Xiao Yu; Fucheng Liao

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A TRACKING CONTROLLER DESIGN WITH PREVIEW ACTION FOR A CLASS OF NONLINEAR LUR'E SYSTEMS WITH TIME-VARYING DELAYS AND EXTERNAL DISTURBANCES

Xiao Yu and Fucheng Liao

In this paper, the tracking control problem for a class of discrete-time nonlinear Lur'e systems with time-varying delays and external disturbances is studied via a preview control method. First, a novel translation approach is introduced to construct the augmented error system for Lur'e systems. The output tracking problem is thereby transformed into a guaranteed cost H_∞ controller design problem. To produce an integral control action that can eliminate the static error, a discrete integrator is included. Next, a memory state feedback controller is developed, and the sufficient conditions for asymptotic stability and guaranteed cost H_∞ performance of the closed-loop system are established by applying a suitable Lyapunov-Krasovskii functional and the linear matrix inequality (LMI) technique. Based on this, the tracking control scheme with preview action for the original system is presented. Finally, the effectiveness of our proposed control method is illustrated via a numerical example.

Keywords: tracking control, preview control, nonlinear Lur'e system, linear matrix inequality

Classification: 93Cxx, 93Dxx

1. INTRODUCTION

Preview control, which is an important and effective control technique, is able to fully utilize known future information about reference signals or disturbances to improve the closed-loop transient response and enhance tracking quality [2, 36, 43]. In the 1960s, three models of preview control were first proposed in [31]. Subsequently, the basic idea of preview control gradually became popular, and considerable research results have been reported in the literature. Based on linear quadratic regulation (LQR) theory and the error system method, an optimal state feedback controller with disturbance preview action, which could enhance the robustness of the closed-loop system, was proposed for linear discrete-time systems in [35]. Following the approach of [35], the preview control problem of linear discrete-time systems, under the assumption that the reference signal was able to be previewed, was addressed in [12]. The result was further developed in [11] where the preview controller was proposed for linear continuous-time systems. As

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an extension of the above results, in [20, 21], the preview controller design for linear descriptor systems was investigated in both discrete- and continuous-time domains. In practical applications, some phenomena, such as time delays and external disturbances, are usually unavoidable. For this reason, some recent studies have taken these complexities into account. For example, according to the H_∞ control principle and the operator Riccati equation, the preview controller for linear time-delay systems was discussed in [16]. In addition, some mixed control methods, such as LQR control, preview control and H_∞ control, have also been proposed to address the tracking problem of linear uncertain systems, such as those in [33] and [44]. However, most of the existing preview controller design methods focus on certain linear systems, whereas very few results can be immediately applied to nonlinear control systems. Moreover, when facing a variety of complex nonlinear dynamics, researchers are still unable to find convenient and effective universal approaches to design the preview controller. The preview control of nonlinear systems, therefore, is a challenging problem that exactly explains the importance of this current study.

The Lur'e system refers to a nonlinear dynamic system that consists of a linear system and a nonlinear feedback loop satisfying certain sector constraints [13, 14, 25]. In fact, this class of nonlinear systems covers various important physical systems, such as genetic oscillators [18], Chua's circuits [26, 27] and cellular neural networks [6, 8]. Owing to the theoretical and practical significance of these systems in control engineering, a tremendous amount of research has been done on the control problems of nonlinear Lur'e systems. For instance, by using the convex combination method, the H_∞ controller design was presented for a class of continuous-time Lur'e systems in [29]. Considering the influence of time-varying delays, the H_∞ control problem of continuous-time Lur'e systems was revisited in [38], in which a novel proportional derivative feedback technique was constructed. Generally, state feedback control strategies were used in [29] and [38]. From the point of view of applications, static and dynamic output feedback control strategies were preferred by engineers [15, 41]. Furthermore, several synchronization control problems were studied for chaotic Lur'e systems in [4, 10, 17, 22, 23]. This was especially true in [19], where the globally synchronised regions of linearly coupled Lur'e systems were analysed via a decomposition method. More recently, the tracking control with preview action was investigated for a class of continuous-time Lur'e nonlinear systems in [39]. In addition to these works, some other control problems, including adaptive pinning control [7, 32, 34] and consensus tracking control [24, 42], were successfully developed for nonlinear systems in the Lur'e form. However, to the best of our knowledge, until now, the preview controller design for nonlinear Lur'e systems has been relatively rare. In addition, the external disturbances and time delays are frequently encountered in various dynamical systems, and they may degrade the performance of the systems or even lead to instability [5, 30]. Therefore, the preview controller design for Lur'e systems with time-varying delays and external disturbances is of great significance, and it has not yet been reported in the literature.

Motivated by the above discussion, in this paper, the preview controller design for a class of discrete-time disturbed nonlinear Lur'e systems with time-varying delays is investigated. First, according to the intrinsic characteristics of the system under consideration, an appropriate translation method is chosen to construct an augmented error system with available future reference information. Then, the output tracking problem is reduced to a guaranteed cost H_∞ control problem. To make full use of the stored current and previous state information, a memory state feedback controller is developed. Meanwhile, based on the Lyapunov–Krasovskii functional approach and the LMI technique, the sufficient conditions for asymptotic stability and guaranteed cost H_∞ performance of the resulting closed-loop system are systematically discussed. Finally, the preview controller design method for the original system is presented. Compared with the previous works [39, 40], the main contributions of the present paper are outlined as follows: (i) for the first time, a novel preview control scheme is proposed for the output tracking problem of discrete-time disturbed nonlinear Lur'e systems with time-varying delays via a translation approach; (ii) because the tracking error is not used as usual in the state augmentation process, the construction of the augmented error system is simplified, which effectively reduces the complexity of the controller design; and (iii) sufficient linear matrix inequality conditions are established, which ensure that the closed-loop system is not only asymptotically stable but also satisfies the requirement of guaranteed cost H_∞ performance.

This paper is organized as follows. In section 2, the system description and some preliminaries are given. The construction of the augmented error system is presented in Section 3. The design method of preview controller is demonstrated in Section 4. Section 5 gives a simulation example to verify the performance of our method. Section 6 concludes the paper.

Notations: R^n denotes the n-dimensional Euclidean space; $R^{n\times m}$ denotes the $n \times m$ matrix space; For a matrix P, $P > 0$ ($P < 0$) means that P is a real symmetric positive (or negative) definite matrix; $P > Q$ stands for $P - Q > 0$; I and 0 denote the identity matrix and the zero matrix with appropriate dimension, respectively; $l_2\,[0, \infty)$ refers to the space of square summable infinite vector sequences, and for $\omega(k) \in l_2\,[0,\infty)$, its norm is given by $\|\omega\|_2 = \sqrt{\sum_{k=0}^{\infty} \omega^T(k)\omega(k)}$. M^T denotes the transpose of the matrix M. M^{-T} denotes the inverse of the matrix M^T, namely $(M^T)^{-1}$. The symbol "*" is used to represent the symmetric term in a symmetric matrix.

2. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider the discrete-time nonlinear system

$$\begin{cases} x(k+1) = Ax(k) + A_d x(k - d(k)) + Bu(k) + Df(y(k)) + E\omega(k), \\ y(k) = Cx(k), \end{cases} \tag{1}$$

where $x(k) \in R^n$ is the state vector, $u(k) \in R^m$ is the control input, and $y(k) \in R^p$ is the output vector. $\omega(k) \in R^q$ is the external disturbance vector. A, A_d, B, D, E and C are known real constant matrices of appropriate dimensions. It is assumed that the time-varying delay $d(k)$ is known and satisfies $0 \le d_1 \le d(k) \le d_2$. $f(y) \in R^p$ denotes a memoryless time-invariant nonlinearity.

In this paper, the following assumptions are required:

Assumption 1. The nonlinearity $f(\cdot) \in R^p$ is in the form of

$$f(y) = \begin{bmatrix} f_1(y_1) & f_2(y_2) & \dots & f_p(y_p) \end{bmatrix}^T,$$

where $f_i(0) = 0$, and there exist constants $\underline{k}_i, \bar{k}_i \in R$ and $\underline{k}_i < \bar{k}_i$ such that

$$\left(f_i(s_2) - f_i(s_1) - \underline{k}_i(s_2 - s_1)\right)\left(f_i(s_2) - f_i(s_1) - \bar{k}_i(s_2 - s_1)\right) \le 0, s_1 \neq s_2, \tag{2}$$

for $i = 1, 2, \dots, p$.

Remark 2.1. As shown in [13], under Assumption 1, the nonlinear term $f_i(\cdot)$ belongs to sector $[\underline{k}_i, \bar{k}_i]$. In Assumption 1, if the inequality constraint (2) is replaced with

$$\underline{k}_i \le \frac{f_i(s_2) - f_i(s_1)}{s_2 - s_1} \le \bar{k}_i, s_2 \neq s_1, \tag{3}$$

the result obtained in this paper is still valid. Indeed, all nonlinearities satisfying the inequality (3) satisfy inequality condition (2).

Assumption 2.

$$rank \begin{bmatrix} A + A_d - I & B \\ C & 0 \end{bmatrix} = n + p. \text{ (full row rank)}$$

Assumption 3. The external disturbance $\omega(k)$ converges to a constant vector ω as $k \to \infty$, i. e., $\lim_{k\to\infty} \omega(k) = \omega$. Additionally, the difference vector between the disturbance and its limit belongs to $l_2\,[0,\infty)$, i. e., $\omega(k) - \omega \in l_2\,[0,\infty)$.

Assumption 4. The reference signal $r(k)$ converges to a constant vector r as $k \to \infty$, i. e., $\lim_{k\to\infty} r(k) = r$. Furthermore, it is assumed that $r(k)$ is able to be previewed, and the preview length is M_r, that is, at each time k, M_r future values $r(k+1), \dots, r(k+M_r)$ as well as the present and past values of the reference signal are available. Additionally, the future values of the reference signal beyond $k + M_r$ are assumed to be a constant vector r, namely,

$$r(k + i) = r, \;\; i = M_r + 1, \;\; M_r + 2, \dots.$$

Remark 2.2. Assumption 4 describes the preview property of the reference signal $r(k)$, and it is an important and basic assumption in the field of preview control. Future information about the reference signal is less important as it exceeds the preview range. Therefore, it is usually assumed that the values beyond the preview length are constant [2, 31, 35, 36, 43].

The following technical lemmas are employed to establish our main results.

Lemma 2.3. (Schur complement lemma, Schur et al. [3]) The symmetric matrix $\begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix} < 0$ if and only if one of the following two conditions is satisfied:

(i) $S_{11} < 0, \;\; S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(ii) $S_{22} < 0, \;\; S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 2.4. (Nasiri et al. [28]) If there exist matrices $P > 0$ and G with appropriate dimensions, then

$$-G^T P^{-1} G \le P - G - G^T.$$

3. CONSTRUCTION OF THE AUGMENTED ERROR SYSTEM

In this section, we construct the augmented error system of system (1) via a translation approach.

Under Assumptions 3 and 4, if the closed-loop system of system (1) can track the reference signal, there are steady-state values $x(\infty)$ and $u(\infty)$ such that

$$\begin{cases} x(\infty) = Ax(\infty) + A_d x(\infty) + Bu(\infty) + Df(r) + E\omega, \\ r = Cx(\infty), \end{cases} \tag{4}$$

that is,

$$\begin{bmatrix} A + A_d - I & B \\ C & 0 \end{bmatrix} \begin{bmatrix} x(\infty) \\ u(\infty) \end{bmatrix} = \begin{bmatrix} -Df(r) - E\omega \\ r \end{bmatrix}. \tag{5}$$

When Assumption 2 holds, equation (5) obtains the solution $x(\infty), u(\infty)$.

Define the new variables:

$$\begin{cases} \tilde{x}(k) = x(k) - x(\infty), \\ \tilde{u}(k) = u(k) - u(\infty), \\ \tilde{y}(k) = y(k) - r, \\ \tilde{r}(k) = r(k) - r, \\ \tilde{\omega}(k) = \omega(k) - \omega. \end{cases} \tag{6}$$

From $(1), (4)$ and (6), we obtain the following dynamics:

$$\begin{cases} \tilde{x}(k+1) = A\tilde{x}(k) + A_d\tilde{x}(k - d(k)) + B\tilde{u}(k) + D\tilde{f}(\tilde{y}(k)) + E\tilde{\omega}(k), \\ \tilde{y}(k) = C\tilde{x}(k), \end{cases} \tag{7}$$

where $\tilde{f}(\tilde{y}(k)) = f(y(k)) - f(r)$, and from Assumption 3 one obtains $\tilde{\omega}(k) \in l_2\,[0, \infty)$.

Denote $\tilde{y}_i = y_i - r_i$ and $\tilde{f}_i(\tilde{y}_i) = f_i(y_i) - f_i(r_i)$. By Assumption 1, one can see that

$$(\tilde{f}_i(\tilde{y}_i) - \underline{k}_i\tilde{y}_i)(\tilde{f}_i(\tilde{y}_i) - \bar{k}_i\tilde{y}_i) \leq 0. \tag{8}$$

Equation (8) yields

$$(\tilde{f}(\tilde{y}) - \underline{K}\tilde{y})^T(\tilde{f}(\tilde{y}) - \bar{K}\tilde{y}) \leq 0, \tag{9}$$

where $\underline{K} = diag(\underline{k}_1, \underline{k}_2, \ldots, \underline{k}_p)$ and $\bar{K} = diag(\bar{k}_1, \bar{k}_2, \ldots, \bar{k}_p)$.

By employing the loop transformation [13], the restriction equation (9) is transformed into the following condition

$$\phi^T(\tilde{y})\left(\phi(\tilde{y}) - (\bar{K} - \underline{K})\tilde{y}\right) \leq 0, \tag{10}$$

where $\phi(\tilde{y}) = \tilde{f}(\tilde{y}) - \underline{K}\tilde{y}$. Hence, system (7) can be rewritten as

$$\begin{cases} \tilde{x}(k+1) = (A + D\underline{K}C)\tilde{x}(k) + A_d\tilde{x}(k - d(k)) + B\tilde{u}(k) + D\phi(\tilde{y}(k)) + E\tilde{\omega}(k), \\ \tilde{y}(k) = C\tilde{x}(k), \end{cases} \tag{11}$$

where $\phi(\tilde{y})$ satisfies (10), that is, $\phi(\tilde{y})$ belongs to the sector $[0, \bar{K} - \underline{K}]$.

To make full use of the known future knowledge about the reference signal $r(k)$, we define the following vector:

$$x_r(k) = \begin{bmatrix} \tilde{r}(k) \\ \tilde{r}(k+1) \\ \vdots \\ \tilde{r}(k+M_r) \end{bmatrix}.$$

Under Assumption 4, it is easily seen that

$$x_r(k+1) = A_r x_r(k), \tag{12}$$

where

$$A_r = \begin{bmatrix} 0 & I_p & 0 & \dots & 0 \\ 0 & 0 & I_p & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & I_p \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}.$$

Note that the vector $x_r(k)$ contains all available future information about the reference signal at the current time k. By means of the lifting technique [20, 44], equations (11) and (12) can be combined together, and then such useful information can be successfully input into the control input.

Define the tracking error as $e(k) = y(k) - r(k)$. As noted in [40], the integral control action of the tracking error $e(k)$ possesses the capability to effectively eliminate static error. Since the control input in system (11) is $\tilde{u}(k)$ rather than the difference of $\tilde{u}(k)$, the designed controller for the augmented system consisting of (11) and (12) does not include the integral control action of the tracking error. Therefore, to avoid the static error, we add the following integrator:

$$v(k+1) = v(k) + e(k). \tag{13}$$

If $v(k)$ possesses a steady-state value $v(\infty)$ such that $\lim_{k\to\infty} v(k) = v(\infty)$, then it follows that $\lim_{k\to\infty} e(k) = 0$. To this end, denote $\tilde{v}(k) = v(k) - v(\infty)$, and from (13), the dynamics of $\tilde{v}(k)$ can be expressed as

$$\begin{aligned} \tilde{v}(k+1) &= \tilde{v}(k) + e(k) \\ &= \tilde{v}(k) + (y(k) - r) - (r(k) - r) \\ &= \tilde{v}(k) + \tilde{y}(k) - \tilde{r}(k) \\ &= \tilde{v}(k) + C\tilde{x}(k) - \tilde{r}(k). \end{aligned} \tag{14}$$

To synthesize a tracking controller with preview action plus the above integrator, we define the augmented state vector $\bar{x}(k) = \begin{bmatrix} \tilde{x}^T(k) & x_r^T(k) & \tilde{v}^T(k) \end{bmatrix}^T$. Combining (11), (12) and (14) yields

$$\begin{cases} \bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{A}_d\bar{x}(k-d(k)) + \bar{B}\tilde{u}(k) + \bar{D}\phi(\tilde{y}(k)) + \bar{E}\tilde{\omega}(k), \\ \tilde{y}(k) = \widetilde{C}\bar{x}(k), \end{cases} \tag{15}$$

where

$$\bar{A}=\begin{bmatrix} A+D\underline{K}C & 0 & 0 \\ 0 & A_r & 0 \\ C & G_r & I \end{bmatrix},\ G_r=\begin{bmatrix} -I & 0 & \dots & 0 \end{bmatrix},\ \bar{A}_d=\begin{bmatrix} A_d & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\bar{B}=\begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix},\ \bar{E}=\begin{bmatrix} E \\ 0 \\ 0 \end{bmatrix},\ \bar{D}=\begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix},\ \tilde{C}=\begin{bmatrix} C & 0 & 0 \end{bmatrix}.$$

For performance assessment, we adopt the following quadratic cost function:

$$J=\sum_{k=0}^{\infty}\left[\tilde{v}^T(k)Q_v\tilde{v}(k)+\tilde{u}^T(k)R\tilde{u}(k)\right], \tag{16}$$

where $Q_v > 0$ and $R > 0$ are given weighting matrices. In J, the first term embodies the requirement of small accumulative errors, and the second term reflects the limitation of the control range; thus, the physical meaning is clear.

Furthermore, the cost function (16) can also be rewritten as a square of the two-norm of the following performance signal:

$$z(k)=M\bar{x}(k)+N\tilde{u}(k), \tag{17}$$

where

$$M=\begin{bmatrix} 0 & 0 & Q_v^{1/2} \\ 0 & 0 & 0 \end{bmatrix},\ N=\begin{bmatrix} 0 \\ R^{1/2} \end{bmatrix}.$$

In other words,

$$J=\sum_{k=0}^{\infty} z^T(k)z(k)=\|z\|_2^2. \tag{18}$$

Combining (15) and (17) yields the following Lur'e-type augmented error system

$$\begin{cases} \bar{x}(k+1)=\bar{A}\bar{x}(k)+\bar{A}_d\bar{x}(k-d(k))+\bar{B}\tilde{u}(k)+\bar{D}\phi(\tilde{y}(k))+\bar{E}\tilde{\omega}(k), \\ \tilde{y}(k)=\tilde{C}\bar{x}(k), \\ z(k)=M\bar{x}(k)+N\tilde{u}(k), \end{cases} \tag{19}$$

where $\phi(\tilde{y})$ satisfies the sector condition (10).

Remark 3.1. In the existing literature regarding preview control, there are two convenient and effective approaches, namely, the difference approach [20, 33, 44] and the auxiliary method [40], for constructing the required preview control system. However, the drawback of these two methods is that they fail to deal with systems with time-varying delays. To overcome this difficulty, a novel translation technique, which makes use of the steady-state values related to the state vector and control input, is provided. Moreover, compared with [40], the tracking error is not embedded into the augmented error system (19) as a state component. This treatment helps to reduce the dimension of the augmented system and simplify the structure of the preview controller.

4. DESIGN OF THE PREVIEW CONTROLLER

In this section, inspired by [29, 33, 40], we wish to design a preview controller such that: (i) the closed-loop output $y(k)$ asymptotically tracks the reference signal $r(k)$, that is, $\lim_{k\to\infty} e(k) = 0$; and (ii) the guaranteed cost H_∞ control performance is satisfied, that is,

$$J = \|z\|_2^2 \leq \gamma^2 \|\tilde{\omega}\|_2^2 + J^*,$$

where $\gamma > 0$ is a prescribed H_∞ performance level, and J^* is a certain upper bound of the cost function.

We design a memory state feedback controller in the following form:

$$\tilde{u}(k) = K\bar{x}(k) + L\bar{x}(k - d(k)), \tag{20}$$

where K and L are the controller gains to be determined.

Remark 4.1. As commented in [37], because they consider more information, memory controllers usually lead to better performances than those of memoryless controllers for the designed systems. However, in this paper, the time delay is assumed to be known, which is restrictive to some extent. Especially when the delayed states are inaccessible or too expensive to be measured, memory controller realization becomes difficult. A possible choice is to use the maximum delay bound d_2 for control. Another way is to implement the conventional memoryless state feedback control strategy by setting $L = 0$.

Substituting the controller (20) into system (19), we obtain the closed-loop system as follows:

$$\begin{cases} \bar{x}(k+1) = (\bar{A} + \bar{B}K)\bar{x}(k) + (\bar{A}_d + \bar{B}L)\bar{x}(k - d(k)) + \bar{D}\phi(\tilde{y}(k)) + \bar{E}\tilde{\omega}(k), \\ \tilde{y}(k) = \tilde{C}\bar{x}(k), \\ z(k) = (M + NK)\bar{x}(k) + NL\bar{x}(k - d(k)). \end{cases} \tag{21}$$

To facilitate the presentation, we denote

$$K_\Delta = diag(\frac{\bar{k}_1 - \underline{k}_1}{2}, \frac{\bar{k}_2 - \underline{k}_2}{2}, \cdots, \frac{\bar{k}_p - \underline{k}_p}{2}),\ Z = diag(\mu_1, \mu_2, \cdots, \mu_p).$$

Theorem 4.2. Suppose that Assumptions 1-4 are satisfied. If for a given $Z > 0$ there exist matrices $P > 0,\ Q > 0,\ S,\ S_d,\ U$ and V such that

$$\begin{bmatrix} P - S - S^T & * & * & * & * & * & * \\ 0 & Q - S_d - {S_d}^T & * & * & * & * & * \\ K_\Delta Z\tilde{C}S & 0 & -Z & * & * & * & * \\ 0 & 0 & 0 & -\gamma^2 I & * & * & * \\ \bar{A}S + \bar{B}U & \bar{A}_d S_d + \bar{B}V & \bar{D} & \bar{E} & -P & * & * \\ MS + NU & NV & 0 & 0 & 0 & -I & * \\ S & 0 & 0 & 0 & 0 & 0 & -(d_2 - d_1 + 1)^{-1}Q \end{bmatrix} < 0, \tag{22}$$

then the closed-loop system (21) is asymptotically stable, and the performance index J satisfies

$$J=\|z\|_2^2\leq\gamma^2\|\omega\|_2^2+\bar{x}^T(0)P^{-1}\bar{x}(0)+\sum_{i=-d_2}^{-1}\bar{x}^T(i)Q^{-1}\bar{x}(i)+\sum_{i=-d_2+1}^{-d_1}\sum_{j=i}^{-1}\bar{x}^T(j)Q^{-1}\bar{x}(j). \quad (23)$$

Furthermore, the controller gains can be obtained by $K=US^{-1}$ and $L=VS_d{}^{-1}$.

Proof. We first consider system (19) with $\tilde{\omega}(k)=0$, namely,

$$\bar{x}(k+1)=(\bar{A}+\bar{B}K)\bar{x}(k)+(\bar{A}_d+\bar{B}L)\bar{x}(k-d(k))+\bar{D}\phi(\tilde{y}(k)). \quad (24)$$

For system (24), we adopt the following Lyapunov–Krasovskii functional:

$$V(k)=V_1(k)+V_2(k)+V_3(k), \quad (25)$$

where

$$V_1(k)=\bar{x}^T(k)P^{-1}\bar{x}(k),\ V_2(k)=\sum_{i=k-d(k)}^{k-1}\bar{x}^T(i)Q^{-1}\bar{x}(i),\ V_3(k)=\sum_{i=-d_2+1}^{-d_1}\sum_{j=k+i}^{k-1}\bar{x}^T(j)Q^{-1}\bar{x}(j).$$

It is clear that $V(k)$ in (25) is positive definite because of $P>0$ and $Q>0$. Taking the forward difference of $V(k)$ along the trajectories of system (24) yields

$$\Delta V(k)=\Delta V_1(k)+\Delta V_2(k)+\Delta V_3(k). \quad (26)$$

To make the mathematical derivations clear, the three items on the right side of equation (26) are calculated separately. First, we obtain

$$\begin{aligned}\Delta V_1(k)&=\bar{x}^T(k+1)P^{-1}\bar{x}(k+1)-\bar{x}^T(k)P^{-1}\bar{x}(k)\\&=\bar{x}^T(k)\left[(\bar{A}+\bar{B}K)^TP^{-1}(\bar{A}+\bar{B}K)-P^{-1}\right]\bar{x}(k)\\&\quad+2\bar{x}^T(k)(\bar{A}+\bar{B}K)^TP^{-1}(\bar{A}_d+\bar{B}L)\bar{x}(k-d(k))+2\bar{x}^T(k)(\bar{A}+\bar{B}K)^TP^{-1}\bar{D}\phi(\tilde{y})\\&\quad+\bar{x}^T(k-d(k))(\bar{A}_d+\bar{B}L)^TP^{-1}(\bar{A}_d+\bar{B}L)\bar{x}(k-d(k))\\&\quad+2\bar{x}^T(k-d(k))(\bar{A}_d+\bar{B}L)^TP^{-1}\bar{D}\phi(\tilde{y})+\phi^T(\tilde{y})\bar{D}^TP^{-1}\bar{D}\phi(\tilde{y}).\end{aligned} \quad (27)$$

Then $\Delta V_2(k)$ is given by

$$\begin{aligned}V_2(k)&=\sum_{i=k+1-d(k+1)}^{k}\bar{x}^T(i)Q^{-1}\bar{x}(i)-\sum_{i=k-d(k)}^{k-1}\bar{x}^T(i)Q^{-1}\bar{x}(i)\\&=\sum_{i=k+1-d(k+1)}^{k-d_1}\bar{x}^T(i)Q^{-1}\bar{x}(i)+\sum_{i=k-d_1+1}^{k-1}\bar{x}^T(i)Q^{-1}\bar{x}(i)+\bar{x}^T(k)Q^{-1}\bar{x}(k)\\&\quad-\sum_{i=k-d(k)+1}^{k-1}\bar{x}^T(i)Q^{-1}\bar{x}(i)-\bar{x}^T(k-d(k))Q^{-1}\bar{x}(k-d(k)).\end{aligned}$$

Due to the fact that $d(k) \geq d_1$, thus $\sum_{i=k-d_1+1}^{k-1} \bar{x}^T(i)Q^{-1}\bar{x}(i) - \sum_{i=k-d(k)+1}^{k-1} \bar{x}^T(i)Q^{-1}\bar{x}(i) \leq 0$. Hence,

$$\Delta V_2(k) \leq \sum_{i=k+1-d(k+1)}^{k-d_1} \bar{x}^T(i)Q^{-1}\bar{x}(i) + \bar{x}^T(k)Q^{-1}\bar{x}(k) - \bar{x}^T(k-d(k))Q^{-1}\bar{x}(k-d(k)). \tag{28}$$

Consider the forward difference of function $V_3(k)$:

$$\begin{aligned}\Delta V_3(k) &= \sum_{i=-d_2+1}^{-d_1} \sum_{j=k+1+i}^{k} \bar{x}^T(j)Q^{-1}\bar{x}(j) - \sum_{i=-d_2+1}^{-d_1} \sum_{j=k+i}^{k-1} \bar{x}^T(j)Q^{-1}\bar{x}(j) \\ &= \sum_{i=-d_2+1}^{-d_1} \left[\sum_{j=k+1+i}^{k-1} \bar{x}^T(j)Q^{-1}\bar{x}(j) + \bar{x}^T(k)Q^{-1}\bar{x}(k) - \sum_{j=k+1+i}^{k-1} \bar{x}^T(j)Q^{-1}\bar{x}(j) - \bar{x}^T(k+i)Q^{-1}\bar{x}(k+i) \right] \\ &= \sum_{i=-d_2+1}^{-d_1} \left[\bar{x}^T(k)Q^{-1}\bar{x}(k) - \bar{x}^T(k+i)Q^{-1}\bar{x}(k+i) \right] \\ &= (d_2-d_1)\bar{x}^T(k)Q^{-1}\bar{x}(k) - \sum_{i=k-d_2+1}^{k-d_1} \bar{x}^T(i)Q^{-1}\bar{x}(i).\end{aligned} \tag{29}$$

Because $d(k) \leq d_2$, one obtains from (28) and (29) that

$$\Delta V_2(k) + \Delta V_3(k) \leq (d_2-d_1+1)\bar{x}^T(k)Q^{-1}\bar{x}(k) - \bar{x}^T(k-d(k))Q^{-1}\bar{x}(k-d(k)). \tag{30}$$

From (26), (27) and (30), the following inequality can be derived:

$$\begin{aligned}\Delta V(k) \leq{} & \bar{x}^T(k)\left[(\bar{A}+\bar{B}K)^T P^{-1}(\bar{A}+\bar{B}K) - P^{-1}\right]\bar{x}(k) \\ & + 2\bar{x}^T(k)(\bar{A}+\bar{B}K)^T P^{-1}(\bar{A}_d+\bar{B}L)\bar{x}(k-d(k)) + 2\bar{x}^T(k)(\bar{A}+\bar{B}K)^T P^{-1}\bar{D}\phi(\tilde{y}) \\ & + \bar{x}^T(k-d(k))(\bar{A}_d+\bar{B}L)^T P^{-1}(\bar{A}_d+\bar{B}L)\bar{x}(k-d(k)) \\ & + (d_2-d_1+1)\bar{x}^T(k)Q^{-1}\bar{x}(k) - \bar{x}^T(k-d(k))Q^{-1}\bar{x}(k-d(k)) \\ & + 2\bar{x}^T(k-d(k))(\bar{A}_d+\bar{B}L)^T P^{-1}\bar{D}\phi(\tilde{y}) + \phi^T(\tilde{y})\bar{D}^T P^{-1}\bar{D}\phi(\tilde{y}).\end{aligned} \tag{31}$$

For a given matrix $Z = diag(\mu_1, \mu_2, \ldots, \mu_p) > 0$, according to the condition (10), it is straightforward to show that

$$\phi(\tilde{y})^T Z\phi(\tilde{y}) - 2\tilde{y}^T Z K_\Delta \phi(\tilde{y}) \leq 0. \tag{32}$$

Therefore, the upper bound of $\Delta V(k)$ can be estimated as

$$\begin{aligned}\Delta V(k) \leq\ & \bar{x}^T(k)\left[(\bar{A}+\bar{B}K)^T P^{-1}(\bar{A}+\bar{B}K) - P^{-1} + (d_2-d_1+1)Q^{-1}\right]\bar{x}(k) \\ & + 2\bar{x}^T(k)(\bar{A}+\bar{B}K)^T P^{-1}(\bar{A}_d+\bar{B}L)\bar{x}(k-d(k)) + 2\bar{x}^T(k)(\bar{A}+\bar{B}K)^T P^{-1}\bar{D}\phi(\tilde{y}) \\ & + \bar{x}^T(k-d(k))((\bar{A}_d+\bar{B}L)^T P^{-1}(\bar{A}_d+\bar{B}L) - Q^{-1})\bar{x}(k-d(k)) \\ & + 2\bar{x}^T(k-d(k))(\bar{A}_d+\bar{B}L)^T P^{-1}\bar{D}\phi(\tilde{y}) + \phi^T(\tilde{y})\bar{D}^T P^{-1}\bar{D}\phi(\tilde{y}) \\ & - \phi^T(\tilde{y})Z\phi(\tilde{y}) + 2\bar{x}^T(k)\tilde{C}^T Z K_\Delta\phi(\tilde{y}) \\ =\ & \begin{bmatrix} \bar{x}^T(k) & \bar{x}^T(k-d(k)) & \phi^T(\tilde{y}) \end{bmatrix} \Pi \begin{bmatrix} \bar{x}^T(k) & \bar{x}^T(k-d(k)) & \phi^T(\tilde{y}) \end{bmatrix}^T,\end{aligned}$$

where

$$\Pi = \begin{bmatrix} \Pi_{11} & * & * \\ (\bar{A}_d+\bar{B}L)^T P^{-1}(\bar{A}+\bar{B}K) & (\bar{A}_d+\bar{B}L)^T P^{-1}(\bar{A}_d+\bar{B}L) - Q^{-1} & * \\ \bar{D}^T P^{-1}(\bar{A}+\bar{B}K) + K_\Delta Z\tilde{C} & \bar{D}^T P^{-1}(\bar{A}_d+\bar{B}L) & \bar{D}^T P^{-1}\bar{D} - Z \end{bmatrix}$$

with $\Pi_{11} = (\bar{A}+\bar{B}K)^T P^{-1}(\bar{A}+\bar{B}K) - P^{-1} + (d_2-d_1+1)Q^{-1}$.

Now, we prove that condition (22) of Theorem 4.2 leads to $\Pi < 0$.

Applying Lemma 2.4, condition (22) ensures

$$\begin{bmatrix} -S^T P^{-1} S & * & * & * & * & * & * \\ 0 & -S_d{}^T Q^{-1} S_d & * & * & * & * & * \\ K_\Delta Z\tilde{C}S & 0 & -Z & * & * & * & * \\ 0 & 0 & 0 & -\gamma^2 I & * & * & * \\ \bar{A}S+\bar{B}U & \bar{A}_d S_d + \bar{B}V & \bar{D} & \bar{E} & -P & * & * \\ MS+NU & NV & 0 & 0 & 0 & -I & * \\ S & 0 & 0 & 0 & 0 & 0 & -(d_2-d_1+1)^{-1}Q \end{bmatrix} < 0. \tag{33}$$

For convenience, denote

$$\Gamma_1 = \begin{bmatrix} 0 & 0 & 0 & I \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ I & 0 & 0 & 0 \end{bmatrix}.$$

Performing the congruence transformation on the matrix inequality (33), that is, pre-multiplying and post-multiplying $Block\text{-}diag(S^{-T}, S_d{}^{-T}, I, \Gamma_1)$ and its transpose, respectively, and by using $K = US^{-1}$ and $L = VS_d{}^{-1}$, one can obtain that

$$\begin{bmatrix} -P^{-1} & * & * & * & * & * & * \\ 0 & -Q^{-1} & * & * & * & * & * \\ K_\Delta Z\tilde{C} & 0 & -Z & * & * & * & * \\ I & 0 & 0 & -(d_2-d_1+1)^{-1}Q & * & * & * \\ \bar{A}+\bar{B}K & \bar{A}_d+\bar{B}L & \bar{D} & 0 & -P & * & * \\ M+NK & NL & 0 & 0 & 0 & -I & * \\ 0 & 0 & 0 & 0 & \bar{E}^T & 0 & -\gamma^2 I \end{bmatrix} < 0. \tag{34}$$

Then, we can derive from inequality (34) that

$$\begin{bmatrix} -P^{-1} & 0 & \tilde{C}^T Z K_\Delta & I & (\bar{A}+\bar{B}K)^T \\ 0 & -Q^{-1} & 0 & 0 & (\bar{A}_d+\bar{B}L)^T \\ K_\Delta Z \tilde{C} & 0 & -Z & 0 & \bar{D}^T \\ I & 0 & 0 & -(d_2-d_1+1)^{-1}Q & 0 \\ \bar{A}+\bar{B}K & \bar{A}_d+\bar{B}L & \bar{D} & 0 & -P \end{bmatrix} < 0. \quad (35)$$

Applying Lemma 2.3 again, it can be inferred that $\Pi < 0$. Thus, according to Lyapunov stability theory, system (24) is asymptotically stable.

Next, guaranteed cost control with H_∞ disturbance attenuation is considered.

For the simplicity of the vector and matrix presentation, we define

$$X(k) = \begin{bmatrix} \bar{x}^T(k) & \bar{x}^T(k-d(k)) & \phi^T(\tilde{y}) & \tilde{\omega}^T(k) \end{bmatrix}^T,$$

$$\Omega_{11} = (\bar{A}+\bar{B}K)^T P^{-1}(\bar{A}+\bar{B}K) - P^{-1} + (d_2-d_1+1)Q^{-1} + (M+NK)^T(M+NK),$$

$$\Omega_{21} = (\bar{A}_d+\bar{B}L)^T P^{-1}(\bar{A}+\bar{B}K) + L^T N^T (M+NK),$$

$$\Omega_{22} = (\bar{A}_d+\bar{B}L)^T P^{-1}(\bar{A}_d+\bar{B}L) - Q^{-1} + L^T N^T N L,$$

$$\Omega = \begin{bmatrix} \Omega_{11} & * & * & * \\ \Omega_{21} & \Omega_{22} & * & * \\ \bar{D}^T P^{-1}(\bar{A}+\bar{B}K) + K_\Delta Z \tilde{C} & \bar{D}^T P^{-1}(\bar{A}_d+\bar{B}L) & \bar{D}^T P^{-1}\bar{D} - Z & * \\ \bar{E}^T P^{-1}(\bar{A}+\bar{B}K) & \bar{E}^T P^{-1}(\bar{A}_d+\bar{B}L) & \bar{E}^T P^{-1}\bar{D} & \bar{E}^T P^{-1}\bar{E} - \gamma^2 I \end{bmatrix}.$$

Using Lemmas 2.3 and 2.4, it can be easily proven that condition (22) guarantees $\Omega < 0$. We still consider the Lyapunov–Krasovskii functional (25). Through some mathematical manipulation, we obtain that

$$X^T(k)\Omega X(k) = \Delta V(k) + z^T(k)z(k) - \gamma^2 \tilde{\omega}^T(k)\tilde{\omega}(k) \le 0.$$

Furthermore, we see that

$$z^T(k)z(k) \le -\Delta V(k) + \gamma^2 \tilde{\omega}^T(k)\tilde{\omega}(k).$$

Summing both sides of the above inequality from 0 to ∞, and taking $d(0) \le d_2$ into account, it follows that

$$J = \|z\|_2^2 \le \gamma^2 \|\tilde{\omega}\|_2^2 + \bar{x}^T(0)P^{-1}\bar{x}(0) + \sum_{i=-d_2}^{-1} \bar{x}^T(i)Q^{-1}\bar{x}(i) + \sum_{i=-d_2+1}^{-d_1} \sum_{j=i}^{-1} \bar{x}^T(j)Q^{-1}\bar{x}(j).$$

This implies that the H_∞ norm from $\tilde{\omega}(k)$ to $z(k)$ is less than the prescribed level γ, and the term $\bar{x}^T(0)P^{-1}\bar{x}(0) + \sum_{i=-d_2}^{-1} \bar{x}^T(i)Q^{-1}\bar{x}(i) + \sum_{i=-d_2+1}^{-d_1} \sum_{j=i}^{-1} \bar{x}^T(j)Q^{-1}\bar{x}(j)$ represents the upper bound of the cost function J when $\tilde{\omega}(k) = 0$; thus, guaranteed cost control is achieved. Additionally, when the initial state $\bar{x}(i) = 0,\ i = -d_2, -d_2+1, \ldots, -1, 0$, we have $\|z\|_2^2 \le \gamma^2 \|\tilde{\omega}\|_2^2$, that is, classical H_∞ control is achieved. This completes the proof.

□

Remark 4.3. In Theorem 4.2, the matrix inequality condition (22) is established for asymptotic stability of system (21) with guaranteed cost performance. When the diagonal matrix Z is fixed beforehand, this inequality is a linear matrix inequality, which can be solved numerically by resorting to MATLAB LMI toolbox. Moreover, it should be mentioned that the Lyapunov–Krasovskii functional (25) may not be the best choice for stability analysis. For the discrete-time Lur'e nonlinear time-delay systems considered in this paper, we could further reduce the possible conservatism of the theoretical results by making an effort towards the construction of Lyapunov–Krasovskii functionals with more general forms that contain more information about time delays (see, e. g., [30]), which is an interesting research issue for further investigation.

Let

$$J^* = \bar{x}^T(0)P^{-1}\bar{x}(0) + \sum_{i=-d_2}^{-1} \bar{x}^T(i)Q^{-1}\bar{x}(i) + \sum_{i=-d_2+1}^{-d_1} \sum_{j=i}^{-1} \bar{x}^T(j)Q^{-1}\bar{x}(j).$$

Then, equation (23) can be written as

$$J \le \gamma^2 \|\tilde{\omega}\|_2^2 + J^*. \tag{36}$$

It is noted that the bound J^* in Theorem 4.2 depends explicitly upon the initial conditions of system (21). To remove this dependence on the initial condition, we adopt the method generally used in the literature (see, e. g., [9]). Suppose that the initial state is arbitrary but belongs to the set $\varsigma = \left\{\bar{x}(i) : \bar{x}(i) = G\upsilon_i, \upsilon_i^T\upsilon_i \le 1, i = -d_2, -d_2+1, \ldots, 0\right\}$, where G is a given matrix; then, the cost bound leads to

$$\begin{aligned} J^* &= \bar{x}^T(0)P^{-1}\bar{x}(0) + \sum_{i=-d_2}^{-1} \bar{x}^T(i)Q^{-1}\bar{x}(i) + \sum_{i=-d_2+1}^{-d_1} \sum_{j=i}^{-1} \bar{x}^T(j)Q^{-1}\bar{x}(j) \\ &\le \upsilon_0^T G^T P^{-1} G\upsilon_0 + \sum_{i=-d_2}^{-1} \upsilon_i^T G^T Q^{-1} G\upsilon_i + \sum_{i=-d_2+1}^{-d_1} \sum_{j=i}^{-1} \upsilon_j^T G^T Q^{-1} G\upsilon_j \\ &\le \lambda_{\max}(G^T P^{-1} G)\upsilon_0^T\upsilon_0 + \sum_{i=-d_2}^{-1} \lambda_{\max}(G^T Q^{-1} G)\upsilon_i^T\upsilon_i + \sum_{i=-d_2+1}^{-d_1} \sum_{j=i}^{-1} \lambda_{\max}(G^T Q^{-1} G)\upsilon_j^T\upsilon_j \\ &\le \lambda_{\max}(G^T P^{-1} G) + \sum_{i=-d_2}^{-1} \lambda_{\max}(G^T Q^{-1} G) + \sum_{i=-d_2+1}^{-d_1} \sum_{j=i}^{-1} \lambda_{\max}(G^T Q^{-1} G). \end{aligned} \tag{37}$$

Theorem 4.4. Suppose that Assumptions 1-4 are satisfied. If for a given $Z > 0$ there exist matrices $P > 0$, $Q > 0$, S, S_d, U and V, and scalars α and β such that the optimization problem

$$\begin{aligned}
&\min \alpha + [d_2 + \tfrac{(d_2-d_1)(d_2+d_1-1)}{2}]\beta \\
&\text{s.t. (I) LMI(22),} \\
&\quad \text{(II)} \begin{bmatrix} -\alpha I & G^T \\ G & -P \end{bmatrix} < 0, \\
&\quad \text{(III)} \begin{bmatrix} -\beta I & G^T \\ G & -Q \end{bmatrix} < 0,
\end{aligned}$$

obtains a solution $(\alpha,\ \beta,\ P,\ Q,\ S,\ S_d,\ U,\ V)$, then the resulting closed-loop system (21) is asymptotically stable and J satisfies

$$J = \|z\|_2^2 \leq \gamma^2 \|\tilde{\omega}\|_2^2 + \alpha + [d_2 + \frac{(d_2-d_1)(d_2+d_1-1)}{2}]\beta. \tag{38}$$

Furthermore, the controller gains can be obtained by $K = US^{-1}$ and $L = V{S_d}^{-1}$.

Proof. According to Theorem 4.2, (I) guarantees that the closed-loop system (21) is asymptotically stable and the cost function J satisfies (23). By Lemma 2.3, (II) and (III) are equivalent to $G^T P^{-1} G < \alpha I$ and $G^T Q^{-1} G < \beta I$, respectively. Then, it follows from (37) that

$$\begin{aligned}
J^* &\leq \lambda_{\max}(G^T P^{-1} G) + \left[d_2 + \frac{(d_2-d_1)(d_2+d_1-1)}{2}\right] \lambda_{\max}(G^T Q^{-1} G) \\
&\leq \alpha + [d_2 + \frac{(d_2-d_1)(d_2+d_1-1)}{2}]\beta.
\end{aligned}$$

By using (36), we can obtain (38). This completes the proof. □

Remark 4.5. If the H_∞ performance level γ is fixed in advance, then a suitable guaranteed cost controller can be derived by solving the optimization problem in Theorem 4.4. Otherwise, denote $\lambda = \gamma^2$ and solve the following optimization problem:

$$\begin{aligned}
&\min \lambda \\
&\text{s.t. LMI(22)}
\end{aligned}$$

If the optimization problem mentioned above obtains a solution $(P,\ Q,\ S,\ S_d,\ U,\ V)$, then the optimal H_∞ performance level $\gamma^* = \sqrt{\lambda}$ under a certain cost function can be obtained.

Theorem 4.4 presents the design method of the controller gain matrices that ensure the asymptotic stability of the closed-loop system. Indeed, if the closed-loop system is asymptotically stable, then we obtain $x(k) \to x(\infty)$ and $v(k) \to v(\infty)$ as $k \to \infty$, furthermore we obtain

$$\lim_{k\to\infty} e(k) = \lim_{k\to\infty} (v(k+1) - v(k)) = 0,$$

which implies that the asymptotic output tracking is achieved.

To clarify the proposed tracking controller structure, we partition the gain matrices $K = US^{-1}$ and $L = VS_d{}^{-1}$ determined in Theorem 4.4 as

$$K = \begin{bmatrix} K_x & K_r & K_v \end{bmatrix}, \tag{39}$$

$$K_r = \begin{bmatrix} k_r(0) & k_r(1) & \dots & k_r(M_r) \end{bmatrix}, \tag{40}$$

$$L = \begin{bmatrix} L_x & L_r & L_v \end{bmatrix}, \tag{41}$$

$$L_r = \begin{bmatrix} l_r(0) & l_r(1) & \dots & l_r(M_r) \end{bmatrix}. \tag{42}$$

Thus, the controller (20) can be expressed as

$$\tilde{u}(k) = K_x\tilde{x}(k) + K_r x_r(k) + K_v\tilde{v}(k) + L_x\tilde{x}(k-d(k)) + L_r x_r(k-d(k)) + L_v\tilde{v}(k-d(k)).$$

From (14), we can derive that $\tilde{v}(k) = \sum_{i=0}^{k-1} e(i) + \tilde{v}(0)$. Recalling the definition (6) of $\tilde{u}(k)$, the main result in this paper is derived immediately.

Theorem 4.6. Suppose that Assumptions 1-4 are satisfied. If the optimization problem in Theorem 4.4 is solvable, then the preview controller for system (1) is

$$\begin{aligned} u(k) = {} & K_x(x(k) - x(\infty)) + \sum_{i=0}^{M_r} k_r(i)(r(k+i) - r) + K_v(\sum_{i=0}^{k-1} e(i) + v(0) - v(\infty)) \\ & + L_x(x(k-d(k)) - x(\infty)) + \sum_{i=0}^{M_r} l_r(i)(r(k-d(k)+i) - r) \\ & + L_v(\sum_{i=0}^{k-d(k)-1} e(i) + v(0) - v(\infty)) + u(\infty), \end{aligned} \tag{43}$$

where the related controller gain matrices $K_x,\ K_v,\ L_x,\ L_v,\ k_r(i),\ l_r(i)\ (i = 0, \dots, M_r)$ are determined by (39) – (42). Under this controller scheme, the output vector $y(k)$ can achieve asymptotic tracking of the reference signal $r(k)$.

Remark 4.7. It is worth pointing out that in the designed controller (43), the terms $\sum_{i=0}^{M_r} k_r(i)r(k+i)$ and $\sum_{i=0}^{M_r} l_r(i)r(k-d(k)+i)$ represent the preview compensation action with respect to the reference signal. The introduction of these two terms is the key reason for the tracking performance improvement of the closed-loop system, which has not been taken into account in [14, 19, 24, 42]. In addition, in [39], the authors investigated the tracking control with preview action for continuous-time Lur'e nonlinear systems with external disturbances, and they did not consider discrete-time systems and the time delay factor. In this paper, for the first time, a preview tracking controller with guaranteed cost H_∞ control performance is proposed for discrete-time Lur'e systems with disturbances and time-varying delays.

5. NUMERICAL SIMULATIONS

Example 1 Consider system (1) with

$$A=\begin{bmatrix} 1.33 & 0 \\ -0.3 & 0.55 \end{bmatrix},\ A_d=\begin{bmatrix} 0.2 & 0 \\ 0 & 0.07 \end{bmatrix},\ B=\begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix},$$

$$D=\begin{bmatrix} 0.01 \\ 0 \end{bmatrix},\ E=\begin{bmatrix} 0.05 \\ 0 \end{bmatrix},\ C=\begin{bmatrix} 0.5 & -1 \end{bmatrix}.$$

The nonlinearity is $f(y) = 0.5(|y+1| - |y-1|)$, and the time delay is assumed to be $d(k) = \text{Round}\,(\sin(0.01k) + 2)$, where $\text{Round}(x)$ denotes the nearest integer to the real number x.

It is easy to check that $f(y)$ satisfies Assumption 1 with $\underline{k} = 0$ and $\bar{k} = 1$. Meanwhile, Assumption 2 also holds since $rank \begin{bmatrix} A+A_d-I & B \\ C & 0 \end{bmatrix} = 3$. Furthermore, assume that the external disturbance and the reference signal verify Assumptions 3 and 4. Thus, the basic requirements of this paper are satisfied.

For the purpose of the simulation, the external disturbance $\omega(k)$ is taken as $\omega(k) = e^{-2.5(k+1)}$. Set $\gamma = 8$, $R = 0.05$ and $Q_v = 0.05$. By resorting to the MATLAB LMI toolbox, the solution of the optimization problem in Theorem 4.4 can be computed. To quantify the effect of the preview action on output tracking performance, the following three cases are taken into consideration.

Case ①. When $M_r = 0$ (no preview), the controller gain matrices are

$$K_x = [-11.1156\ 2.4603],\ K_v = -1.0680,$$

$$L_x = [-1.4769\ 0.1724],\ L_v = 4.9707\times 10^{-9}.$$

Case ②. When $M_r = 2$, the controller gain matrices are

$$K_x = [-7.5131\ 1.1958],\ K_v = -0.4796,$$

$$K_r = [0.4794\ 0.4794\ 0.5053],$$

$$L_x = [-1.0352\ 0.0879],\ L_v = 1.5584\times 10^{-10},$$

$$L_r = 10^{-9}\times[0.1776\ 0.0591\ 0.0114].$$

Case ③. When $M_r = 5$, the controller gain matrices are

$$K_x = [-7.5102\ 1.1946],\ K_v = -0.4789,$$

$$K_r = [0.4789\ 0.4789\ 0.5051\ 0.4731\ 0.4131\ 0.3493],$$

$$L_x = [-1.0356\ 0.0880],\ L_v = -5.0040\times 10^{-10},$$

$$L_r = 10^{-9}\times[-0.2310\ 0.1318\ 0.1362\ 0.1054\ 0.0533\ 0.0202].$$

Simulations are performed with the following two reference signals.

(i) The reference signal is taken as

$$r(k) = \begin{cases} 3, & k \geq 30 \\ 0, & k < 30. \end{cases} \tag{44}$$

Figure 1 depicts the output trajectories of the closed-loop system. Figure 2 plots the time response of the tracking error. The time responses of the closed-loop states and the control input are illustrated in Figures 3 and 4, respectively.

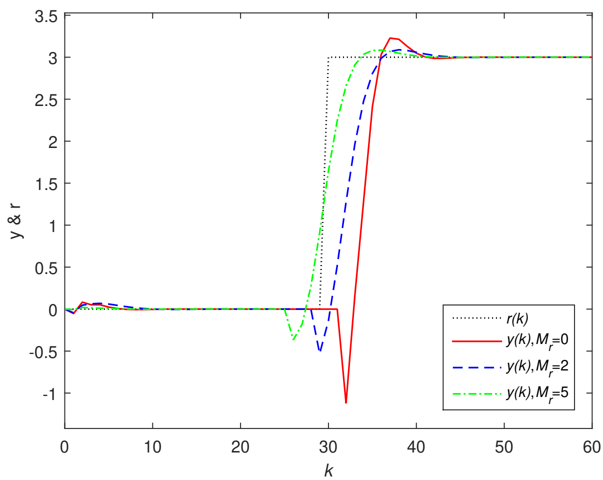

Fig. 1. Time response of the closed-loop output and reference signal (44).

From Figure 1, we can see that the controllers designed in these three cases can force the system output to track the desired reference signal, and compared to the traditional controller without preview (i.e., $M_r = 0$), the preview controller causes the system to produce a better transient responses. To be more specific, the overshoot of the output is dramatically reduced and the settling time is shortened. From Figure 2, the proposed preview controller can significantly decrease the output tracking error, thereby improving the tracking precision of the closed-loop system. Moreover, the tracking behaviour can be further improved by appropriately increasing the preview length. However, such manipulation may incur some computational complexity. In fact, due to the introduction of M_r-step preview information about the reference signal, the dimension of the augmented error system is increased, which undoubtedly results in the requirement of some additional computations for solving the LMI problem. Fortunately, since the selected preview length is usually not very large, the impact caused by such computations can be almost ignored. Numerical simulation shows that the solving time for the case with preview is slightly longer than that of the case without preview, but no significant time difference occurs. Figure 3 shows that the improvement in system performance

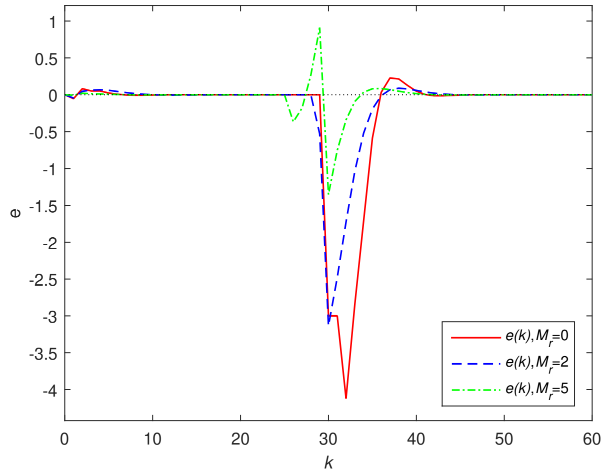

Fig. 2. Time response of the tracking error.

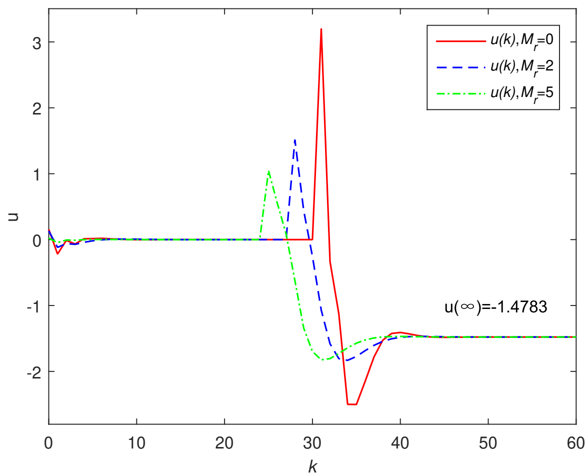

Fig. 3. Time response of the control input.

via the preview controller is not at the expense of increasing the input amplitude. Furthermore, it can be computed from (5) that the steady-state values $x(\infty)$ and $u(\infty)$ in this example are $x(\infty) = \begin{bmatrix} 0.8180 \\ -2.5910 \end{bmatrix}$ and $u(\infty) = -1.4783$. As shown in Figures 3 and 4, the control signal and states of the closed-loop system ultimately tend to their steady-state values. Therefore, the simulation results are completely consistent with the theoretical conclusions.

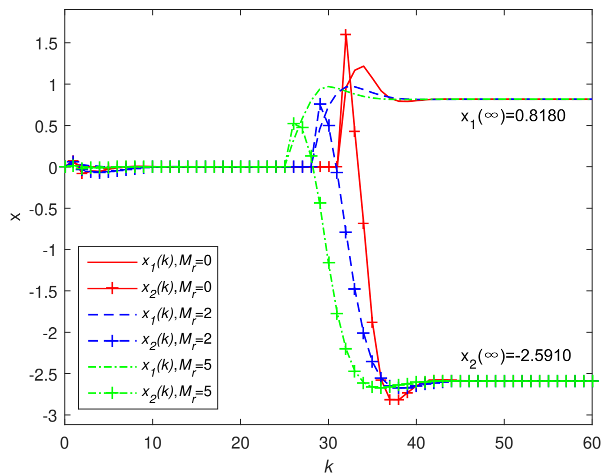

Fig. 4. Time response of the closed-loop states.

(ii) The reference signal is taken as

$$r(k) = \begin{cases} 0, & k < 30 \\ 0.15(k-30), & 30 \leq k \leq 50 \\ 3, & k > 50. \end{cases} \tag{45}$$

Figure 5 shows the output response of the closed-loop system. The time responses of the tracking error and the control input are illustrated in Figures 6 and 7, respectively. The closed-loop state response is demonstrated in Figure 8.

As shown in Figure 5, the controllers proposed in these three cases cause the system output to track the reference signal accurately, despite the existence of external disturbances and time delays. The difference lies in the fact that the preview controller exhibits superior tracking quality, produces faster responses, and achieves lower overshoots than those of the traditional controller. This is mainly because the preview controller not only possesses a certain disturbance attenuation ability, but also includes a preview compensator used to improve the output tracking effect. Figure 6 reveals that the tracking error using the preview control method is noticeably smaller, and the output tracking behaviour is quite perfect. It can also be seen from Figure 7 that the incorporation of the preview action can still ensure that the control input is maintained within a certain range. In other words, the control signal possesses a reasonable amplitude. Meanwhile, Figures 7 and 8 also illustrate that the input vector and state vector of the closed-loop system ultimately tend to their steady-state values, which shows the effectiveness of the proposed method.

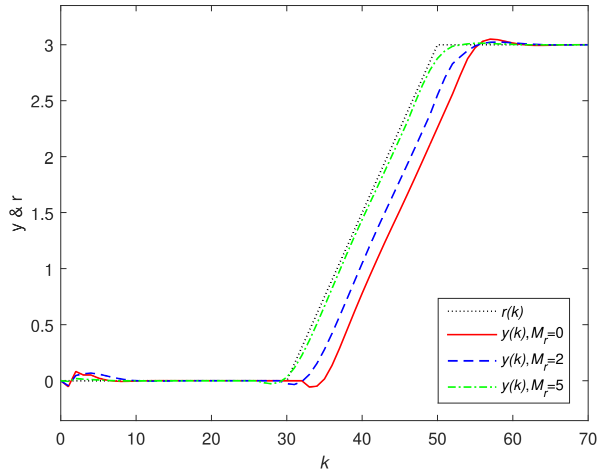

Fig. 5. Time response of the closed-loop output and signal (45).

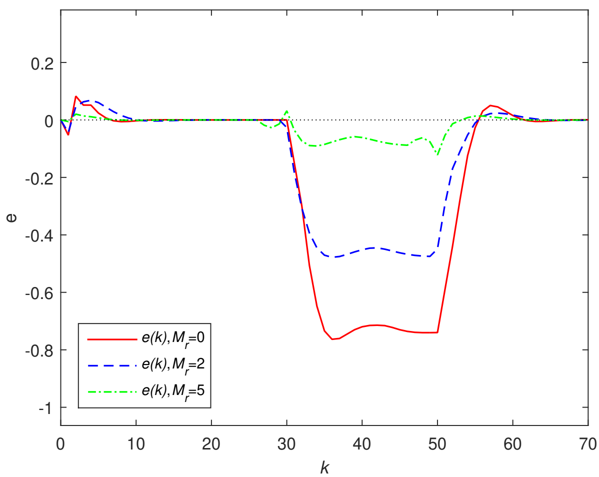

Fig. 6. Time response of the tracking error.

6. CONCLUSIONS

The problem of preview tracking control for a class of discrete-time Lur'e time-delay systems with external disturbance is studied in this paper. First, the translation technique using the steady-state values is adopted to derive the augmented error system with tracking error and reference preview information. In order to avoid the static error phenomenon, a discrete integrator is introduced. Following Lyapunov stability theory,

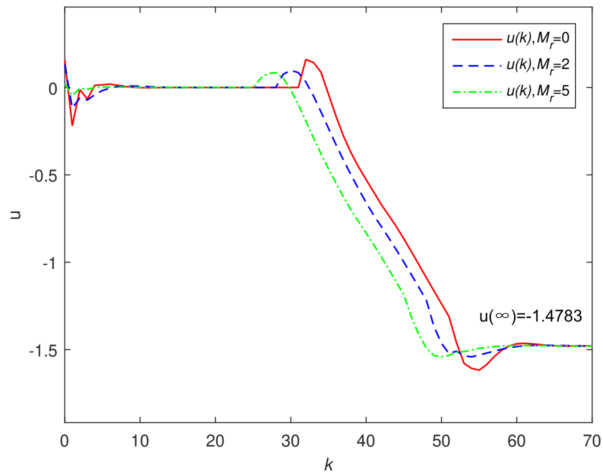

Fig. 7. Time response of the control input.

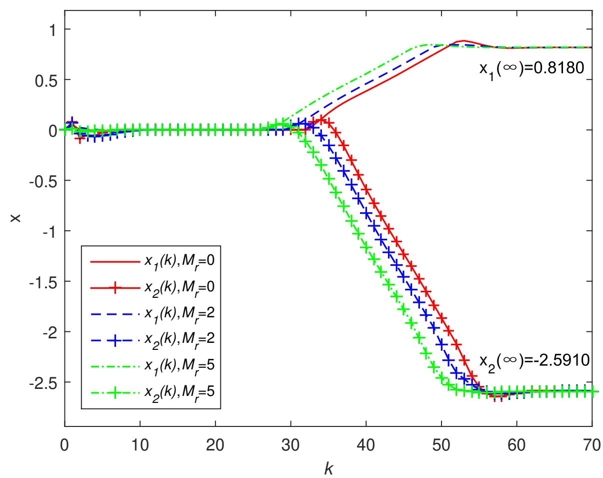

Fig. 8. Time response of the closed-loop states.

the design of a memory state feedback controller ensuring asymptotic stability and guaranteed cost H_∞ control performance of the closed-loop system is formulated in terms of linear matrix inequalities. Based on this, the preview control scheme for the original Lur'e system is presented. Finally, numerical simulation results are displayed to show the effectiveness of the proposed preview controller. A challenging and interesting future topic is how to generalize the results of this paper to more types of discrete-time nonlinear systems by employing some novel techniques in [1, 9].

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Xiao Yu, School of Science, Shandong Jianzhu University, Jinan 250101. P. R. China.
e-mail: yuxiao19@sdjzu.edu.cn

Fucheng Liao, School of Mathematics and Physics, University of Science and Technology Beijing, Beijing 100083. P. R. China.
e-mail: fcliao@ustb.edu.cn