two positive parameters $\pi_1 > 0$,

$$\begin{bmatrix} (A+\alpha I)^T Q^{-1} + Q^{-1}(A+\alpha I) - C^T C & Q^{-1}G \\ G^T Q^{-1} & -\pi_1 I \end{bmatrix} \le 0, \tag{41}$$

$$-\alpha Q^{-1} + \gamma^2 \epsilon_1 I < 0, \tag{42}$$

hold. Moreover, for the matrix Q^{-1}, and the parameters ϵ_1 and π_1 solved by (41) and (42), if the sampling period T satisfies the following condition

$$\begin{bmatrix} -\alpha Q^{-1} - C^T C + \frac{\gamma^2 \epsilon}{\pi} I & 0 & C^T C & 0 & (A - QC^T C)^T \\ 0 & -\pi I & 0 & I & 0 \\ C^T C & 0 & -(\frac{1}{T} - \alpha) Q^{-1} & 0 & (QC^T C)^T \\ 0 & I & 0 & -\frac{\epsilon}{\pi} I & G^T \\ A - QC^T C & 0 & QC^T C & G & -\frac{Q}{T} \end{bmatrix} < 0, \tag{43}$$

where $\pi = \frac{1}{\pi_1}$ and $\epsilon = \frac{\epsilon_1}{\pi_1}$, then, the estimation error system (34) converges to a small region of the origin. The size of the region approaches to zero when the design parameter π approaches infinity.

P r o o f. From Theorem 4.1, the proof can be easily obtained. □

Based on Theorem 4.2, we give the following algorithm sketch to derive the optimal sampling period T^*.

Step 1: Select the parameter α such that $(A+\alpha I, G, C)$ is minimum-phase.

Step 2: Solve the following optimal problem

$$\min \quad -\pi_1$$

$$\text{s.t.} \begin{cases} \begin{bmatrix} (A+\alpha I)^T Q^{-1} + Q^{-1}(A+\alpha I) - C^T C & Q^{-1}G \\ G^T Q^{-1} & -\pi_1 I \end{bmatrix} \le 0, \\ \\ -\alpha Q^{-1} + \gamma^2 \epsilon_1 I < 0. \end{cases} \tag{44}$$

Step 3: Calculate $\pi = \frac{1}{\pi_1}$ and $\epsilon = \frac{\epsilon_1}{\pi_1}$;

Step 4: Solve the following optimal problem

$$\min \quad T_1$$

$$\text{s.t.} \begin{bmatrix} -\alpha Q^{-1} - C^T C + \frac{\gamma^2 \epsilon}{\pi} I & 0 & C^T C & 0 & (A - QC^T C)^T \\ 0 & -\pi I & 0 & I & 0 \\ C^T C & 0 & -(T_1 - \alpha) Q^{-1} & 0 & (QC^T C)^T \\ 0 & I & 0 & -\frac{\epsilon}{\pi} I & G_1^T \\ A - QC^T C & 0 & QC^T C & G_1 & -T_1 Q \end{bmatrix} < 0. \tag{45}$$

The optimal sampling period $T^* = \frac{1}{T_1}$, where T_1 is the optimal solution to the optimal problem (45).

Remark 4.3. There is a minimum-phase condition in both Theorem 3.7 and Theorem 4.2. However, for example, if the minimum-phase condition is removed in Theorem 4.2, and there exist Q, ϵ_1, π_1 and T such that the conditions (41), (42) and (43) hold, then, the error system (34) still converges to a small region of the origin.

Remark 4.4. In [23, 30, 31], sampled-data observers are designed for nonlinear systems by using the high gain observer techniques. The upper bounds of the sampling period T are dependent on the high gains. Due to use of conservative estimations in the proofs, the sampling periods are very small, for example, in Example 3.1 [31], $T = 2 \times 10^{-5} s$. In [4], the observer gain is obtained by solving a LQ Riccati equation. In order to relieve the peaking phenomenon, the design parameter π is determined by the method of stepwise schedule. Whereas, the observer gain and the optimal design parameter π can be derived by solving LMI-based optimal problem (27) or (45), which can be settled numerically tractable and efficiently by Matlab LMI Toolbox [25].

5. SIMULATION RESULTS

Consider a flexible joint robot borrowed from [15]. The system matrices are given as follows

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & -1 \end{bmatrix}, \; G_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \; B = G_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \; C^T = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix},$$

the control input $u(t) = 0$, the nonlinear function $f(x) = -3\sin(x_1)$, and the external disturbance $d(t) = 5$. It is obvious that $f(x)$ is globally Lipschitz with the Lipschitz constant $\gamma = 3$. We choose the design parameter $\alpha = 3$, then solve the optimal problem (44) by Matlab LMI Toolbox, we obtain the parameters $\pi = 3.9370e + 3$, $\epsilon = 1.0001$, sampling period $T = \frac{1}{4.8079e+4}(s)$, and the sampled robust observer gain

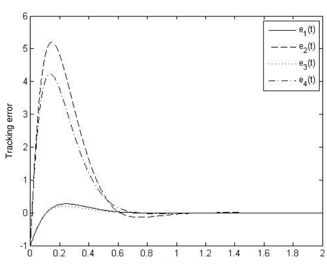
$$L = \begin{bmatrix} 0.0537 & 17.9633 \\ 0.9104 & 107.4391 \\ 19.2613 & 0.0537 \\ 103.7872 & 0.9837 \end{bmatrix}.$$

From the optimal problem (45), we can obtain that the optimal sampling period T is determined by π. However, the optimal design parameter π may not yield the optimal sampling period. We make a list of several schemes in Table 1 by choosing different design parameters π and solving the optimal problem (45). The simulation diagrams are shown in Figure 1 with the initial conditions $x(0) = [1; 1; 1; 1]$, $\hat{x}(0) = [2; 2; 2; 2]$. $e_1(t), \dots, e_4(t)$ are the state estimation errors of flexible joint robot system. And Figure (a)$-$(f) in Figure 1 are the state estimation errors simulation diagrams corresponding to Scheme 1$-$6, respectively. From the simulation results, we can obtain the following conclusions:

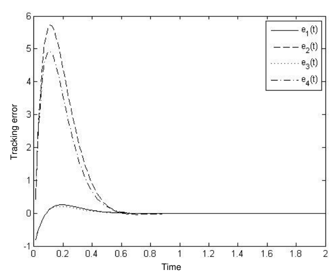
The simulation diagrams of state estimation errors are shown in Figure 1. From Figure 1, we can obtain the following conclusions. The sampling frequency will be too large if the design parameter π is too small or too large. The peaking phenomenon of error dynamics system will become more and more serious along with the increasing of design parameter π. However, a large value of π will lead to a fast convergence speed of state estimation errors.

scheme	parameter(π)	parameter(ϵ)	sampling frequency(f)
1	4e+3	1.0031	2000Hz
2	7e+3	1.0949	96Hz
3	1e+4	1.1806	83Hz
4	1e+5	1.7149	155Hz
5	1e+6	2.4612	397Hz
6	1e+7	3.9091	1118Hz

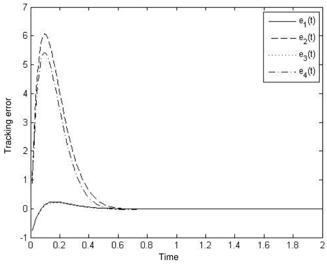
Tab. 1. Parameters correspond to each scheme.



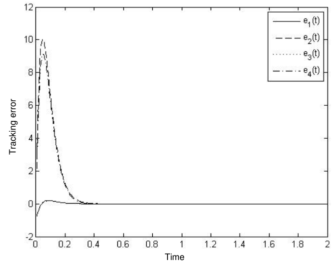
(a) Scheme 1



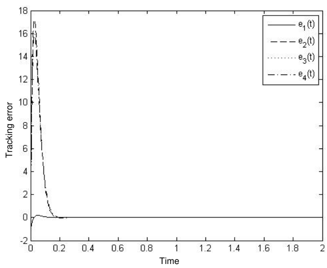
(b) Scheme 2



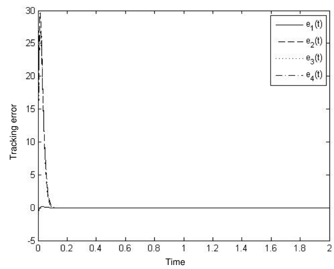
(c) Scheme 3



(d) Scheme 4



(e) Scheme 5



(f) Scheme 6

Fig. 1. State estimation errors simulation diagram.

6. CONCLUSION

In this paper, a robust sampled-data observer was proposed for Lipschitz nonlinear systems. Under the minimum-phase condition, it was shown that there always exists a sampling period such that the estimation errors converge to zero for whatever large Lipschitz constant. The optimal sampling period could also be achieved by solving a LMI-based optimal problem. The design methods were extended to Lipschitz nonlinear systems with large external disturbances as well. In such a case, the estimation errors converged to a small region of the origin. The size of the region could be small enough by selecting a proper parameter. Compared with the existing results, the design parameters could be easily obtained by solving LMIs.

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